Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

★ ★ ★ ★ **Newsletter** ★ ★ ★ ★

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Contents

From the Editor

A new year has come and a new issue of the Newsletter is here. Again much material has been collected, and I hope you get valuable information.

I apologize that some of the members received their October issues only in early November. SIAM sent the copies out by mid-October, but mailing was slow to several countries. I hope this problem can be resolved with the current issue by a different mailing procedure. Please let me know if you have not received your copy by the end of February.

The officers of the Activity Group have decided to give public access to back issues of the Newsletter in electronic form. At the time of their appearance, new issues of the Newsletter will not be accessible electronically since it should be the privilege of the members of the Activity Group to receive them. However, as an additional benefit for the members, previous issues can now be obtained as `dvi` or `PostScript` files from my WWW homepage:

http://www.zib-berlin.de/~bzfkoepf/

or by anonymous `ftp` at `ftp.zib-berlin.de` in the directory `pub/UserHome/Koepf/SIAM`. You probably will prefer to receive the `dvi` files since these are much smaller in size, because they do not include the logo. Note that the WWW address of my homepage has changed!

Since these back issues are available also for nonmembers, this might prove to be a welcome advertisement for the Group. We warmly recommend

SIAM Activity Group
on
Orthogonal Polynomials and Special Functions

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Elected Officers
Charles Dunkl, *Chair*
Tom H. Koornwinder, *Vice Chair*
Willard Miller, *Program Director*
Nico M. Temme, *Secretary*

Appointed Officer
Wolfram Koepf, *Editor of the Newsletter*
Martin E. Muldoon, *Webmaster*

◇

The purpose *of the Activity Group is*

—to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.

that you post this information in your departments.

I would like to encourage you again to submit material for the Newsletter. Particularly short reports about the use of orthogonal polynomials and special functions in applications would be warmly appreciated. For the submission procedure see *How to Contribute to the Newsletter* on page 15. Many thanks to all contributors to the current issue! Again contributors are cited by their names and email connections while their mailing addresses can be obtained from the Editor by request.

Furthermore I would like to point your attention to the *Problems Section* (p. 11). Note that problems can be submitted with or without solution. Different from the Problems Sections in other periodicals we do not mark problems to be solved or unsolved. This makes their solution more challenging and should also avoid the barrier in front of a supposedly difficult problem.

This time three new problems were submitted, and there are still some open problems from previous issues. Have a look at them, and don't hesitate to send in a solution!

The results of the the recent election for officers of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions* are finalized. The newly-elected officers are:

Chair: Charles F. Dunkl

Vice Chair: Tom H. Koornwinder

Secretary: Nico M. Temme

Program Director: Willard Miller, Jr.

Each will be serving a three-year term beginning January 1, 1996.

On behalf of all members, I would like to thank all the candidates for their willingness to run, and I welcome the new officers of this Group warmly!

Furthermore, I would like to thank George Gasper and Martin Muldoon, who are quitting as officers, for their service for the Activity Group. Martin Muldoon will remain involved in our Activity Group as editor of OP-SF Net (joint with Tom Koornwinder, starting with issue 3.2) and as Webmaster, managing our new WWW home page

http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/

which is also accessible via the SIAM home page

http://www.siam.org/

The election was extremely close and required a careful recount. Also, for your information, 43% of the membership voted. This is extremely high. In the SIAM elections, usually about 20% vote.

As unedited quotes I would like to mention some comments and suggestions for future activities of the Group that were submitted by the members with the ballots.

Future Activities (selection):

- Seminars on introductory research for postgraduate students.
- Some activities in western European countries as well as in Latin America.
- Please start discussions with SIAM about the possibility to start publication of a monograph on orthogonal polynomials and special functions.

- Sponsorship for new families of special functions and new sets of orthogonal polynomials.
- Define a program of self study for the non-mathematician, i.e., for an engineer mathematically inclined that uses OP and SF. In other words, could the group indicate a limited set of books and papers (including survey paper) that could give an adequate basis to understand survey papers that would give the current trends and needs in OP-SF.

Other Comments (selection):

- A hard copy of the newsletter is important to those members who don't have e-mail. Otherwise, they can't really participate. I am retired (and I suspect others in the group area also retired) but still active in research so a hard copy of the group's newsletter is important to me. Hence my vote for N.M. Temme for secretary.
- Do we need competitive candidates? I hate to have to pick between such well-qualified candidates.
- Great newsletter!
- For the beginners in the field of OP-SF, where does an engineer that is mathematically oriented start his own self-study program? Could the group indicate some survey papers that would give a picture of the current research done in the field of OP-SF?

The following suggestions for future nominees were given:

Chair: Richard Askey, T.H. Koornwinder

Vice Chair: G. Gasper, Willard Miller, L. Vinet

Secretary: J.S. Geronimo, M. Ismail

Program Director: B.C. Carlson, Walter Van Assche

Beginning with their terms, the newly elected officers of the Activity Group were appointing me as Editor of the Newsletter for the full three-year term until the end of 1988. I gladly accepted this appointment, and I would like to express my thanks for their confidence in my work.

January 31, 1996 Wolfram Koepf

New Year's Greetings from the Chair

Speaking in behalf of the Group I extend sincere thanks and appreciation to George Gasper and Martin Muldoon for their work as officers. George is stepping down from Vice Chair after six years; he was always quick to offer help and good advice. We must have sent hundreds of e-mails back and forth. Martin was Program Director for three years and did a great job organizing the annual minisymposia. He has offered to continue as webmaster for the group's home page. We welcome Bill Miller and Nico Temme to the positions of Program Director and Secretary for the next three years. Rounding out the elected officers is Tom Koornwinder, moving to Vice Chair from Secretary. I look forward to working with all of you for the advancement of our field and profession.

More personally, I will mention some of my hopes and concerns for the group. I am pleased to see the membership grow, attracting people from all around the world. Our capability for distributing news is excellent, thanks to the efforts of Tom, Eugene Tomer, and Wolfram Koepf. I think in this regard we are definitely living up to our charter to further communication in our field. We have not yet organized an actual meeting, although our members have been involved with some outstanding conferences, for example, the Fields Institute conference on q-series and applications at the University of Toronto, in June '95. Also there seems to be a steady stream of meetings in Belgium and the Netherlands, so one could say that our field has enough conferences in various places to serve peoples' needs, and one of our main tasks is to provide publicity.

Mathematically, what is going on? There is the part of our activity dedicated to providing information to potential users of special functions. One is reminded of questions sent in on Usenet newsgroups like sci.math.research and sci.math.symbolic which can easily be answered by experts. In a more sophisticated direction, Dan Lozier and his co-workers at NBS are launching a service to test and validate software for special functions via the Web. Handbooks are beginning to appear in CD-ROM form, as another example of electronic dissemination of knowledge.

I think our group will continue to be productively involved in these areas. The other main concern of ours is research. For myself I find anything other than face-to-face contact with colleagues a lesser substitute. It is certainly very useful to be able to exchange e-mail messages with colleagues wherever they may be, and to get relatively simple questions answered (sometimes) but for me it does not take the place of sitting in a colleague's office, writing on the blackboard, and jointly brainstorming. It is indeed easier now than it was ten years ago to work electronically on joint papers with others, but I suspect most research starts with actual personal contact. This gives emphasis to the need for conferences, yet our world-wide membership does make it difficult to bring together enough participants within the SIAM meeting framework.

To conclude, I am pleased with the contribution the group has been able to make in our area: the Newsletter, OP-SF Net, cooperating with the ftp site which was constructed by Waleed Al-Salam and which has now been transferred to the care of Hans Haubold (see p. 12). I hope to become personally, or at least electronically, acquainted with more of you. Lastly, remember to send in news items, remarks, suggestions for book reports, and problems, to the Newsletters.

Charles F. Dunkl
(cfd5z@virginia.edu)

Introducing the New Officers

Program Director: Willard Miller

Willard Miller earned a PhD. in Applied mathematics from the University of California, Berkeley, in 1963 and spent the following two years as a postdoc at the Courant Institute, New York University. In 1965 he joined the faculty of the School of Mathematics, University of Minnesota, where he remains to this day. In 1973 he was promoted to full professor and from 1978 to 1986 he was head of the department.

Willard was one of the three principal investigators on the grant to the Institute for Mathematics and its Applications (IMA) which was established at the University of Minnesota in 1982 by the US National Science Foundation. An important part of the mission of the IMA is motivate mathematicians to apply their tools to problems that arise in the other sciences, engineering, and in particular, industry. From 1987 to 1994 Willard served as the associate director of the IMA with the director Avner Friedman.

Since mid-1994 Willard has been Associate Dean for Finance and Planning in the Institute of Technology. This unique college includes the mathematics, basic sciences and engineering departments of the University. For a period of 4.5 months in the summer and fall of 1995 he was Acting Dean while a search for a new dean proceded.

Willard's primary research interests lie in the exploitation of symmetry in physical theories. A long time interest is the study of Lie symmetry techniques to characterize special solutions of the scalar and spinor equations of mathematical physics, particularly those that are obtainable via separation of variables. For more than 20 years he has been collaborating in this regard with Professor Ernie Kalnins of the University of Waikato, Hamilton, New Zealand. Special functions arise in this study as solutions of the PDEs and with a symmetry characterization. Willard has published nearly 130 papers, many of them with Kalnins and other collaborators, and 3 books. Recent work has been on topics such as q-series, R-matrices, supergroups, superintegrable systems, and radar, all from a symmetry point of view.

Both at the IMA and the Dean's Office, Willard has made special efforts to link industry and academia, and he plans to continue these efforts as Program Director of SIAG/OS.

Secretary: Nico M. Temme

Nico Temme earned a "doctorandus" degree in Applied mathematics from the University of Amsterdam, The Netherlands, in 1968 and joined the Mathematical Centre in Amsterdam at the Department of Applied Mathematics. The MC is a foundation with a research institute that is not connected to one of the Dutch universities, although strong connections with universities exist.

In 1976 Nico defended his thesis *Some Aspects of Applied Analysis: Asymptotics, Special Functions and their Numerical Computation*. He was involved in the daily management of the department and continued his research in asymptotics and special functions.

In 1981 the institute of the Foundation Mathematical Centre assumed a different name: *CWI, Centrum voor Wiskunde en Informatica* (Centre for Mathematics and Computer Science), to stress the growing importance of computer science in the institute and to satisfy the computer science people in the country. In 1988 the Department of Pure Mathematics and the Department of Applied Mathematics were put together with a new name: *Department of Analysis, Algebra and Geometry*. Nico got involved in management and science policy of CWI.

At CWI free research, in particular analysis, got into a tight corner, and it was difficult to hire new people on the institute's budget. It was a very lucky moment that Adri Olde Daalhuis joined the analysis group in 1989; under Nico's supervision he finished his thesis *Uniform, Hyper-, and q−Asymptotics* in January 1993. At present it is unthinkable to appoint PhD. students at CWI without external funds. By writing a proposal on wavelets with applications to seismic signals it was possible to start an externally funded project in analysis with one PhD. and one post-doc, and with Nico as project leader, but his heart is in asymptotics and special functions.

Nico is member of the editorial board of "Mathematics of Computation", "Methods and Applications in Analysis", "Zeitschrift für angewandte und mathematische Physik", "CWI Quarterly", and he has written numerous papers on asymptotics, in particular uniform asymptotics of integrals, and on special functions; he just finished a book on the classical special functions which will be published by Wiley in January 1996.

Reports from Meetings and Conferences

1. Annual SIAM Meeting, Charlotte, N. Carolina, October 23-26, 1995

The Minisymposium *Computational Aspects of Special Functions and Orthogonal Polynomials*, sponsored by the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*, chaired by Walter Gautschi and the writer of this note, was attended by approximately 15 to 20 people, and had the following talks:

– John Boyd (University of Michigan) talked about Hermite expansions, the anharmonic oscillator, and perturbation theory.

– Walter Gautschi (Purdue University) discussed an algorithm for generating polynomials orthogonal with respect to a Sobolev norm. This theory has significant differences from the ordinary one, for example, there is no three-term recurrence.

– Dan Lozier (Nat'l Inst. of Standards and Technology) gave a survey on software for special functions. He is working on a project to provide accuracy checks for algorithms. The idea would be for researchers to send in a list of pairs $(x, f(x))$ where f is a special function, and then the NIST computers check these values independently and generate an error report for the researcher. Dan has set up a Web site and asks that the special function community read over his suggestions and send feedback. The URL is http://math.nist.gov/nesf.

– Nico Temme (CWI, Amsterdam) talked about the problems of accurate computations of probability distribution functions in regions of rapid increase, for example, incomplete beta integrals with very large parameters. Often such computations require much more work than routine problems.

– The writer of this note presented joint work with Don Ramirez on computation of surface measures of ellipsoids in N-space via Lauricella F_D functions and an application to optimal designs in statistics.

Charles F. Dunkl
(cfd5z@virginia.edu)

Forthcoming Meetings and Conferences

1. Umbral Calculus Workshop: MIT, April 22-23, 1996

In honor of Gian-Carlo Rota's 64th birthday, the *RotaFest* conference is being organized by Richard Stanley, Neil White and Richard Ehrenborg at the *Massachusetts Institute of Technology*, Cambridge, Massachusetts, USA from April 17 to April 20, 1996.

This conference will be followed by a two day workshop devoted to Rota's Umbral Calculus. This special session will be held Monday and Tuesday, April 22 and 23, 1996, and is organized by Alessandro Di Bucchianico, Daniel Loeb, and Nigel Ray.

The confirmed speakers at the Umbral Calculus Workshop thus far are

– George Andrews, Pennsylvania State University
– Marilena Barnabei, Università di Bologna, Italy

- William Y. C. Chen, Los Alamos National Laboratories and Nanking University, China
- Ottavio D'Antona, Università di Milano, Italy
- Alessandro Di Bucchianico, Eindhoven University of Technology, The Netherlands
- Philip Feinsilver, Southern Illinois University at Carbondale
- Henryk Gzyl, Universidad Simon Bolivar, Venezuela
- Mourad Ismail, University of South Florida at Tampa
- Daniel E. Loeb, Université de Bordeaux, France
- Heinrich Niederhausen, Florida Atlantic University
- Nigel Ray, University of Manchester, United Kingdom
- Brian Taylor, Massachusetts Institute of Technology
- Luis Verde-Star, Universidad Autónoma Metropolitana, Mexico
- Jet Wimp, Drexel University

Full details about the workshop are available via WWW: http://www-math.mit.edu/~loeb/rotafest.html

Daniel Loeb
(loeb@labri.u-bordeaux.fr)

2. CRM Workshop on the Theory of Special Functions: CRM, May 6-26, 1996

The *Centre de Recherches Mathematiques* is hosting a year long program in applied and numerical analysis in 1995–1996.

Between May 13-26, 1996, the *Workshop on the Theory of Special Functions* will be organized.

The first workshop (13–18 May 1996) has the title *Theory of nonlinear special functions: the Painlevé transcendents*. The list of invited speakers includes: M. Ablowitz, P.A. Clarkson, C.M. Cosgrove (∗), B. Dubrovin, A.S. Fokas, B. Grammaticos (∗), J. Harnad, A.R. Its, N. Joshi, A.V. Kitaev, V.E. Korepin, M.D. Kruskal (∗), V.B. Matveev, F. Nijhoff, K. Okamoto (∗), C. Rogers, F. Smirnov, H. Umemura.

The second workshop (20–26 May, 1996) has the title *Algebraic methods and q-special functions*. The list of invited speakers includes: W. Al-Salam (∗), G.E. Andrews, R. Askey, D. Bressoud, I. Cherednik (∗), D. Chudnovsky, G. Chudnovsky, C.F. Dunkl, P. Etingof, R. Floreanini, I.B. Frenkel (∗), M. Ismail, E.G. Kalnins, T.H. Koornwinder, I.G. Macdonald, D.R. Masson (∗), M. Noumi, E. Opdam, M. Rahman (∗), D. Stanton (∗), S.K. Suslov, A. Veselov.

In both lists (∗) means: to be confirmed.

Please register by March, 1996 for participation and accommodation or by May 1, 1996 for attendance only.

Louis Pelletier
(PelletL@CRM.UMontreal.ca)

3. International Joint Mathematics Meeting: Antwerp, Belgium, May 22-24, 1996

The first joint meeting of the AMS and the mathematical societies of the Benelux countries (Belgium, the Netherlands, and Luxemburg) will be held on May 22–24, 1996 in Antwerp, Belgium.

Walter Van Assche (Katholieke Universiteit Leuven) and Marcel G. de Bruin (Technische Universiteit Delft) are organizing a contributed paper session on *Special Functions* (to be confirmed by the Belgian Mathematical Society) with eight 20-minutes lectures.

Program

M. G. de Bruin, Technical University Delft, The Netherlands: *Continuous Sobolev-Laguerre polynomials and their continued fractions.*

H. G. Meijer, Technical University Delft, The Netherlands: *Coherent pairs for Sobolev orthogonal polynomials.*

H. Bavinck, Technical University Delft, The Netherlands
Linear perturbations of differential and difference operators with polynomial eigenfunctions.

J. Van Der Jeugt, Universiteit Gent, Belgium
Transformation and summation formulas for multiple hypergeometric functions.

M. Schlosser, Ch. Krattenthaler, Universität Wien, Austria: *Multidimensional matrix inversion and A_r and C_r basic hypergeometric functions.*

W. Koepf, Konrad-Zuse-Zentrum, Berlin, Germany
Efficient computation of orthogonal polynomials in computer algebra.

W. Koepf, Konrad-Zuse-Zentrum, Berlin, Germany
Orthogonal polynomial identities in Maple V.4.

R.F. Swarttouw, Computer Algebra Nederland, Amsterdam, The Netherlands
An interactive electronic version of the Askey-Wilson table for classical orthogonal polynomials.

Please look at the meetings part of the AMS e-MATH WWW home page

http://e-math.ams.org/committee/meetings/

for latest information or consult a news bulletin of one of the organizing organizations (e.g. Notices AMS).

Walter Van Assche
(Walter.VanAssche@wis.kuleuven.ac.be)

4. Joint Summer Research Conference on Random Matrices, Statistical Mechanics, and Painlevé Transcendents: South Hadley, June 23-27, 1996

This conference will take place at the *Mount Holyoke College*, South Hadley, Massachusetts, USA. It is chaired

jointly by Pavel Bleher and Alexander Its, both of Indiana University-Purdue University at Indianapolis.

The analysis of correlation functions for exactly solvable quantum models and of the partition functions in the theory of random matrices has gradually become an exciting new branch of mathematical physics which has deep connections with both the classical theory of special functions and orthogonal polynomials and the modern theory of quantum groups and topological quantum field theory.

Further details can be obtained from Notices AMS, November 1995, pp. 1454–1457 or by WWW: http://www.ams.org/committee/meetings/src.html

People interested in attending should send

1. Title and dates of conference desired
2. Full name
3. Mailing address
4. Area code and phone number for office, home, and fax
5. Email address
6. Scientific background relevant to the topic of the conference
7. Financial assistance requested (or indicate if support is not required)

to

Summer Research Conference Coordinator
Conferences Department
American Mathematical Society
P.O. Box 6887
Providence, RI 02940
USA
fax: +1-401-445 4140
email wsd@ams.edu

Deadline for receipt of requests for information is March 1, 1996.

Richard Askey
(askey@math.wisc.edu)

5. Meeting on Symmetries and Integrability of Difference Equations: Canterbury, July 1-5, 1996

A meeting on *Symmetries and Integrability of Difference Equations* (SIDE) will be held at the University of Kent at Canterbury from Monday 1st July to Friday 5th July 1996. This conference is the successor of the meeting on the same topics held in Esterel, Quebec, Canada in May 1994.

The conference is being organised by Professor Peter Clarkson, Institute of Mathematics & Statistics, University of Kent (P.A.Clarkson@ukc.ac.uk) and Dr. Frank Nijhoff, Department of Applied Mathematical Studies, Unversity of Leeds (frank@amsta.leeds.ac.uk). The scientific committee consists of Peter Clarkson, Frank Nijhoff, Thanasis Fokas, Loughborough University, U.K. (A.S.Fokas@lut.ac.uk) and Pavel Winternitz, University of Montreal, Canada (wintern@ere.umontreal.ca). The local management of the meeting will be run by Peter Clarkson and Alan Common (A.K.Common@ukc.ac.uk), of the Institute of Mathematics & Statistics.

The conference finances will be run on a minimal basis with delegates charged a small registration fee of 50 pounds and a residential fee of about 40 pounds per day to cover bed, breakfast, lunch, coffee, tea and evening meal. Anyone registering after 31 March 1996 will be charged an additional late registration fee of 50 pounds. Accommodation is limited so early registration is encouraged. We will attempt to attract some supporting grants. We are grateful to the Institute of Mathematics & Statistics for financial support.

The organisers would like to encourage all communications regarding the conference to be carried out by email as far as possible. Please email the organisers as soon as possible if you are interested in participating in this meeting; this is not a definite commitment.

Peter Clarkson
(P.A.Clarkson@ukc.ac.uk)

6. XXI. International Colloquium on Group Theoretical Methods in Physics: Goslar, July 15-20, 1996

Within this Colloquium there will be a.o. a symposium on Quantum Groups covering the following topics: quantum groups and their representations, quantum spaces and quantum symmetries, differential calculus on quantum spaces and quantum groups, non-standard deformations, Yangians, braided Hopf algebras, relations to noncommutative Geometry, q-analogues of special functions and partial differential equations.

The Local Organizing Committee consists of H-D. Doebner, W. Scherer and P. Kramer. For the quantum groups symposium V.K. Dobrev is a co-organizer.

For up-to-date and additional information see the home page WWW: http://www.pt.tu-clausthal.de/~group21/

conference organizers
(group21@pt.tu-clausthal.de)

7. SIAM Annual Meeting: Kansas City, July 22-26, 1996

The 1996 SIAM Annual Meeting will be held in Kansas City, Missouri, in the Hyatt Regency Crown Center. See http://www.siam.org/meetings/an96/an96home.htm for further information, or send an email to meetings@siam.org. A major theme is *New Tools of Applied Mathematics*.

Our Activity Group probably will not sponsor or organize any Minisymposium during this Annual Meeting.

Our Program Director Martin Muldoon (who was in office until December 1995) has explored several options for topics, but all failed for some reason. In particular, the idea of organizing a Minisymposium on *Handbooks of Special Functions* could not be realized, because some of the key lecturers on such a topic already had commitments for a conference ICCAM 95, which will be held in Leuven, Belgium in the same week (see below). Hopefully our Activity Group can organize a Minisymposium with this theme on a later occasion.

Tom H. Koornwinder
(thk@fwi.uva.nl)

8. Seventh International Congress on Computational and Applied Mathematics: Leuven, July 21-26, 1996

The Congress *ICCAM 96* will take place at the *Katholieke Universiteit Leuven* in Belgium and will concentrate on the analysis of computational techniques for solving real scientific problems.

Invited Speakers: H. Brunner, M.E.H. Ismail, F. Marcellan, M. Nakao, J. Nedoma, W. Sweldens, P. Toint.

Participants who would like to hold a short communication (20 minutes) should submit a title and a short abstract (at most 1 page) not later than March 1, 1996 to Prof. M.J. Goovaerts. Please also contact this address for the full announcement and an application form.

M.J. Goovaerts
(fdbaa35@cc1.kuleuven.ac.be)

9. Workshop Transform Methods & Special Functions, II: Varna, August 23-30, 1996

First Announcement (shortened): The Second International Workshop *Transform Methods & Special Functions* will be devoted to the 100th Anniversary of the Bulgarian mathematician Acad. Nikola Obrechkoff (1886–1963). He has left an enormous and valuable heritage of more than 250 papers and several monographs and manuals in various topics: Analysis, Algebra, Number Theory, Numerical Analysis, Summation of Divergent Series, Probablities and Statistics etc. New edition and translation of the Obrechkoff's selected papers are also planned.

By the moment more than 80 foreign experts in *TM&SF* from Americas, Asia, Australia, Europe have confirmed their preliminary interest in taking part in the meeting. The meeting will take place near the town of Varna (birthplace of N. Obrechkoff), in the Black Sea resort *Golden Sands*, close to the seashore. Varna Airport receives direct flights from all over the world as well as regular domestic flights from Sofia. One could arrive also by a number of express trains starting from Berlin, Warsaw, Prague, Moscow, etc, by international highways or to the seaport.

Organizing Committee: Prof. Dr. Petar Rusev, Prof. Dr. Ivan Dimovski, Prof. Dr. Shyam L. Kalla (Kuwait University), Asso. Prof. Dr. Virginia Kiryakova, Asso. Prof. Dr. Lyubomir Boyadjiev.

Topics: Integral Transforms, Special Functions, Series Expansions, Fractional Calculus, Algebraical Analysis, Generalized Functions, Operational Calculus, Univalent Functions; Applications of these topics to Complex Analysis, Differential and Integral Equations.

Scientific programme: includes invited lectures and short presentations. Proceedings of the Workshop will be published. The requirements and the deadlines for the abstracts and papers will be specified in the Second Announcement.

Registration Fees: US$150, for accompanying persons: US$60. These fees are to cover the necessary organization and administrative expenses of the Workshop, the Abstracts and Proceedings *TM&SF'96*, the social programme (coffee breaks, excursion, welcome party) as well as the local transport from/to Varna Airport (about 27 km farway). We have no possibilities to support financially even the invited lecturers, so all the participants are kindly asked to arrange, through their institutions or funding organizations, the payment of their living expenses and the participation fees. The Organizing Committee should book the necessary rooms and halls and pay in advance some 30% of the total expenses in the hotels. That is why, we request those who plan to attend the Workshop, to inform us in the necessary terms what kind of accommodation they would prefer.

Please, fill in the Registration Form and send it as soon as possible to the mailing address of the Organizing Committee or by Fax and email. The deadline is 15 March 1996 (extended for the Activity Group from 15 January 1996).

The same deadline is for sending the participation fees in advance, which is preferable and well appreciated. Payments should be arranged by cheques, sent to the Org. Committee and payable to the Institute of Mathematics - Bulg. Acad. Sci. Since the number of rooms is limited, they will be available with privilege, to those who sent the participation fees in advance, and then in order of precedence of received registration forms.

Looking forward to hearing soon from you and thanking for your interest and commitment, yours sincerely.

Virginia Kiryakova
(virginia@bgearn.bitnet)

10. UN/ESA Workshop: Bonn, September 9-13, 1996

In the past four years the United Nations (UN) in cooperation with the European Space Agency (ESA) organized a series of Workshops on *Basic Space Science* for the benefit

of Third World countries in four regions on Earth.

The Workshop for 1996 is the sixth in the series of UN/ESA Workshops on Basic Space Science and will focus on the principal three windows towards the Universe. The Workshop will also assess the accomplishments of this series of Workshops. It will be hosted by Germany, and will take place at September 9–13, 1996 at the

Gustav-Stresemann-Institut e.V.
Langer Grabenweg 68
D-53175 Bonn, Germany
phone: +49-228-8107-0
fax: +49-228-8107-197

Local organizer is the

Max-Planck-Institut für Radioastronomie
Auf dem Hügel 69
D-53121 Bonn, Germany
phone.: +49-228-525-0
fax: +49-228-525-438
email: bass@mpifr-bonn.mpg.de

Further information can be obtained by WWW at ftp://ecf.hq.eso.org/pub/un/gerannounce.html.

Hans J. Haubold
(haubold@ekpvs2.dnet.tuwien.ac.at)

11. III. International Conference on Functional Analysis and Approximation Theory: Acquafredda di Maratea, September 23-28, 1996

The meeting will be devoted to some significant aspects of contemporary mathematical research on Functional Analysis and Approximation Theory including the applications of these fields in other areas. Suggested topics include:

- Banach spaces, Banach lattices, function spaces
- (Positive) linear operators, semigroups of (positive) linear operators, evolution equations
- Integral equations, interpolation, approximate quadratures
- Approximation methods in abstract spaces and in function spaces, approximation by (positive) operators
- Constructive approximation
- Orthogonal polynomials

The scientific program will consist of invited survey talks (45 min.) and short communications (20 min.). The abstracts of all contributions and the program of the meeting will be available at the beginning of the meeting. The list of invited speakers and a preliminary list of all participants will be sent together with the second announcement. It is expected that the proceedings of the Conference will be published.

The meeting is organized in the sphere of activities of the *Center for Studies in Functional Analysis and Approximation Theory* of the University of Basilicata (Potenza), in collaboration with several other Italian Universities and Organisations.

The organizing committee consists of F. Altomare (University of Bari), M. Campiti (Polytechnic of Bari), G. Criscuolo (University of Napoli), B. Della Vecchia (University of Roma, La Sapienza), G. Mastroianni (University of Basilicata).

The full First Announcement, together with Preliminary Registration Form for participation and for giving a lecture, can be obtained from G. Mastroianni. The completed form should reach the following address as soon as possible.

Prof. Michele Campiti
Dipartimento di Matematica, Universita di Bari
Campus Universitario
Via Edoardo Orabona 4
70125 Bari, Italy
email: campiti@pascal.uniba.it

G. Mastroianni
(mastroianni@pzvx85.cisit.unibas.it)

12. XVth Workshop on Geometric Methods in Physics, Bialowieza, Poland, July 1-7, 1997

The main emphasis will be placed on the following topics: Geometric quantization, coherent states, q-special functions, quantum groups, theory of singularities, wavelets, symplectic and Poisson structures, quantization via $*$-products.

If you are interested to receive further information, please send an email to A. Strasburger (secretary of organizing committee).

Alexander Strasburger
(alekstra@fuw.edu.pl)

Books and Journals

1. Annals of Numerical Mathematics 2, 1–4
2. Numerical Algorithms 10, 1–2
Edited by G. Allasia
Editor-in-Chief: Claude Brezinski
Baltzer Science Publishers, Amsterdam, The Netherlands, 1995.

These books contain the Proceedings of an International Symposium on Special Functions, held at the University of Torino in October 1993 on the occasion of the 70th birthday of Luigi Gatteschi. The two volumes include 11 invited lectures and 31 contributed papers by 66 authors from many countries, for a grand total of over 700 pages.

The first volume opens with a brief Preface including an extract from a letter by Richard Askey describing the connection between Torino and Special Functions going back to the work of Lagrange continued in the 20th century by Tricomi and more recently by Gatteschi. This is followed

by a photograph of Gatteschi and a list of his publications. Walter Gautschi's opening article "Luigi Gatteschi's work on special functions and numerical analysis" goes beyond its title to include some remarks of a biographical nature. Gatteschi's substantial contribution to the subject of error terms in asymptotic approximations of special functions is partially reviewed in Roderick Wong's paper "Error bounds for asymptotic approximations of special functions". Among the other more substantial invited papers are one on the weights of positive quadrature formulas for ultraspherical weight functions by K.–J. Förster and one on orthogonal polynomials on weighted Sobolev spaces by F. Marcellán, T.E. Pérez and M.A. Piñar. Among the contributed papers in the first volume, those which caught the eye of this reader were "Maximum principles and inequalities for special functions" by D. Kershaw, "A monotonicity property of ultraspherical Christoffel numbers" by J. Korevaar and "The Bieberbach conjecture", a mainly expository paper with a conjecture on bi–univalent functions, by H.V. Smith.

The tables of contents of these volumes may be found on the World Wide Web:
http://www.NL.net/~baltzer/anuma.2.html
http://www.NL.net/~baltzer/numa.html

These volumes form a fitting tribute to Luigi Gatteschi; we are indebted to Giampietro Allasia for making their publication possible.

Martin Muldoon
(muldoon@mathstat.yorku.ca)

3. H. T. Lau: A Numerical Library in C for Scientists and Engineers
CRC Press, Inc., Boca Raton, Florida, 1995, hardcover, 816 pp., ISBN 0-8493-7376-X, US$69.95, outside USA $84.00.

This book contains an extensive coverage of computer programs in numerical computing, with several hundred procedures including areas in

- Linear Algebra
- Ordinary and Partial Differential Equations (stiff and non-stiff systems)
- Optimization
- Parameter Estimation
- Special Functions in Mathematical Physics.

A diskette is included with all the source code.

Address of the publisher:

CRC Press, Inc.
2000 Corporate Blvd., N.W.
Boca Raton, FL 33431
USA
phone: +1-407-994 0555

In Europe the book is distributed by

Times Mirror Int'l Publishers Ltd
Customer Service Centre, Unit 1
3 Sheldon Way, Larkfield, Aylesford
Kent ME20 6SF
United Kingdom
phone: +44-171-391 6464
fax: +44-171-391 6599

T. Hanks
(be478@freenet.carleton.ca)

4. Zahar, R.: Approximation and Computation. A Festschrift in Honor of Walter Gautschi
Birkhäuser, Basel, Switzerland, 1995, hardcover, 640 pp., ISBN 3-7643-3753-2, sFr. 168.00.

In recognition of Walter Gautschi's many contributions to mathematics and computer sciences, and to celebrate his sixty-fifth birthday, an international conference was held in his honor at Purdue University in December 1993. From that meeting has arisen this collection of invited papers, providing an intimate view of the work of 60 outstanding scientists from sixteen countries, and standing as a tribute by his colleagues and students to Walter Gautschi.

The book provides an extensive survey of the most current topics in the fields of approximation, orthogonal polynomials, quadrature, and special functions. It is a uniquely uniform representation of a broad spectrum of mathematical subjects.

Wolfram Koepf
(koepf@zib-berlin.de)

Software Announcements

1. Software Testing Service for Special Functions

This is a project of Daniel W. Lozier. Its purpose is to begin to develop software at the National Institute of Standards and Technology for use in testing the accuracy, or numerical precision, of mathematical software for special functions. The service would use the World Wide Web to receive test requests and return test results. The tests would be run on a network of workstations at the Institute. It is hoped that the service will be of practical utility to anyone who uses special functions in physics or other applications, and that it will stimulate the interest of applied mathematicians who are interested in computation of special functions as well as computer scientists who are interested in innovative uses of the Internet.

References (Chronological Order):

[1] Numerical Evaluation of Special Functions, by D.W. Lozier and F.W.J. Olver, gives a simple classification of special functions and a cross-index to published algorithms and software for their numerical evaluation. The original reference is W. Gautschi, Mathematics of Computation 1943–1993: A Half-Century of Computational Mathematics, AMS Proceedings of Symposia in Applied Mathematics 48 (1994), 79–125. It is presented in hyper-

text on WWW to assist in locating information about specific special functions. See: http://math.nist.gov/nesf/.

[2] Software Needs in Special Functions, by D.W. Lozier, will appear in the Journal of Computational and Applied Mathematics in 1996. A preprint, NISTIR 5490, National Institute of Standards and Technology, August 1994, is available from the author.

[3] Software Testing Service for Special Functions: A Proposal and Request for Comments, by D.W. Lozier, provides details of the proposed testing service and asks for comments on its feasibility and utility. This document is an informal working paper that is not intended for ordinary publication. Date: October 1995. See WWW: http://math.nist.gov/stssf/.

The views of all interested individuals about this proposal are sought. Comments on the feasibility, utility, or any other aspect of the proposal can be sent to the author.

Daniel W. Lozier
(lozier@nist.gov)

2. Math Constants Web Resource

The Favorite Mathematical Constants web page
http://www.mathsoft.com/asolve/constant/constant.html
is an evolving collection of essays about constants, other than π and e, which appear throughout mathematics. Well-known examples are due to Euler, Catalan, Khintchine and Feigenbaum, but there are many others too.

The essays are accessible to advanced undergraduates and intended to be starting points for continuing research (so bibliographies are as complete as possible).

Please drop by and, of course, leave comments or suggestions on how to improve these pages. I strongly recommend using Netscape 1.1 or higher.

As a teaser, I offer the following: Can the infinite product $1^{1/1} \cdot 3^{1/9} \cdot 5^{1/25} \cdot 7^{1/49} \cdot 9^{1/81} \cdots$ be expressed in terms of known constants?

Steven Finch
(sfinch@mathsoft.com)

Problems and Solutions

Thus far 13 problems have been submitted five of which have been solved (#**1**, **4**, **6**, **7**, **10**).

2. Is it true that

$$x^2 t^x \, {}_2F_1(x+1,\, x+1;\, 2;\, 1-t)$$

is a convex function of x whenever $-\infty < x < \infty$ and $0 < t < 1$?

Submitted by George Gasper, August 19, 1992.
(george@math.nwu.edu)

3. The following Toeplitz matrix arises in several applications. Define for $i \neq j$

$$A_{ij}(\alpha) = \frac{\sin \alpha\pi(i-j)}{\pi(i-j)},$$

and set $A_{ii} = \alpha$. Conjecture: the matrix

$$M = (I - A)^{-1}$$

has positive entries. A proof is known for $1/2 \leq \alpha < 1$. Can one extend this to $0 < \alpha < 1$?

Submitted by Alberto Grünbaum, November 3, 1992.
(grunbaum@math.berkeley.edu)

5. The result of Problem #**4** can be generalized to

$$\begin{aligned} S_m &= \sum_{n=0}^{\infty} \frac{(-1)^n (mn + 1/2)!}{\sqrt{\pi}\,(mn+1)!} \\ &= \frac{1}{m} \sum_{k=0}^{m-1} \frac{\sin\left(5(2k+1)\pi/(4m) + \pi/4\right)}{[2\sin\left((2k+1)\pi/(2m)\right)]^{1/2}} \end{aligned}$$

valid for integral $m \geq 2$.

Submitted by J. Boersma and P.J. de Doelder, July 12, 1993.
(wstanal@win.tue.nl)

8. Can the real and imaginary parts of a hypergeometric series of type ${}_pF_q$ with one complex parameter (either in the numerator or the denominator) be expressed by means of multiple hypergeometric series?

Submitted by Ernst D. Krupnikov, July 25, 1993.
(ernst@net.neic.nsk.su)

9. Prove or disprove: The functions

$$H_n(t) = (-1)^n \, F_n(nt)$$

that are defined in terms of the *Bateman functions*

$$\begin{aligned} F_n(t) &= e^{-t} \left(L_n(2t) - L_{n-1}(2t) \right) \\ &= -e^{-t} \, \frac{2t}{n} \, L_{n-1}^{(1)}(2t) \\ &= (-1)^n \frac{2}{\pi} \int_0^{\pi/2} \cos\left(t \, \tan\theta - 2\,n\,\theta\right) d\theta \, , \end{aligned}$$

$L_n^{(\alpha)}(t)$ denoting the *generalized Laguerre polynomials*, have the property that $H_n(t_0)$ is strictly decreasing with increasing n at the point $t_0 = 2$. Note that this is not true for any $t_0 < 2$, but on the other hand at $t_0 = 2$ seems to be numerically evident. Note further that $H_n(t)$ satisfy the simple differential equation

$$t\, H_n''(t) = n^2 \, (t-2) \, H_n(t) \, .$$

The differential equation demonstrates the importance of the point $t_0 = 2$.

These functions occur particularly in the study of nonvanishing analytic functions of the unit disk. For more details on these functions, see Koepf, W. and Schmersau, D.: Bounded nonvanishing functions and Bateman functions, Complex Variables **25** (1994), 237–259.

Submitted by Wolfram Koepf, February 10, 1995.
(koepf@zib-berlin.de)

11. A Bessel–Laplace Transform. For $p \geq 2$ show that

$$I = \int_0^\infty e^{-pt} J_0(t) K_0(at) dt = \frac{1}{2p} \mathbf{K}(k)$$

where

$$a = \sqrt{p^2 - 1 \pm p\sqrt{p^2 - 4}}$$

$$k = \frac{1}{p}\sqrt{\frac{1}{2}p^2 + 1 \mp \frac{1}{2}p\sqrt{p^2 - 4}}\,.$$

Submitted by Lawrence Glasser, Nov 9, 1995.
(laryg@sun.mcs.clarkson.edu)

12. Absolutely monotonic function. The functions $t_n(x)$ generated by

$$\frac{1}{(1-z)^2\left(1 - z + \sqrt{1 - 2xz + z^2}\right)^\alpha} = \sum_{n=0}^{\infty} t_n(x)\, z^n$$

are nonnegative for $\alpha \geq 0$, $x \in [-1, 1]$. Can this be proved by elementary means?

Submitted by Dieter Schmersau, January 8, 1996.
(schmersau@math.fu-berlin.de)

13. Product of Chebyshev Polynomials. For any pair of positive even $n, m \in \mathbb{N}$ let

$$\begin{aligned} F^{(n,m)}(x) &= 2^n \prod_{k=0}^{n-1} \cosh\left(\frac{m}{2} \operatorname{arccosh}\left(x - \cos\frac{(2k+1)\pi}{n}\right)\right) \\ &= 2^n \prod_{k=0}^{n-1} T_{m/2}\left(x - \cos\left(\frac{(2k+1)\pi}{n}\right)\right)\,, \end{aligned}$$

where $T_m(x)$ denotes the Chebyshev polynomials of the first kind. These functions occur in statistical physics. They constitute polynomials in x

$$F^{(n,m)}(x) = \sum_{j=0}^{nm/4} A_j(n,m)\, x^{2j}$$

whose coefficients $A_j(n,m)$ are integers. Show the symmetry

$$F^{(n,m)}(x) = F^{(m,n)}(x)\,,$$

and give a representation of the coefficients $A_j(n,m)$.

Submitted by Christian Hege, January 16, 1996.
(hege@zib-berlin.de)

Miscellaneous

1. Special Functions ftp Site

The anonymous ftp site at `euler.math.ualberta.ca` has been transferred to the new anonymous ftp site at `unvie6.un.or.at`, directory `siam`.

The new ftp site can also be accessed by using a World Wide Web browser (Mosaic, Netscape, or their X Window implementations) at the address `ftp://unvie6.un.or.at/siam`.

Manuscripts can be obtained from directory `opfs` and subdirectories. In particular, examine the file `00Contents.ftpsite` in the `submissions` directory.

You are invited to submit one or more of your not-yet-in-print manuscripts which you wish to make available to the OPSF community. They should be in TEX, LATEX, AMSTEX, or AMSLATEX format. These manuscripts can be submitted by one of the following methods:

a) anonymous ftp to `unvie6.un.or.at`;
then `cd siam/submissions`;
then `put file` where `file` is the name of the file you wish to deposit.
b) email to `haubold@ekpvs2.dnet.tuwien.ac.at`.

Hans J. Haubold
(haubold@ekpvs2.dnet.tuwien.ac.at)

2. CD-ROM Version of Gradshteyn & Ryzhik

At the end of January 1996 Academic Press will release a CD-ROM version of the book *Table of Integrals, Series, and Products*, Fifth Edition by I.S. Gradshteyn and I.M. Ryzhik. It will be available for the most common platforms (MS-Windows, Macintosh and Unix).

See further information in recent advertisements of Academic Press or mail to a Product Manager of Academic Press: Nomi Schalit (nschalit@acad.com) for North America, or Rachel Bridgman, (rachael.bridgman@hbuk.co.uk) for Europe, Middle East and Africa.

Tom H. Koornwinder
(thk@fwi.uva.nl)

3. WWW Home Page

Our Activity Group has started its own home page at the World Wide Web. The WWW address is:
http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/
At the moment Martin Muldoon (muldoon@mathstat.yorku.ca) is in charge of it. If you may have already your own home page, you might consider to link it to ours.

The home page contains a comprehensive list of ftp and WWW addresses relevant for our field under "List of WWW pages of interest to members".

This list will be regularly updated. Please mail corrections and additions to Martin Muldoon.

Martin Muldoon
(muldoon@mathstat.yorku.ca)

4. Szegő Memorial

Pictures of the Szego bust can be viewed on WWW: http://www.math.ohio-state.edu/~nevai/SZEGO.

Please also consult that site for further articles and documentation in connection with Gábor Szegő's centennial birthday.

Paul Nevai
(nevai@math.ohio-state.edu)

5. Askey-Wilson Computer Algebra Mini-Project

During the period January-June 1996 René Swarttouw is working at *RIACA*, Eindhoven, The Netherlands, on a project *A computer implementation of the Askey-Wilson scheme*. This project was proposed by Tom Koornwinder. The purpose of the project is to make a start with bringing the report

R. Koekoek and R.F. Swarttouw
The Askey-scheme of hypergeometric orthogonal polynomials and its q-analogue
Report 94–05, Technical University Delft, Faculty TWI, 1994

in the form of an interactive book, with facilities for symbolic manipulation of formulas.

Further information from Tom Koornwinder or from René Swarttouw (renes@can.nl).

By the way, the Delft TWI Report 94-05 mentioned above can now be electronically retrieved by ftp:

ftp ftp.twi.tudelft.nl
cd TWI/publications/tech-reports/1994
binary
get DUT-TWI-94-05.ps.gz

which can also be reached by WWW:
ftp://ftp.twi.tudelft.nl/TWI/publications/tech-reports/1994/

You will thus obtain the compressed postscript file of the original hardcopy version. Later errata are not yet incorporated.

Tom H. Koornwinder
(thk@fwi.uva.nl)

6. Research Problems Section of Constructive Approximation on WWW

The *Journal of Constructive Approximation* has started on its WWW home page http://www.math.usf.edu/CA/index.html a Research Problems section, edited by Peter Borwein, Ingrid Daubechies and Vilmos Totik.

The Research Problems section will publish open problems. Ideally, these problems are easy to state, substantial but not impossible, and of wide appeal. Problems need not be new.

Manuscripts should be between one and four TEX pages. Enough references should be included that the reader can access the recent literature. All submissions will be refereed and reprints will be supplied. Solutions will not, as a matter of course, be published in *Constructive Approximation*.

Submissions for the Research Problems section should be made electronically (Research_Problems@cecm.sfu.ca) in any standard version of TEX or LATEX or by using the *Constructive Approximation* macros (please avoid using unusual macros or style files). Queries may also be directed to the above email address.

Tom H. Koornwinder
(thk@fwi.uva.nl)

Impression from the Mittag-Leffler Institute, Sweden

by Tom H. Koornwinder
(thk@fwi.uva.nl)

During almost two months I am staying here at the Mittag-Leffler Institute in Djursholm, Sweden. Djursholm is a suburb north of Stockholm, with luxurious villas on big lots, and beautifully located at the Stockholm archipelago. The institute is housed in the villa which was built in 1890 for the Swedish mathematician Gösta Mittag-Leffler (1846–1927) and which was drastically reconstructed and extended several times during the next fifteen years. Mittag-Leffler and his Finnish wife Signe, née af Lindfors, lived here for the rest of their life. The villa is towering on a hill and makes the impression of a small castle. From the beginning the library took a central place in the villa, and the whole architecture is a function of the necessity to house a library which was the largest private mathematics library in the world.

Mittag-Leffler is well-known by his work in function theory, but his greatest merit for mathematics is probably in his extensive international contacts with the top mathematicians of that time and in his founding (in 1881) and editing of the journal Acta Mathematica. From the beginning on, first-rate contributions from leading mathematicians in France and Germany were obtained, and the journal is still considered as one of the highest rank mathematics journals in the world.

In 1916 Mittag-Leffler and his wife set up a foundation to promote research in pure mathematics in the Scandinavian countries (see G. & S. Mittag-Leffler, Testament 16/3 1916, Acta Math. 40 (1916), III-X). The foundation was to maintain the large library in the villa and to support a research institute there with several professors, and with fellowships for younger mathematicians. In 1916 the plans for the Institute were realistic at least in that Mittag-Leffler's financial resources were adequate for the task. However, in 1922 there was a large financial crash related to the economic crisis in Europe at the time. The crash brought Mittag-Leffler near bankruptcy and at his death in 1927 the resources did not allow the realisation of his original intentions.

The Royal Swedish Academy of Sciences, which had incorporated the Institute in 1919, appointed Torsten Carleman as Director of the Institute. Until 1969, the activities of the Institute were mainly restricted to maintaining the library and editing Acta Mathematica. In 1969 outward funding was obtained by which the new director Lennart Carleson could finally realise Mittag-Leffler's intentions. From then on, a topic is chosen for every year. Within this field experts are invited to work at the Institute for periods of one or more months, and fellowships are made available for post-docs and graduate students. When one looks at the list of programs of the past twenty-five years, one sees topics from analysis dominating, while algebraic geometry has also been repeatedly a topic of concentration. The program

for the present academic year is Analysis on Lie Groups.

The library is really marvellous, both as a piece of architecture and because of its wealth of older books and journals. It also tries to keep up with the present-day flood of books and journals, but it succeeds of course only partially. Applied mathematics is not represented very strongly. The library does not subscribe to any of the SIAM journals, even not to the SIAM Journal on Mathematical Analysis.

As for Acta Mathematica, it is very interesting to look at the *Table générale des tomés 1–35* which was published in 1913. For all authors in alphabetical order one finds there a vitae and a list of publications which appeared in Acta. There is a picture gallery of the authors as well. Various well-known authors in orthogonal polynomials and special functions can be found there, for instance Thomas Stieltjes. In Volume 2 (1883) one already finds a paper by E. Goursat *Sur une classe de fonctions representées par des integrales définies*, and in Volume 3 (1883) a short note by H. Mellin *Eine Verallgemeinerung der Gleichung* $\Gamma(1+x)\Gamma(1-x) = \pi x/\sin \pi x$. Even in the just published issue of Acta Mathematica special functions do occur in E. Opdam's paper *Harmonic analysis for certain representations of graded Hecke algebra*, Acta Math. 175 (1995), 75-121. Although the outsider would not guess this from the title, what really happens here is generalizing the Plancherel formula and the Paley-Wiener theorem for the Mehler-Fock transform to an integral kernel involving Jacobi functions associated with an arbitrary root system (the so-called Heckman-Opdam hypergeometric functions).

Further information about the Mittag-Leffler Institute and its history can be obtained from WWW: http://www.ml.kva.se.

Discrete Grosjean Orthogonal Polynomials

by I. Area, E. Godoy, A. Ronveaux, A. Zarzo
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Andre.Ronveaux@fundp.ac.be, azarzo@ccupm.upm.es)

The Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ with $\alpha+\beta+1=0$ and $-1<\alpha<0$ called *Grosjean polynomials of the first kind* $G_n^{\alpha}(x)$ are the only one parameter family of Jacobi polynomials such that the first associated family $(P_n^{(\alpha,\beta)}(x))^{(1)}$ is inside the Jacobi family $\alpha+\beta-1=0$ (see [1]).

From the well known limit relation between Hahn polynomials $h_n^{(\alpha,\beta)}(x;N)$ and the Jacobi polynomials ([1], (2.6.2))

$$\lim_{n\to\infty} \frac{1}{N^n} h_n^{(\alpha,\beta)}\left(\frac{N}{2}(1+s);N\right) = P_n^{(\alpha,\beta)}(s)\ ,$$

a simple question arises naturally: Is this property true for Hahn polynomials? The answer is yes but with slight differences and it is easily seen if the problem is presented in this way:

Let $\gamma_n^H(\alpha,\beta,N)$ and $\beta_n^H(\alpha,\beta,N)$ be the recurrence coefficients for monic Hahn polynomials $H_n^{(\alpha,\beta)}(x;N)$ of degree n obtained by the three term recurrence relation:

$$\begin{aligned} xH_n^{(\alpha,\beta)}(x;N) &= H_{n+1}^{(\alpha,\beta)}(x;N) + \beta_n^H(\alpha,\beta,N)\, H_n^{(\alpha,\beta)}(x;N) + \\ &\quad \gamma_n^H(\alpha,\beta,N)\, H_{n-1}^{(\alpha,\beta)}(x;N), \quad (n\geq 2) \end{aligned}$$

with $H_0^{(\alpha,\beta)}(x;N)=1$ and $H_1^{(\alpha,\beta)}(x;N)=x-\beta_0^H$. The aforementioned question is equivalent to find the set of parameters $\bar\alpha,\bar\beta,\bar N$ where $\bar\alpha>-1$, $\bar\beta>-1$ and $\bar N$ integer such that:

$$\begin{aligned} \gamma_{n+1}^H(\alpha,\beta,N) &= \gamma_n^H(\bar\alpha,\bar\beta,\bar N) \\ \beta_{n+1}^H(\alpha,\beta,N) &= \beta_n^H(\bar\alpha,\bar\beta,\bar N) + X(\alpha,\beta,N), \end{aligned}$$

where the explicit values of $\gamma_n^H(\alpha,\beta,N)$ and $\beta_n^H(\alpha,\beta,N)$ are

$$\begin{aligned} \beta_n^H &= \frac{\alpha-\beta+2N-2}{4} + \frac{(\beta^2-\alpha^2)(\alpha+\beta+2N)}{4(\alpha+\beta+2n)(\alpha+\beta+2n+2)} \\ \gamma_n^H &= \frac{n(N-n)(\alpha+n)(\beta+n)(\alpha+\beta+n)(\alpha+\beta+N+n)}{(\alpha+\beta+2n-1)(\alpha+\beta+2n)^2(\alpha+\beta+2n+1)}\ . \end{aligned}$$

Then, we found after some computations:

$$\begin{aligned} \bar\alpha+\bar\beta &= \alpha+\beta+2=1 \\ \bar N &= N-1\ , \end{aligned}$$

and so, the following relation holds

$$[H_n^{(\alpha,-1-\alpha)}(x;N)]^{(1)} = H_{n-1}^{(-\alpha,\alpha+1)}(x-\alpha-1;N-1)\ .$$

As in the Grosjean case, the first associated of

$$H_n^{(\alpha,-1-\alpha)}(x;N) := GD_n^{\alpha}(x)$$

(let us denote by GD_n^{α} the *discrete Grosjean polynomials of first kind*) is, in the (α,β) Hahn plane, symmetrical with respect to the origin (0,0). Mimicking the Jacobi plane we call gD_n^{α} the *discrete Grosjean polynomials of second kind*, located on the line $\alpha+\beta-1=0$.

It is interesting to notice that the three discrete families, in the (α,β) Hahn plane, $(\frac{+1}{2},\frac{+1}{2}),(\frac{+1}{2},\frac{-1}{2}),(\frac{-1}{2},\frac{1}{2})$ corresponding to the other Chebyshev families which are not contained in the line $\alpha+\beta+1=0$ for which the associated is $(\frac{+1}{2},\frac{+1}{2})$ in the Jacobi case, have no analogous in the Hahn case.

Any modification inside the Jacobi matrix built with $GD_n^{\alpha}(x)$ can now be written as a combination of the polynomials $GD_n^{\alpha}(x;N)$ and $gD_n^{-\alpha}(x-\alpha-1;N-1)$.

The technique used in order to find the orthogonality measure in the (continuous) Grosjean case, coming from asymptotics is no more available in this case ($n<N$).

Is there an alternative way to approach this problem?

Acknowledgement: We are pleased to thank W. Van Assche for helpful indications.

References

[1] Nikiforov, A.F., Suslov S.K. and Uvarov, V.B.: *Classical Orthogonal Polynomials of a Discrete Variable*. Springer-Verlag, Berlin, 1991.

[2] Ronveaux, A. and Van Assche, W.: Upward extension of Jacobi matrix for orthogonal polynomials. J. Approx. Theory (in print).

A Note on the Zeroes of Grosjean Polynomials

by A. Ronveaux, J.S. Dehesa, A. Zarzo and R.J. Yáñez
(Andre.Ronveaux@fundp.ac.be, dehesa@ugr.es,
azarzo@upm.es, Carlos1@ugr.es)

The Grosjean polynomials of the first kind $G_n^{\alpha}(x)$ are Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ with $\alpha+\beta+1=0$ $(-1<\alpha<0)$. These polynomials share with Chebyshev polynomials of the first kind $T_n(x)$ two linked properties: the first associated of $G_n^{\alpha}(x)$ is, in the Jacobi (α,β) plane, symmetric with respect to the origin (cfr. Fig. 1), and for an arbitrary finite perturbation of the Jacobi matrix corresponding to $G_n^{\alpha}(x)$, the orthogonality measure can be given explicitly [1].

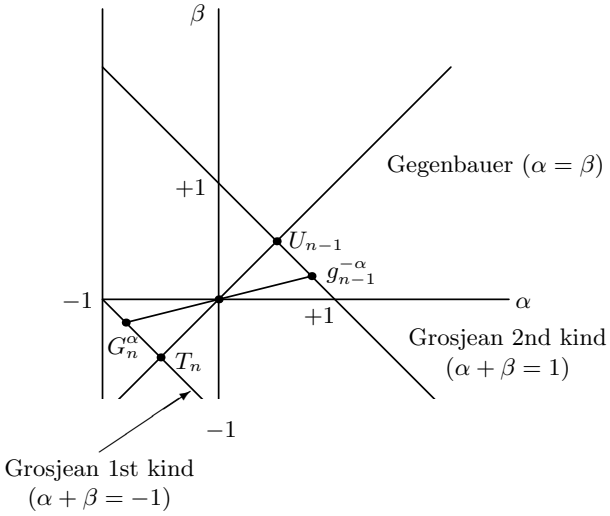

Figure 1: Jacobi (α,β) plane.

The zeroes of $G_n^{\alpha}(x)$ are also connected to the zeroes of $T_n(x)$ via the Laplace-Heine (L-H) formula:

$$\sqrt{n\pi}P_n^{(\alpha,\beta)}(\cos\theta)=\left(\sin\frac{\theta}{2}\right)^{-(\alpha+1/2)}\left(\cos\frac{\theta}{2}\right)^{-(\beta+1/2)}\times$$
$$\cos\left((n+(\alpha+\beta+1)/2)\theta-(\alpha+1/2)\frac{\pi}{2}\right)+\mathrm{O}(\frac{1}{n})$$

in which, for the Grosjean case, the argument in the cosine function can be written as: $n\left(\theta-(\alpha+1/2)\frac{\pi}{2n}\right)$. The rest in the L-H formula is exactly equal to zero when $\alpha=-1/2$, i.e. for Chebyshev polynomials, and this is the reason why the zeroes $x_{j,n}^{T}$ of $T_n(x)$ are given by the projection on the x-axis of the points $M_{j,n}$ of the unit circle defined by the angle $\theta_{j,n}=\frac{\pi}{2n}+j\frac{\pi}{n}$, $(j=0,\ldots,n-1)$. It is therefore tempting to ask: How far are the zeroes $x_{j,n}^{G}$ of Grosjean polynomials from the projection of $M_{j,n}$ on the x-axis "rotated" by an angle of $(\alpha+1/2)\frac{\pi}{2n}$? (cfr. Fig. 2). Of course, the answer to this question is closely related with the following one: How small is the rest in the Laplace-Heine formula in the Grosjean case?

A crude analysis of the moment of order k of the zeroes

$$\mu_k^{(n)}=\frac{1}{n}\sum_{j=1}^{n}(x_{j,n})^k$$

in accordance with the Van Assche-Teugels theorem, shows that

$$\mu G_{2k+1}^{(n)}\approx\frac{\lambda}{n}\quad(\lambda=\alpha+\frac{1}{2})\ \text{ and }\ \mu G_{2k}^{(n)}=\mu T_{2k}^{(n)}+\mathrm{O}(\frac{1}{n^2})\,.$$

where μG and μT denote the moments for Grosjean and Chebyshev polynomials, respectively. This behaviour (for even k) is better, for instance, than in the Gegenbauer case, and could also be the best one among all straight lines crossing the point $(-1/2,-1/2)$ in the Jacobi (α,β) plane, for Jacobi polynomials of the form $P_n^{(\alpha,\beta)}(x)$ with $\beta=t(\alpha+1/2)-1/2$.

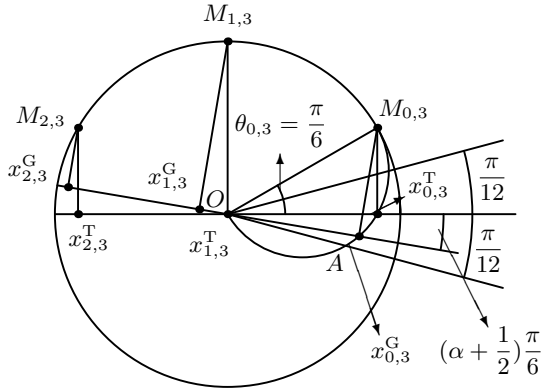

Figure 2: Semi-circles of Grosjean Roots: *For each $\theta_{j,n}$, the zeros $x_{j,n}^{G}$ of the Grosjean Polynomials (for all α with $-1<\alpha<0$) are "close" to the circle of diameter $\overline{OM_{j,n}}$. The arc $\overset{\frown}{x_{0,3}^{T}A}$ of the $M_{0,3}$-circle corresponds to the range $-1/2<\alpha<0$.*

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References

[1] Ronveaux, A. and Van Assche, W.: Upward extension of Jacobi matrix for orthogonal polynomials. J. Approx. Theory (in print).

How to Contribute to the Newsletter

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preferably by email, and in LaTeX format. Other formats are also acceptable and can be submitted by email, regular mail or fax.

Deadline for submissions to be included in the June issue 1996 is May 15, 1996.

The Activity Group also sponsors an electronic news net, called the **OP-SF Net**, which is transmitted periodically by SIAM. The Net provides a rather fast turnaround compared to the Newsletter. To *receive* transmissions, just send your name and email address to poly-request@siam.org (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net *contributions* should be sent to poly@siam.org. Please note that submissions to OP-SF Net are automatically

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The Net is organized by Tom Koornwinder (thk@fwi.uva.nl). Back issues of OP-SF Net can be obtained by anonymous ftp from `ftp.fwi.uva.nl`, in the directory

pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

or by WWW at the addresses

ftp://ftp.fwi.uva.nl/pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

http://www.math.ohio-state.edu/JAT

Martin Muldoon has agreed to become a co-editor of OP-SF Net, starting with the next issue. He, moreover, manages our home page

http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/

on World Wide Web.

We are considering to make better use of World Wide Web for our purposes. Possibly, back issues of OP-SF Net and the Newsletter will be integrated, marked up and hyperlinked so that you can easily browse the Web for matters relevant to our field. Our plans are still somewhat indefinite, but we recommend that you visit our home pages regularly so as to keep abreast of new information as we install it.

Activity Group: Addresses

The *SIAM Activity Group on Orthogonal Polynomials and Special Functions* consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We now have around 150 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/ is managed by Martin Muldoon (muldoon@mathstat.yorku.ca).

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