Fachbereich Mathematik **WiSe 2012/2013**
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Numerik steifer Probleme

Aufgabenblatt 11

Aufgabe 1
Consider the linear system $\mathbf{Ax} = \mathbf{b}$ and suppose that the diagonal elements of $\mathbf{A}$ are all equal. Show that in this case the explicit Euler method used as RK smoother for the above linear system is equivalent to relaxed Jacobi. (4 P)

Aufgabe 2
Consider the difference equation

$$-\epsilon\left(u_{j-1,k} - 2u_{j,k} + u_{j+1,k}\right) - \left(u_{j,k-1} - 2u_{j,k} + u_{j,k+1}\right) = 0$$

which results from a FD discretization of an anisotropic diffusion equation. Collecting the $u_{j,k}$'s in a vector $\mathbf{u}$, we obtain a linear system $\mathbf{Au} = \mathbf{0}$.

Compute the Fourier smoothing factors for the relaxed Jacobi method applied to this system by setting $u_{j,k} = e^{i(j\theta_1 + k\theta_2)}$. (4 P)

Aufgabe 3
Write a MATLAB code to numerically optimize the smoothing properties of the explicit Euler method for the linear advection example as done in the lecture for higher oder RK schemes. In analogy to the lecture notes, list the optimal values of c for the CFL numbers $CFL = 1, 3, 6, 9, 12, 24$ as well as the corresponding smoothing factors. (4 P)

Abgabe: Montag, 28.1.2012, in der Vorlesung