Fachbereich Mathematik
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Numerik steifer Probleme

Aufgabenblatt 4

Aufgabe 1

a) Prove that the stability function of an s-stage Runge-Kutta method satisfies

$$R(z) = 1 + z\mathbf{b}^T(\mathbf{I} - z\mathbf{A})^{-1}e$$

where $\mathbf{b}^T = (b_1, \ldots, b_s)$, $\mathbf{A} = (a_{ij})$, $i, j = 1, \ldots, s$, and $\mathbf{e} = (1, \ldots, 1)^T$.

b) Use this to prove Lemma 2.

(4 P)

Aufgabe 2

Consider the embedded SDIRK-2 method

$$\begin{array}{c|cc} \alpha & \alpha & 0 \\ 1 & 1-\alpha & \alpha \\ \hline & 1-\alpha & \alpha \\ & 1-\hat{\alpha} & \hat{\alpha} \end{array}$$

with $\alpha = 1 - \frac{\sqrt{2}}{2}$ and $\hat{\alpha} = 2 - \frac{5\sqrt{2}}{4}$. Check whether the embedded method is A-stable or L-stable.
Hint: Plotting the stability region will be accepted as a check in terms of A-stability. (4 P)

Aufgabe 3

Program a fixed time step SDIRK time integration method for the ODE system

$$\dot{\mathbf{y}}(t) = \mathbf{f}(\mathbf{y}(t)), \quad \mathbf{y}(t_0) = \mathbf{y}_0, \quad t \in [t_0, t_{end}].$$

For the solution of the appearing systems, assume for the time being that $\mathbf{f}$ is linear and solve the systems directly.

To test this, consider the linear heat equation in 1-D with Dirichlet data

$$u_t = au_{xx}, \quad x \in [0,1], \quad t > 0, a > 0$$

$$u(0,t) = u(1,t) = 0, \quad t > 0,$$

$$u(x,0) = u_0(x) = \sin(\pi x), \quad x \in [0,1].$$

Discretize this with constant mesh width Δx and finite differences. As SDIRK time integration methods, use the implicit Euler method and the SDIRK-2 method given in exercise 2, e.g.

$$\begin{array}{c|cc} \alpha & \alpha & 0 \\ 1 & 1-\alpha & \alpha \\ \hline & 1-\alpha & \alpha \end{array}$$

with $\alpha = 1 - \sqrt{2}/2$.

Visualize the solutions in time and space for $a = 1, 10, 100$ and $t \in [0, 1]$. Play a bit with the Δt. What can be said?

Note: This code will be used again in the future, where we will refine several parts. (4 P)

Abgabe: Montag, 19.11.2012, in der Vorlesung