Fachbereich Mathematik **WiSe 2012/2013**
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Numerik steifer Probleme

Aufgabenblatt 13

Aufgabe 1
Replace the RK smoother in your FAS-MG code from assignment 12, exercise 3, by the nonlinear Jacobi method. For a nonlinear system $\mathbf{f}(\mathbf{u}) = \mathbf{s}$ with $\mathbf{u}, \mathbf{s} \in \mathbb{R}^n$ this method is given by

`Function nonlinJacobi` $(\mathbf{u}, \mathbf{s})$

- Compute correction $\mathbf{h} \in \mathbb{R}^n$:
  For $i = 1 : n$
  Solve $f_i(\mathbf{u} + h_i \mathbf{e}_i) = s_i$
- $\mathbf{u} = \mathbf{u} + \mathbf{h}$

To solve the above scalar nonlinear equations use Newton's method with an approximation of the derivative as

$$\frac{f_i(\mathbf{u} + (h_i + \epsilon)\mathbf{e}_i) - f_i(\mathbf{u} + h_i \mathbf{e}_i)}{\epsilon}.$$

Carry out one time step to solve Burgers' equation as in assignment 12, exercise 3, and check if the FAS-MG converges with this smoother. (4 P)

Aufgabe 2
To solve Burgers' equation (see last assignment sheet) in time for $t \in [0, 0.5]$ now use the time adaptive method of Cash. As in assignment 9, exercise 3, implement a Jacobian-free Newton-GMRES method to solve the resulting nonlinear systems of equations.

Furthermore, as in exercise 3 of last assignment, replace this nonlinear solver by the FAS-MG method and compare both approaches. (4 P)

Aufgabe 3
Finally, consider still another modification to the last exercise.

When you apply Newton's method to the nonlinear systems of equations resulting from the RK stages of the method of Cash you obtain *linear* systems. Now solve these systems by *linear* MG instead of using the GMRES method. This approach is called Newton-MG.

Just like the GMRES method, linear MG with RK smoothers needs the system matrix only to carry out matrix-vector products. Therefore, we may implement this again in the Jacobian-free sense as

$$\frac{\partial \mathbf{F}(\mathbf{y})}{\partial \mathbf{y}} \mathbf{q} \approx \frac{\mathbf{F}(\mathbf{y} + \epsilon \mathbf{q}) - \mathbf{F}(\mathbf{y})}{\epsilon}.$$

Do so and compare your Newton-MG code to the other two approaches. (4 P)

Abgabe: Bis Montag, 11.2.2013, per E-Mail an ortleb@mathematik.uni-kassel.de