Algorithms for the computation of formal Fourier series (FFS)

Let $f : [a, b] \to \mathbb{R}$ be an integrable function in the interval $[a, b]$.
Fourier Series $\mathcal{F}(f)(t) := \frac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \cos(n\,\omega\,t) + \sum\limits_{n=1}^{\infty} b_n \sin(n\,\omega\,t) = \sum\limits_{n=-\infty}^{\infty} c_n\, e^{in\omega t}$ play a very important role in acoustics and other areas of physics and engineering. The determination of the Fourier coefficients is connected with the calculation of some particular integrals.
Modern computer algebra systems (C.A.S.) can generally compute the Fourier coefficients a_k, b_k, and c_k for specific values of $k \in \mathbb{Z}$, of a function f satisfying certain conditions. If one is interested in the computation of those coefficients for all $k \in \mathbb{Z}$, then k is now considered as a symbolic variable, and in this case the computation of the Fourier coefficients is much more difficult, and general purpose C.A.S. like Maple are successful only in rare cases.

In this talk, we give an algorithm for the computation of the Fourier coefficients for some family of functions, whose Fourier coefficients cannot be computed directly by any C.A.S. And to do so, we derive an identity for the Fourier coefficients of a differentiable function f in terms of the Fourier coefficients of its derivative f'. This yields an algorithm to compute the Fourier coefficients of f whenever the Fourier coefficients of f' are known, and vice versa. Next, we generalise this result in order to compute the Fourier coefficients of a n times differentiable function f from those of its n^{th} derivative and vice versa. Finally we use this generalisation to generate an algorithm for the computation of the Fourier coefficients of trigonometric rational functions.