Fachbereich Mathematik **WiSe 2012/2013**
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Numerik steifer Probleme

Aufgabenblatt 7

Aufgabe 1

Let $\mathbf{A} \in \mathbb{R}^{n\times n}$ be a symmetric and positive definite matrix with the spectrum having the property

$$\sigma(\mathbf{A}) \subset (1, 1.5) \cup (399, 400).$$

Estimate the number of CG iterations needed, when starting with $\mathbf{x}_0 = 0$, to reduce the $\mathbf{A}$-norm of the error by a factor of 10^{-4}. Use both the criterion based on condition number and that on polynomials. (4 P)

Aufgabe 2

a) Take $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. We are going to solve $\mathbf{Ax} = \mathbf{b}$.

   (i) Show that the CG method applied to this system should converge in 1 or 2 iterations (using the convergence theory).

   (ii) Choose $\mathbf{x}_0 = (0,0,0)^T$ and do two CG iterations.

b) Suppose that the $\mathbf{A}$ is nonsingular, symmetric, and indefinite. Give an example to show that the CG method can break down.

(4 P)

Aufgabe 3

Consider again the equation from the last assignment

$$-u''(x) = \sin(u(x)) + f(x),\ u(0) = u(1) = 0,$$

with $f(x) = 2 - \sin(x(1-x))$ and $x \in (0,1)$.

Discretize the equation with central differences with $\Delta x = 1/100$ and $\Delta x = 1/1000$. Use an inexact Newton method with initial iterate $u_0 = 0$ and a relative tolerance $TOL = 10^{-10}$ to solve the nonlinear equation systems. As subsolver, use the GMRES method as shown in class and implement that yourselves with Givens rotations. To provide the tolerances, use a fixed tolerance equal to the tolerance in the Newton solver and then the method of Eisenstat and Walker to define the forcing terms with $\eta_{max} = 0.1$ and $\eta_{max} = 0.999$. Make sure to use a sparse matrix format.

As a measure of the computational work use the number of GMRES iterations and the number of Newton iterations needed and the run time of your code determined the matlab functions tic and toc. (4 P)

Abgabe: **Montag, 10.12.2012, in der Vorlesung**