Machine Learning Parameter Systems, Noether Normalisations and Quasi-stable Positions

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Abstract

We discuss the use of machine learning models for finding "good coordinates" for polynomial ideals. Our main goal is to put ideals into quasi-stable position, as this generic position shares most properties of the generic initial ideal, but can be deterministically reached and verified. Furthermore, it entails a Noether normalisation and provides us with a system of parameters. Traditional approaches use either random choices which typically destroy all sparsity or rather simple human heuristics which are only moderately successful. Our experiments show that machine learning models provide us here with interesting alternatives that most of the time make nearly optimal choices.

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1. Introduction

It is well known that many results in algebraic geometry and commutative algebra considerably simplify in generic coordinates. While from a theoretical point of view one may simply exploit that a random transformation (almost) always achieves a generic position, the situation is less simple from a computational point of view. Random transformations are computationally bad, as they destroy all sparsity typically present in generators of polynomial ideals. Furthermore, for many generic positions – like for example the popular GIN position in which one obtains the generic initial ideal – effective tests are either not known or prohibitively expensive.

As we have demonstrated in several articles, quasi-stable position represents an interesting alternative. It shares most of the properties of the GIN position (Hashemi et al., 2012), but can be effectively verified. It entails a Noether normalisation (Seiler, 2009b) and in quasi-stable position one obtains easily a system of parameters (Seiler,

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