Fachbereich Mathematik **WiSe 2012/2013**
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Numerik steifer Probleme

Aufgabenblatt 10

Aufgabe 1
Consider the following variant of the multigrid method given in the lecture notes on pages 66/67. On the coarsest grid, the smoother is applied instead of the direct solver. A V-cycle is used and only presmoothing is employed. Now let the smoother be given by $S_l(\mathbf{x}_l, \mathbf{b}_l) = \mathbf{S}_l^x \mathbf{x}_l + \mathbf{S}_l^b \mathbf{b}_l$. If the smoother is only applied once, show that for a three-level scheme the iteration matrix of $MG(\mathbf{x}_2, \mathbf{b}_2, 2)$ is given by

$$\mathbf{M} = \mathbf{S}_2^x - \mathbf{P}_{2,1}\left(\mathbf{S}_1^b + \mathbf{P}_{1,0}\mathbf{S}_0^b\mathbf{R}_{0,1}\left(\mathbf{I} - \mathbf{A}_1\mathbf{S}_1^b\right)\right)\mathbf{R}_{1,2}\mathbf{A}_2\mathbf{S}_2^x.$$

(4 P)

Aufgabe 2
Let $\mathbf{G}_l$ denote the iteration matrix of the smoother $S_l(\mathbf{x}_l, \mathbf{b}_l)$. Show that the iteration matrix $\mathbf{M}_l(\nu_1, \nu_2)$ of the multigrid method $MG(\mathbf{x}_l, \mathbf{b}_l, l)$ using ν_1 presmoothings and ν_2 postsmoothings can be defined recursively as

$$\begin{aligned}
\mathbf{M}_0(\nu_1, \nu_2) &= 0,\\
\mathbf{M}_k(\nu_1, \nu_2) &= \mathbf{G}_k^{\nu_2}\left(\mathbf{I} - \mathbf{P}_{k,k-1}(\mathbf{I} - \mathbf{M}_{k-1}(\nu_1, \nu_2))\mathbf{A}_{k-1}^{-1}\mathbf{R}_{k-1,k}\mathbf{A}_k\right)\mathbf{G}_k^{\nu_1}, \qquad k = 1, \dots, l.
\end{aligned}$$

(4 P)

Aufgabe 3
Consider the linear advection equation

$$u_t + u_x = 0,$$

on the interval $x \in [0, 2]$ with initial conditions $u_0(x) = u(0, x) = \sin(10\pi x)$ and periodic boundary conditions. Discretize this equation in space on an equidistant grid by the FV method. For time discretization, use the implicit Euler scheme. To solve the resulting linear systems now use the multigrid method as given on pages 66/67 of the lecture notes. For comparison, use the two different smoothers given below, both applied only once as presmoothers. As example of an RK smoother use Heun's method, furthermore use the relaxed Jacobi method with relaxation parameter $\omega = 0.8$. Carry out l-level methods for various values of l and compare the two smoothers with respect to the number of iterations needed to fulfill the stopping criterion

$$\frac{\|\mathbf{A}_l\mathbf{x}_l - \mathbf{b}_l\|}{\|\mathbf{b}_l\|} < 10^{-9}.$$

(4 P)

Abgabe: **Montag, 21.1.2012, in der Vorlesung**