JET — An AXIOM Environment for Geometric Computations with Differential Equations

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Extended Abstract

Geometric methods play an important role in the analysis of nonlinear differential equations. For example, symmetry methods provide the more or less only systematic approach to the construction of solutions. However, most geometric computations tend to be very tedious. Thus the use of computer algebra systems considerably helps in the application of these methods.

JET is an environment within the computer algebra system AXIOM to perform such computations. The current implementation emphasizes the two key concepts involution and symmetry. It provides some packages for the completion of a given system of differential equations to an equivalent involutive one based on the Cartan-Kuranishi theorem and for setting up the determining equations for classical and non-classical point symmetries.

We stress that JET is an *environment* for such computations and not simply a collection of some special purpose algorithms. Thus it contains general data structures for the jet bundle formalism which can also be used for other tasks than the two above mentioned. Using the generic programming facilities of AXIOM it is possible to provide several representations for jet bundles and for different classes of differential equations. The main application packages are independent of such details.

Involution has important applications in symmetry theory. One should e.g. mention that involutive systems are locally solvable and only for such systems the two widely used definitions of a symmetry coincide. The in calculations applied definition as a transformation leaving the differential equation invariant yields for not locally solvable systems usually less symmetries then the definition as a transformation mapping solutions into solutions. This can easily be seen with the system $u_z + yu_x = u_y = 0$. Obviously it is not invariant under y-translations, but its solution space $u = const$ has this symmetry. This effect has especially implications on the nonclassical method of Bluman and Cole.

Other applications include computing the size of the symmetry group of a differential equation without solving the determining equations. Furthermore it is possible to "correct" the result by subtracting some unwanted effects like e.g. the trivial superposition symmetry of linear equations. In the case of gauge theory the concept of involution leads to an new intrinsic definition for the number of degrees of freedom based on a similar formal correction.

A brief description of an earlier version of JET can be found in Ref. [1]. The current version is described in much detail in Ref. [2]. For more information about the underlying mathematical theory we refer to Ref. [3]. Applications of the concept of involution in symmetry theory are discussed in Ref. [4]. Finally we mention Ref. [5] for some applications in physics. All these publications can be obtained via WWW at the URL:

$$\mathtt{http://avalon.ira.uka.de/iaks-calmet/werner/werner.html}.$$

References

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