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Linear Systems Theory

Exercise Sheet 4

Exercise 1

The *convolution* of two functions $f, g : \mathbb{R} \to \mathbb{R}$ is defined as the function

$$(f \star g)(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau\,.$$

It exists for example, if both f and g have compact support or if the support of both functions is contained in $[0, \infty)$, as then the integral is always finite. The convolution can also be defined for distributions, but in the sequel we only need the Dirac distribution and we will simply pretend that it was regular with the Dirac δ-"function" as kernel.

(i) Show that $(f \star g)' = f' \star g = f \star g'$ and $f \star \delta = f$.

(ii) Let $P(\frac{d}{dt})y = Q(\frac{d}{dt})u$ be the input/output representation of a behaviour with signals $y, u : \mathbb{R} \to \mathbb{R}$ and let h be the *impulse response* of the system, i. e. h solves $P(\frac{d}{dt})h = Q(\frac{d}{dt})\delta$. Show that the convolution $y = h \star u$ provides us with an output for the input u.

(iii) Consider the system $\dot{y} = Ay + u$. Find its impulse response $h(t)$ which vanishes for all $t < 0$. What is an output for an input $u(t)$ which also vanishes for all $t < 0$?

Exercise 2

Consider the autonomous linear system

$$\dddot{y}_1 + \dot{y}_1 + \ddot{y}_2 + \dot{y}_2 + y_2 = 0\,, \qquad \dot{y}_1 + 2y_1 + y_2 = 0\,.$$

(i) Compute the dimension of the corresponding behaviour both by direct manipulation of the given equation and by applying the theory presented in the lecture.

(ii) Rewrite the given system as a first-order one in the "naive" way by introducing the derivatives as latent variables: $\xi = (y_1, y_2, \dot{y}_1, \dot{y}_2, \ddot{y}_1, \ddot{y}_2)^T$. This yields a six-dimensional representation.

(iii) Apply the theory of the lecture to each equation in the system and produce a four-dimensional representation as first-order system.

(iv) Is it possible to find an equivalent first-order system of even smaller dimension?