Fachbereich Mathematik **WiSe 2012/2013**
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Numerik steifer Probleme

Aufgabenblatt 8

Aufgabe 1
Prove the following statement:

Let $\mathbf{A}$ be a nonsingular normal matrix. Let $\mathbf{b}$ be a linear combination of k of the eigenvectors of $\mathbf{A} \in \mathbb{R}^{d \times d}$:

$$\mathbf{b} = \sum_{l=1}^{k} \gamma_l \mathbf{v}_l.$$

Then the GMRES iteration, with $\mathbf{x}_0 = 0$, for $\mathbf{A}\mathbf{x} = \mathbf{b}$ will terminate in at most k iterations.

Hint: Look at the term $p(\mathbf{A})\mathbf{r}_0$.

How do you think the convergence curve of GMRES will look like for this case in practice? (4 P)

Aufgabe 2
Replace the Jacobian in your code from assignment 7, exercise 3 with an approximation $\mathbf{A}(\mathbf{u}) \approx \mathbf{DF}(\mathbf{u})$ with

$$a_{ij}(\mathbf{u}) = \frac{1}{\varepsilon}\left(f_i(\mathbf{u}) - f_i(\mathbf{u} + \varepsilon \mathbf{e_j})\right),$$

where $\mathbf{e}_j$ denotes the j-th unit vector. How does the choice of ε affect the solution? (4 P)

Aufgabe 3
Consider again the problem from assignment 7, exercise 3 (and from the last exercise). Add the possibility of right preconditioning (no left preconditioning) to your code, by adding a call to a generic preconditioning function that takes a vector v and outputs a different vector.

Now redo the experiments from that exercise, where you use

- MATLABs ilu command to obtain an incomplete LU decomposition of A with no additional fill-in.
- Jacobi preconditioning
- Symmetric Gauß-Seidel preconditioning

Compare the numbers of iterations as well as computing times.

Furthermore, compute the eigenvalues of A during the first iteration and compare these to both the left- and the right preconditioned version and plot these. Relate the convergence results for right preconditioned GMRES and unpreconditioned GMRES and these pictures to the different convergence speeds you see for GMRES. (4 P)

Abgabe: **Montag, 17.12.2012, in der Vorlesung**