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Linear Systems Theory

Exercise Sheet 7

Exercise 1

Let $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$ be such that (A, b) is a controllable matrix pair. Using the characteristic polynomial of A,

$$\chi_A = \det(sI - A) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\,,$$

we define recursively vectors $v^{(n)} = b$ and $v^{(i)} = Av^{(i+1)} + a_i b$ for $i = 0, \dots, n-1$.

(i) Show that $\{v^{(1)}, \dots, v^{(n)}\}$ is a basis of $\mathbb{R}^n$ and that $v^{(0)} = 0$.

(ii) Prove that $T = \bigl(v^{(1)}, \dots, v^{(n)}\bigr)$ is a transformation matrix that puts (A, b) in controller form.

(iii) Apply this to

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\,, \qquad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\,.$$

Exercise 2

Show that the matrix pair (A, B) is controllable, if and only if no eigenvector of the transposed matrix A^T lies in the orthogonal complement of $\operatorname{im} B$ (for the standard scalar product in $\mathbb{R}^n$).

Exercise 3

(i) Compute a Kalman decomposition of the system defined by

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}\,, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\,.$$

What are the uncontrollable modes?

(ii) Consider again the “bipendulum” from the last sheet. Study directly (i. e. without introduction of a state space form) whether the system is controllable.