Fachbereich Mathematik
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Numerik steifer Probleme

Aufgabenblatt 6

Aufgabe 1
Prove the following theorem on the convergence speed of the inexact Newton method:

Let the standard assumptions hold. Then there are δ and $\bar{\eta}$, such that if $x^{(0)} \in B(\delta)$, $\{\eta_n\} \in (0, \bar{\eta}]$, then the inexact Newton iteration

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$$

with

$$\|F'(x^{(k)})\Delta x^{(k)} + F(x^{(k)})\| \leq \eta_k \|F(x^{(k)})\|$$

converges linearly to x^*. Furthermore,

- if $\eta_k \to 0$, the convergence is superlinear and $p \in [0, 1]$,
- if $\eta_k \leq K_\eta \|F(x^{(k)}\|^p$ for some $K_\eta > 0$, the convergence is superlinear with convergence order $p + 1$.

(4 P)

Aufgabe 2
Make yourself familiar with the CG algorithm and while doing so, answer briefly the following questions:

- In what way does the matrix A appear in the algorithm? In what way does this make the algorithm attractive for sparse matrics?
- In the algorithm, a division appears. Why is the algorithm nevertheless well defined, meaning, why is there never a division by zero?
- How many matrix-vector products are there, how many scalar products and how many matrices and vectors need to be stored?

(4 P)

Aufgabe 3
Consider the equation

$$-u''(x) = \sin(u(x)) + f(x),\ u(0) = u(1) = 0.$$

with $f(x) = 2 - \sin(x(1 - x))$ and $x \in (0, 1)$.

Discretize the equation with central differences with $\Delta x = 1/100$ and $\Delta x = 1/1000$. Use an inexact Newton method with initial iterate $u_0 = 0$ and $TOL = \tau_r = 10^{-10}, \tau_a = 0$ to solve the nonlinear equation systems. As subsolver, use the GMRES method implemented in MATLAB in its unrestarted form and provide the tolerances. For these, use a fixed tolerance equal to the tolerance in the Newton solver and then the method of Eisenstat and Walker to define the forcing terms with $\eta_{max} = 0.1$ and $\eta_{max} = 0.999$. Make sure to use a sparse matrix format.

As a measure of the computational work use the number of GMRES iterations and the number Newton iterations needed. How does the convergence speed look like? (4 P)

Abgabe: Montag, 3.12.2012, in der Vorlesung