Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

★ ★ ★ ★ **Newsletter** ★ ★ ★ ★

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Contents

From the Editor

The electronic media, and, in particular, the World Wide Web, influence the daily work of researchers all over the world more and more. This is reflected, for example, in the continually increasing number of personal WWW home pages of members of our Activity Group. In particular, both the Chair, Charles Dunkl, and the Vice Chair, Tom H. Koornwinder, now have their own WWW home pages whose addresses are listed on page 19.

I invite everybody to consult some of the WWW pages which can give interesting information about orthogonal polynomials and special functions. Rather than looking a formula up in a mathematical dictionary, the desired information could be online available to you!

Steven Finch is developing a WWW page on *Favorite Mathematical Constants* and their interrelations which was announced in the last issue of the Newsletter. His site includes a wealth of mathematical knowledge and can be visited at http://www.mathsoft.com/cgi-shl/constant.bat.

At http://www.cecm.sfu.ca/projects/ISC/ you have access to the *Inverse Symbolic Calculator* that looks up in a huge database to identify decimal numbers and uses modern algorithmic methods to generate algebraic identities between several of them. Using these methods, the formula

$$\pi = \sum_{k=0}^{\infty} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \left(\frac{1}{16} \right)^k$$

was discovered recently; it gives the opportunity to calculate far out coefficients of π's hexadecimal

expansion without the computation of early ones, using single precision arithmetic!

Victor Adamchik and Stan Wagon present a deduction of a generalization of the above formula using symbolic computation on their WWW page http://www.wri.com/~victor/articles/pi/pi.html.

David H. Bailey and Simon Plouffe who have discovered the above representation together with Peter Borwein have their own WWW page on *Recognizing Real Numbers* at http://www.cecm.sfu.ca/organics/papers/bailey/.

An archive on *Numbers* with details about history, theory, applications, as well as printed and electronic references can be found at http://archives.math.utk.edu/subjects/numbers.html containing e.g., a link to continued fractions.

Concerning families of orthogonal polynomials, René Swarttouw designed an interface to Maple which currently enables the generation of plots as well as an online computation of recurrence, differential and difference equations for orthogonal polynomials of the Askey-Wilson scheme, equipped with any standardization you may wish to use; see also p. 15. Implementations of the editor are used for this purpose. This interface can be visited at http://www.can.nl/~renes/CAOP.html.

You see there is a lot to discover! Strolling through these pages will furthermore link you to many other interesting sites. I hope you all can take advantage of the possibilities which these new media offer!

If you know about interesting WWW pages concerning orthogonal polynomials and special functions that I did not mention, please submit the corresponding addresses to the Editor. Our members should receive such information.

May 31, 1996 Wolfram Koepf

Memorial Note about Waleed Al-Salam 1926-1996

Friends and colleagues are saddened by news of the passing, a few months short of his 70th birthday, of Waleed Al-Salam, Professor Emeritus of Mathematics at the University of Alberta. Born on July 15, 1926 in Baghdad, Iraq, he died on April 14, 1996 in Edmonton, Alberta, Canada.

Waleed studied at the University of California, Berkeley, earning a Bachelor's degree in Engineering Physics in 1950 and a M.A. in Mathematics in 1951. He returned to Baghdad as an Instructor at the College of Science for a few years, before enrolling for the Ph.D. at Duke University. He completed the degree in 1958 with a thesis *On the Bessel Polynomials* written under the supervision of Leonard Carlitz. By this time he was already a regular contributor to the periodical literature with some 20 published articles on a variety of topics in orthogonal polynomials and special functions.

After completing his Ph.D., Waleed returned to the College of Science in Baghdad as Associate Professor. Coming back to North America in 1962, he eventually (1966) took a position at the University of Alberta, where he was Professor

of Mathematics from 1967 until his retirement in 1992. Waleed continued to contribute to several areas related to orthogonal polynomials; his CV lists over 80 articles. Areas covered by his work include characterization theorems (see his survey article in pp. 1–24 of P. Nevai, ed., Orthogonal Polynomials: Theory and Practice, Kluwer, 1990), Turan expressions, generating functions, summation formulas, q-analogs, and fractional operators. He and his collaborators did much work on various special and generalized systems of orthogonal polynomials. The Al-Salam-Carlitz polynomials (1965) are still frequently cited; see, e.g., the papers of R. Askey and S.K. Suslov in Lett. Math. Phys. 29 (1993), 123-132 and J. Phys. A 20 (1993), L693-L698. The Al-Salam-Chihara polynomials (1976) play an important role in the Askey-Wilson scheme of basic hypergeometric orthogonal polynomials.

Waleed supervised the Ph.D. work of Bill Allaway (1972) and Mourad Ismail (1974), and conducted joint work with a variety of people at Alberta and elsewhere. In later years, these included Ted Chihara, A. Verma, Mourad Ismail and Waleed's wife, Nadhla Al-Salam, also on the faculty at Alberta, and a member of this Activity Group.

On his retirement in 1992, Waleed decided to put his expertise and energy at the disposal of the orthogonal polynomials community by starting and maintaining an ftp site for papers in the area. This effort prospered and he continued to oversee it until last year when his failing health made it necessary to pass the task to Hans Haubold at the UN Office in Vienna. Waleed had been diagnosed with leukemia in 1993 and this was to reduce his active participation in conferences in subsequent years. Nevertheless, with Nadhla's constant support, he still managed to travel and old and new friends were able to benefit from his knowledge and enjoy his optimistic and humorous personality.

Martin Muldoon
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Memorial Note about Lawrence C. Biedenharn

Lawrence C. Biedenharn Jr., 73, a longtime member of Duke University's physics faculty and an internationally known researcher in theoretical physics, died Monday, Feb. 12, in Austin, Texas, after a lengthy battle with cancer. He had made his home in Texas in recent years.

Biedenharn became the youngest full professor on the Duke faculty—at age 38—when he was appointed in 1961. He remained at Duke until 1993, when he retired as James B. Duke professor of physics and subsequently moved to the University of Texas at Austin as adjunct professor.

A native of Vicksburg, Miss., he received both his bachelor's degree and his Ph.D. from the Massachusetts Institute of Technology and served in World War II as a Signal Corps officer and later on the staff of Gen. Douglas MacArthur in Japan.

Serving on the faculties of Yale University and Rice University before coming to Duke, he published six books and hundreds of research articles in the fields of nuclear physics and later mathematical physics. He also edited the Journal of Mathematical Physics for many years.

Biedenharn, Holman, and Louck showed how the classical work on ordinary hypergeometric series is intimately related to the irreducible representations of the compact group $SU(2)$. Similarly, they found $U(n)$ multiple series generalizations of one-variable hypergeometric summation and transformation theorems by comparing two ways of computing the matrix elements of multiplicity free Wigner and Racah coefficients in $U(n)$. This work was done in the context of the quantum theory of angular momentum and the special unitary groups $SU(n)$. This work motivated the far-reaching q-analogs that Milne, Gustafson, and their co-workers subsequently found. Applications of this later work include unified proofs of the Macdonald identities, constant term identities, multiple q-beta integrals, $U(n+1)$ and symplectic generalizations of the Bailey Transform and Bailey Lemma, classical matrix inversion results, numerous classical summation and

transformation theorems for one-variable q-series, Rogers-Ramanujan identities, and, finally, new infinite families of identities for sums of squares in classical number theory. His book about *Quantum Group Symmetry and q-tensor Algebras*, jointly written with M.A. Lohe, appeared recently, s. page 13. Biedenharn's work continues to motivate much of this recent research in multivariable orthogonal polynomials, special functions, and their applications.

Lawrence C. Biedenharn is survived by his wife of 45 years, Sarah; his son John; daughter Sally; and two grandchildren.

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Stephen C. Milne
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Reports from Meetings and Conferences

1. Umbral Calculus Workshop: MIT, April 22-23, 1996

A workshop on Umbral Calculus was held at M.I.T. (Cambridge, U.S.A.) on April 22–23 as an addition to the Rotafest (the celebration of Gian-Carlo Rota's 64th birthday). The workshop was organized by Daniel Loeb, Nigel Ray and Alessandro Di Bucchianico with much support from Rotafest organizer Richard Stanley and secretarial staff of M.I.T.

The goal of this workshop was to give an overview of the development since Rota's seminal papers on the subject in the early seventies and to present new developments. Although Umbral Calculus is classified under combinatorics in the AMS classification, it has applications to several fields of mathematics, including special functions. We refer to *Dynamical Survey 3* of the *Electronic Journal of Combinatorics* for more details.

Talks were delivered (in chronological order) by Brian Taylor, Marilena Barnabei, Heinrich Niederhausen, Luis Verde-Star, Jet Wimp, George Andrews, Mourad Ismail, Miguel Méndez, William Chen, Daniel Loeb, Ottavio D'Antona, Nigel Ray, Philip Feinsilver, Henryk Gzyl, Alessandro Di Bucchianico, and Jack Freeman. Below we will briefly summarize those talks related to orthogonal polynomials or special functions.

- Brian Taylor presented joint work with Gian-Carlo Rota on a rigorous foundation of the classical umbral calculus of the previous century. Brian illustrated the high computational power of this approach by giving one-line proofs of properties of the Bernoulli polynomials.
- Luis Verde-Star presented a very general Hopf algebra structure that he showed to be the underlying structure of many analytic methods such as Laplace transforms etc. As a nice application he gave a one-line calculation without induction of the integral $\int_0^\infty x^n\, e^{-x}\, dx$.
- Jet Wimp showed how cut operators on Laurent series form a powerful tool for proving hypergeometric identities.
- Mourad Ismail discussed various properties of the Askey-Wilson operators.
- William Chen showed how elementary identities and a clever use of the operator $\vartheta : p(x) \mapsto \frac{p(qx)-p(x)}{x}$ yields many basic hypergeometric identities and transformations.
- Daniel Loeb presented ongoing joint research with Gian-Carlo Rota and Alessandro Di Bucchianico on a basis-free infinite-dimensional umbral calculus. As a first application, he showed how to calculate integrals of the form $\int_{||x||=1} x_1^{d_1} \ldots x_n^{d_n}\, d\mu(x)$, where μ is an orthogonally invariant measure.
- Philip Feinsilver showed results from joint research with René Schott on linearization and abelianization of Lie algebras with various applications to hypergeometric functions.
- Alessandro Di Bucchianico gave an umbral approach to variance functions of exponential families (joint work with Daniel Loeb). He gave a new proof of the following theorem of Philip Feinsilver: The variance function of a natural exponential family is a polynomial of degree at most 2 if and only if the associated Sheffer polynomials are orthogonal.
- Jack Freeman showed an algebraic transform method that can be used to find orthogonal polynomials (with respect to a linear functional) within a certain class. He applied his method to the classes of Sheffer polynomials and Chebyshev-like polynomials.

The workshop was attended by approximately 40 to 60 people.

Alessandro Di Bucchianico
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2. CRM Workshop on Special Functions: CRM, May 21-26, 1996

This is a report on the *Atelier sur les méthodes algébriques et q-fonctions spéciales* (Workshop on the

Algebraic Methods and q-special Functions), Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada, May 21-26, 1996.

Monday, May 20, was a national holiday in Canada (Victoria Day) and this pushed the start of the workshop to Tuesday morning. Luc Vinet, co-organizer with Pavel Winternitz and director of the Centre, opened the session at 9:00, welcomed the participants and dedicated the workshop to the memory of Waleed Al-Salam, who passed away April 14 of this year (see p. 2 for an obituary). There were approximately seventeen invited speakers who gave hour-long talks; there were about fourteen contributed half-hour talks. David and Gregory Chudnovsky were invited but were unable to attend and their time on the program was taken by other events. Sergei Suslov was invited, could not attend, but Dick Askey delivered his lecture. Dick also gave his own lecture (more details later). There was indeed a heavy emphasis on q-special functions, both of one-variable Askey-Wilson type and of several-variable Macdonald type. The algebraic methods were highly refined and sophisticated, mostly based on root systems and associated mathematical objects such as double affine Hecke algebras. Here is an alphabetical list of the invited speakers and the titles of their lectures:

George Andrews: Plane partitions and MacMahon's partition analysis

Richard Askey: An inequality of Vietoris and some related hypergeometric sums

Ivan Cherednik: Spherical difference Fourier transform

Charles Dunkl: Intertwining operators and polynomials associated with the symmetric group

Pavel Etingof: Macdonald eigenvalue problem and representations of quantum $gl(n)$

Roberto Floreanini: Quantum algebras and generalized hypergeometric functions

Adriano Garsia: Polynomiality of the Macdonald q,t-Kostka coefficients: a short proof

Mourad Ismail: Moment problems and orthogonal polynomials

Tom Koornwinder: The A_1-tableau of Dunkl-Cherednik operators

Boris Kupershmidt: The great powers of q-calculus

Ian Macdonald: Symmetric and non-symmetric orthogonal polynomials

David Masson: Contiguous relations, continued fractions and orthogonality: a ten year journey up the Askey chart

Willard Miller, Jr.: Tensor products of q-superalgebras and q-series identities

Masatoshi Noumi: Raising operators for Macdonald polynomials

Eric Opdam: Spectral analysis of Hecke algebras

Siddhartha Sahi: Recent results on Jack polynomials and Macdonald polynomials

Dennis Stanton: q-orthogonal polynomials as moments

Sergei Suslov (talk delivered by R. Askey): Some basic hypergeometric series and q-Bessel functions.

Contributed talks were given by N. Atakishiyev, R. Chouikha, P. Floris, A. Grünbaum, K. Kadell, M. Kapilevich, J. LeTourneux, K. Mimachi, A. Odzijewicz, V. Spiridonov, A. Strasburger, N. Takayama, F. van Diejen, L. Vinet. Ian Macdonald started his lecture Tuesday morning and finished it on Wednesday. Dick Askey gave an extra half-hour talk on Wednesday containing an overview of Askey-Wilson polynomials. Tom Koornwinder used another one of the hours originally scheduled for the Chudnovsky's to discuss René Swarttouw's web site which gives access to a vast collection of formulas and references for the polynomials contained in the Askey tableaux ($q = 1$ and general q). The URL is http://www.can.nl/~renes/index.html, see also the software announcement on p. 15.

There were approximately sixty participants; as well some members of the Centre dropped in on the lectures. The languages spoken at coffee and in the hallways appeared to be English, Russian, French, Japanese, Dutch, Polish. The weather was mostly delightful with a few evening showers. The University is in a scenic location near the Mont Royal; the Pavillon André-Aisenstadt, which houses the Centre, is a beautifully designed and equipped academic building, with a wonderful view of the northwest of the city. Ian Macdonald gave the first lecture on Tuesday, Adriano Garsia gave the last one on Saturday, thus bracketing an intense period of leading-edge mathematics. It was generally agreed that the workshop was excellent both in organization and inspiration to the participants for future work. At the conclusion all applauded and thanked the organizers for this exciting conference.

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3. Special Functions at the AMS-BeNeLux Meeting: Antwerp, Belgium, May 22-24, 1996

From May 22 to 24, the Americam Mathematical Society and the Mathematical Societies of the Benelux countries (Belgium, The Netherlands, and Luxemburg) had a joint meeting in Antwerp, Belgium. The meeting was rather successful, with about 500 people attending the conference. In the mornings there were plenary speakers. Special sessions and contributed papers were planned during the afternoons, with about 12 parallel sessions each afternoon.

There was a special session on wednesday May 22 on *Special Functions*, organized by Marcel G. de Bruin and Walter Van Assche. About 20 people attended this special session. The first two talks were on Sobolev orthogonal polynomials. Marcel de Bruin talked about *continuous Sobolev-Laguerre polynomials and their generalized continued fractions*. He looked at orthogonal polynomials for the inner product

$$\begin{aligned}\langle f,g\rangle &= \int_0^\infty f(x)g(x)x^\alpha e^{-x}\,dx \\ &\quad + \lambda\int_0^\infty f'(x)g'(x)x^\alpha e^{-x}\,dx.\end{aligned}$$

The monic orthogonal polynomials satisfy a four-term recurrence relation, and from this Marcel de Bruin studied the corresponding generalized continued fraction and the connection with simultaneous rational approximation. Henk G. Meijer then talked about *coherent pairs for Sobolev polynomials*. Coherent pairs were introduced by Iserles et al. in 1991. If (μ_1,μ_2) is a coherent pair of measures, then the Sobolev orthogonal polynomials for the inner product

$$\begin{aligned}\langle f,g\rangle &= \int f(x)g(x)\,d\mu_1(x) \\ &\quad + \lambda\int f'(x)g'(x)\,d\mu_2(x)\end{aligned}$$

can be studied in detail starting from the orthogonal polynomials for the measure μ_1 (or the measure μ_2). The pair (μ,μ), where $d\mu(x)=x^\alpha e^{-x}\,dx$ on $[0,\infty)$, studied by de Bruin in the previous talk, is an example of a coherent pair. Meijer recently succeeded in finding all coherent pairs. He showed that in a coherent pair, one of the measures needs to be classical (Jacobi, Laguerre, or Hermite), and then the other measure is a perturbation of the classical measure (addition of a point mass, multiplication or division by a polynomial of degree one). Herman Bavinck then talked about *A new result for Laguerre, Charlier, and Meixner polynomials*. He showed that for all $x,\alpha\in\mathbb{C}$

$$\sum_{k=j}^{i} k^s L_{i-k}^{(-\alpha-i-1)}(-x)L_{k-j}^{(\alpha+j)}(x)=0,$$

for all $i,j,s\in\mathbb{N}$, provided that $i>2s+j$, and gave similar results for Meixner polynomials and Charlier polynomials.

The next two talks were on multiple hypergeometric series. First Joris Van Der Jeugt talked on *Transformation and summation formulas for multiple hypergeometric functions*. His starting point is the quantum theory of angular momentum. He recalled the 3-j and the 6-j coefficients and indicated that they are related to the Hahn polynomials and the Racah polynomials in the Askey table. Then he looked at the 9-j coefficients, which he considered as an inspiring source of obtaining new relations between hypergeometric series, in particular the double hypergeometric Kampé de Fériet series $F_{1:1;1}^{0:3;3}$ of unit argument. Next Michael Schlosser presented *multidimensional matrix inversions and A_r and D_r basic hypergeometric series* (joint work with Ch. Krattenthaler). They computed the inverse of a specific infinite r-dimensional matrix, which they then applied to obtain many new identities for multidimensional hypergeometric series.

Finally the remaining three talks were on computer algebra and orthogonal polynomials. Our newsletter editor Wolfram Koepf gave two talks. The first dealt with *efficient computation of orthogonal polynomials in computer algebra*. He mentioned various ways to compute orthogonal polynomials (explicit series, generating functions, Rodrigues formula, differential equation, recurrence relation, determinants) and showed how each of these methods performs on a computer algebra system (Maple, Mathematica, Reduce, MuPad), using the Chebyshev polynomials as a guiding example. Then he presented *orthogonal polynomial identities in Maple V.4*. Zeilberger's algorithm gives recurrence relations and differential equations for hypergeometric sums/integrals and is therefore quite useful for orthogonal polynomials in the Askey table. Wolfram gave an overview of Maple's new capabilities. This was demonstrated on the computer. Wolfram implementations were used by René Swarttouw in his *computer implementation of the Askey-Wilson scheme*. René is presently working on a project of creating an interactive book on orthogonal polynomials in the Askey-Wilson scheme, with facilities for symbolic manipulation of formulas. He first showed a clickable Askey-Wilson table, where properties of each family of orthogonal polynomials can be found just by clicking with the mouse on the required family. Also limit transitions can be found by clicking the relevant arrows. René also worked on an interface with Maple, allowing anyone to generate recurrence relations and differential equations of any system of orthogonal polynomials in the Askey-Wilson table, even after an affine transformation and rescaling. Finally he showed that the reverse is also under construction: given the coefficients of a three-term recurrence relation, the user will be informed with which particular system of orthogonal polynomials one is dealing. René Swarttouw's project can be seen on internet at homepage http://www.can.nl/~renes/index.html.

Walter Van Assche
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Forthcoming Meetings and Conferences

1. International Memorial Conference D.S. Mitrinović: Niš, Serbia, June 20-22, 1996

The *International Memorial Conference D.S. Mitrinović* will be held in Niš, Serbia, Yugoslavia from June 20–22,

1996. The conference will be held on the Faculty of Electronic Engineering, Beogradska 14, 18000 Niš. It covers the topics:

- Approximation Theory
- Complex Analysis
- Differential, Integral and Functional Equations
- General Inequalities
- Orthogonal Polynomials and Special Functions

The accepted papers will be published in three volumes dedicated to Professor Mitrinović. The titles of volumes will be as follows:

1. Recent Progress in Inequalities
2. Advances in Mathematical Analysis
3. Topics in Mathematics with Applications

The approximate number of pages per volume will be 400. The appearance of the first volume is expected ahead of the conference, whereas the other two will appear later. All volumes will be comprised of survey and contribution papers. Survey papers, due to their length, content and scientific value may be considered as book chapters, and therefore, could be of great interest to a wide range of mathematicians. Also, the authors are well-known and have made significant contributions to the relevant topics.

On behalf of the Organizing Committee
Prof. Gradimir V. Milovanović, Chair
Faculty of Electronic Engineering
Department of Mathematics
P.O. Box 73, Beogradska 14
18000 Niš, Yugoslavia

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2. Joint Summer Research Conference on Random Matrices, Statistical Mechanics, and Painlevé Transcendents: South Hadley, June 23-27, 1996

This conference will take place at the *Mount Holyoke College*, South Hadley, Massachusetts, USA. It is chaired jointly by Pavel Bleher and Alexander Its, both of Indiana University-Purdue University at Indianapolis.

The analysis of correlation functions for exactly solvable quantum models and of the partition functions in the theory of random matrices has gradually become an exciting new branch of mathematical physics which has deep connections with both the classical theory of special functions and orthogonal polynomials and the modern theory of quantum groups and topological quantum field theory.

Further details can be obtained by WWW:
http://www.ams.org/committee/meetings/src.html

Richard Askey
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3. International Workshop on Orthogonal Polynomials in Mathematical Physics, in Honour of André Ronveaux: Madrid, June 24-26, 1996

Orthogonal polynomials play a crucial role in a lot of areas of Mathematics, Physics, etc.

The aim of the Workshop is that a limited number of invited mathematicians and theoretical physicists discuss and review various recent works of the *Theory of Orthogonal Polynomials* and its applications to one of the most interesting areas of knowledge: *Mathematical Physics*. The Workshop will take place at the *Escuela Politécnica Superior, Universidad Carlos III de Madrid* in Leganés, Madrid.

Invited talks:

N. Atakishiev, *Difference equations and some of their solutions*

Jesus S. Dehesa, *Information theory, quantum entropy and orthogonal polynomials*

Yuri F. Smirnov, *The Wigner-Racah algebra for $SU_q(2)$ and $SU_q(1,1)$ and q-analogs of Hahn, Kravchuk and Racah polynomials. The (quasi) exact solvability concept on the finite difference equations*

H.T. Koelink, *Addition formulas for q-special functions. Hecke algebras and q-Krawtchouk polynomials*

A. Aptekarev, *Toda-type dynamics for the coefficients of recurrence relations*

R. Koekoek, *The q-analogue of the Askey-scheme of hypergeometric orthogonal polynomials*

A. Ronveaux, *Connection coefficients for orthogonal polynomials*

The topics to be considered will be:

1. q-Polynomials and related topics.
2. Entropy of polynomials and related physical problems.
3. Difference equations and their solutions.
4. Finite difference equations and the (quasi) exact solvability concept.
5. Toda lattices and orthogonal polynomials.
6. Orthogonal polynomials in Mathematical Physics.

Organizing Committee: M. Alfaro (Univ. de Zaragoza), R. Álvarez-Nodarse, Secretary (Univ. Carlos III), A.G. García (Univ. Carlos III), G. López Lagomasino (Univ. Carlos III) and F. Marcellán, Chairman (Univ. Carlos III).

R. Álvarez Nodarse
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4. Meeting on Symmetries and Integrability of Difference Equations: Canterbury, July 1-5, 1996

A meeting on *Symmetries and Integrability of Difference Equations* (SIDE) will be held at the University of Kent at Canterbury from Monday 1st July to Friday 5th July 1996. This conference is the successor of the meeting on the same topics held in Esterel, Quebec, Canada in May 1994.

The conference is being organised by Professor Peter Clarkson, Institute of Mathematics & Statistics, University of Kent (P.A.Clarkson@ukc.ac.uk) and Dr. Frank Nijhoff, Department of Applied Mathematical Studies, University of Leeds (frank@amsta.leeds.ac.uk). A second announcement, with a preliminary list of speakers and information about registration is now available at WWW: http://stork.ukc.ac.uk/IMS/maths/SIDE96/

Peter Clarkson
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5. XVth Workshop on Geometric Methods in Physics, Bialowieza, Poland, July 1-7, 1996

The main emphasis of this workshop will be placed on the following topics: Geometric quantization, coherent states, q-special functions, quantum groups, theory of singularities, wavelets, symplectic and Poisson structures, quantization via $*$-products.

If you are interested to receive further information, please send an email to A. Strasburger (secretary of organizing committee).

Alexander Strasburger
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6. XXI. International Colloquium on Group Theoretical Methods in Physics: Goslar, July 15-20, 1996

Within this Colloquium there will be a.o. a symposium on Quantum Groups covering the following topics: quantum groups and their representations, quantum spaces and quantum symmetries, differential calculus on quantum spaces and quantum groups, non-standard deformations, Yangians, braided Hopf algebras, relations to noncommutative Geometry, q-analogues of special functions and partial differential equations.

The Local Organizing Committee consists of H-D. Doebner, W. Scherer and P. Kramer. For the quantum groups symposium V.K. Dobrev is a co-organizer.

For up-to-date and additional information see the home page WWW: http://www.pt.tu-clausthal.de/~group21/

conference organizers
(group21@pt.tu-clausthal.de)

7. SIAM Annual Meeting: Kansas City, July 22-26, 1996

The 1996 SIAM Annual Meeting will be held on July 22–26, 1996 in Kansas City, Missouri, USA. See http://www.siam.org/meetings/an96/an96home.htm for further information, or send an email to `meetings@siam.org`. A major theme is *New Tools of Applied Mathematics*. Carl de Boor will receive the von Neumann prize at this meeting.

In the February issue of the Newsletter, on p. 7, we wrote that our Activity Group probably would not sponsor or organize any Minisymposium during this Annual Meeting. Fortunately, some proposal has still come up:

The SIAM Activity Group on Orthogonal Polynomials and Special Functions will present a Minisymposium on *Modern Topics in Orthogonal Systems*. The minisymposium will be devoted to two modern extensions of the classical theory of orthogonal systems of special functions in one real variable. The first topic is *Wavelets*, organized by Gilbert G. Walter. Talks by Walter, P.R. Massopust, and T.Q. Nguyen will be given at the first meeting of the minisymposium, 8:30-10:30 on Tuesday morning, July 23, and there will be a talk by A. Ron at the second meeting, 8:30-10:30, Friday, July 26. The second topic, also treated at the Friday session, is *Multivariate Orthogonal Polynomials That Generalize Jacobi Polynomials* with Robert Gustafson as the speaker. Both topics are of importance in the approximation of functions and representation of data. Wavelets, in particular, have applications in signal processing, medicine and biology.

Abstracts

Gilbert G. Walter, University of Wisconsin-Milwaukee *Improving Wavelet Approximations*

> *Abstract:* Discrete wavelets are orthogonal systems on the real line (or in n-space) which generally have superior convergence properties compared to classical systems. However, they share a shortcoming—Gibbs' phenomenon—which causes errors at the edge of a truncated signal or image. It was shown by Shim and Volkmer that this always happens for orthogonal approximations for all continuous wavelets with sufficient decay. It also occurs for some interpolating approximations. Ways of avoiding this for both types of approximations will be presented.

Peter R. Massopust, Sam Houston State University, Huntsville, Texas. *Multiwavelets, Multiresolution Schemes, and Hyperbolic Conservation Laws*

> *Abstract:* Multiresolution schemes based on multiwavelets are presented. These schemes employ a combination of interpolation and direct evaluation and are a generalization of earlier work by A. Harten. It is shown how such multires-

olution schemes can be used to obtain accurate and computationally efficient numerical weak solutions of partial differential equations arising in computational fluid dynamics. The emphasis is on one-dimensional problems but extensions to higher dimensions will be indicated. Numerical experiments are given.

Y. Hui, C.W. Kok, **T.Q. Nguyen**, University of Wisconsin - Madison. *Image Coding Using Shift-Invariant Dyadic Wavelet Transform*

Abstract: A new class of wavelet filters, shift-invariant wavelet filters, is proposed for the purpose of image compression. The existing approaches obtain the shift-invariant wavelet transform by finding the path in the decomposition tree that minimizes shift-variance with respect to the given cost function. This procedure is signal dependent and is inefficient for image compression, since the subband decomposition has to be performed for all shifts of input signal during processing time. The proposed shift-invariant wavelet transform has better shift-invariant property compared with the conventional dyadic wavelet transform without changing the structure of it. Two bit allocation schemes, which are suitable for the proposed shift-invariant wavelet transform coding, are proposed and evaluated. Experimental results show that the shift-invariant wavelet transform has better energy compaction property in image coding compared to the conventional wavelet transform.

Robert A. Gustafson, Texas A&M University, College Station, Texas. *Multivariate Symmetric Orthogonal Polynomials Generalizing Jacobi Polynomials and Interpolation on Symmetric Lattices in* $\mathbb{R}^n$

Abstract: We discuss a family of multivariate symmetric orthogonal polynomials which have expansion formulas (as multivariate hypergeometric series) and properties similar to classical Jacobi polynomials. We also discuss discrete analogs of these polynomials. In the construction of these orthogonal polynomials, we use a new family of inhomogeneous symmetric polynomials, generalizing the classical symmetric functions (Schur functions). These new symmetric polynomials can be used in interpolation problems for symmetric functions on multidimensional symmetric lattices.

Amos Ron, University of Wisconsin - Madison, Zuowei Shen, National University of Singapore *Wavelet Frames—a New Approach*

Abstract: We unravel the structure of wavelet system with the aid of two new notions: the affine product, and a quasi-affine system. This leads to a characterization of affine frames; the induced characterization of tight affine frames is in terms of exact orthogonality relations. A general oversampling theorem trivially follow from these characterizations.

Most importantly, the affine product can be factored during a multiresolution analysis construction, and this leads to a very simple sufficient condition for constructing tight frames from multiresolution. Of particular importance are the facts that the underlying scaling function does not need to satisfy any a-priori conditions, and that the freedom offered by redundancy can be fully exploited in these constructions.

Willard Miller, Jr.
(miller@.umn.edu)

8. Seventh International Congress on Computational and Applied Mathematics: Leuven, July 21-26, 1996

The Congress *ICCAM 96* will take place at the *Katholieke Universiteit Leuven* in Belgium and will concentrate on the analysis of computational techniques for solving real scientific problems.

Invited Speakers: H. Brunner, M.E.H. Ismail, F. Marcellán, M. Nakao, J. Nedoma, W. Sweldens, P. Toint.

Please contact M.J. Goovaerts for the full announcement and an application form.

M.J. Goovaerts
(fdbaa35@cc1.kuleuven.ac.be)

9. AMS-MAA Mathfest: Seattle, Washington, USA, August 10-12, 1996

At the AMS-MAA Mathfest (Summer meeting) there will be four lectures dealing with binomial coefficients and theorems:

- **Gian-Carlo Rota**: The many lives of binomial coefficients. August 12, 10:40 (AMS-MAA invited address)
- **Richard A. Askey**: The binomial theorem and some extensions.
  Lecture 1: Some of the history of the binomial theorem and its extensions
  Lecture 2: Refined counting and a noncommutative version of the binomial theorem
  Lecture 3: Integral analogues of the binomial theorem, orthogonal polynomials and education
  10, 11 and 12 August, 9:35 (MAA Earle Raymond Hedrick Lectures)

Tom H. Koornwinder
(thk@fwi.uva.nl)

10. Workshop Transform Methods & Special Functions, II: Varna, August 23-30, 1996

The Second International Workshop *Transform Methods & Special Functions* will be devoted to the 100th Anniversary of the Bulgarian mathematician Acad. Nikola Obrechkoff (1886–1963). He has left an enormous and valuable heritage of more than 250 papers and several monographs and manuals in various topics: Analysis, Algebra, Number Theory, Numerical Analysis, Summation of Divergent Series, Probabilities and Statistics etc. New edition and translation of the Obrechkoff's selected papers are also planned.

The meeting will take place near the town of Varna (birthplace of N. Obrechkoff), in the Black Sea resort *Golden Sands*, close to the seashore.

Organizing Committee: Prof. Dr. Petar Rusev, Prof. Dr. Ivan Dimovski, Prof. Dr. Shyam L. Kalla (Kuwait University), Asso. Prof. Dr. Virginia Kiryakova, Asso. Prof. Dr. Lyubomir Boyadjiev.

Topics: Integral Transforms, Special Functions, Series Expansions, Fractional Calculus, Algebraical Analysis, Generalized Functions, Operational Calculus, Univalent Functions; Applications of these topics to Complex Analysis, Differential and Integral Equations.

Virginia Kiryakova
(virginia@bgearn.bitnet)

11. UN/ESA Workshop: Bonn, September 9-13, 1996

In the past four years the United Nations (UN) in cooperation with the European Space Agency (ESA) organized a series of Workshops on *Basic Space Science* for the benefit of Third World countries in four regions on Earth.

The Workshop for 1996 is the sixth in the series of UN/ESA Workshops on Basic Space Science and will focus on the principal three windows towards the Universe. The Workshop will also assess the accomplishments of this series of Workshops. It will be hosted by Germany, and will take place at September 9–13, 1996 at the

Gustav-Stresemann-Institut e.V.
Langer Grabenweg 68
D-53175 Bonn, Germany
phone: +49-228-8107-0
fax: +49-228-8107-197

Local organizer is the

Max-Planck-Institut für Radioastronomie
Auf dem Hügel 69
D-53121 Bonn, Germany
phone.: +49-228-525-0
fax: +49-228-525-438
email: bass@mpifr-bonn.mpg.de

Further information can be obtained by WWW at ftp://ecf.hq.eso.org/pub/un/gerannounce.html.

Hans J. Haubold
(haubold@ekpvs2.dnet.tuwien.ac.at)

12. III. International Conference on Functional Analysis and Approximation Theory: Acquafredda di Maratea, Italy, September 23-28, 1996

The meeting will be devoted to some significant aspects of contemporary mathematical research on Functional Analysis and Approximation Theory including the applications of these fields in other areas. Suggested topics include:

- Banach spaces, Banach lattices, function spaces
- (Positive) linear operators, semigroups of (positive) linear operators, evolution equations
- Integral equations, interpolation, approximate quadratures
- Approximation methods in abstract spaces and in function spaces, approximation by (positive) operators
- Constructive approximation
- Orthogonal polynomials

The scientific program will consist of invited survey talks (45 min.) and short communications (20 min.). The abstracts of all contributions and the program of the meeting will be available at the beginning of the meeting. The list of invited speakers and a preliminary list of all participants will be sent together with the second announcement. It is expected that the proceedings of the Conference will be published.

The meeting is organized in the sphere of activities of the *Center for Studies in Functional Analysis and Approximation Theory* of the University of Basilicata (Potenza), in collaboration with several other Italian Universities and Organisations.

The organizing committee consists of F. Altomare (Univ. of Bari), M. Campiti (Polytechnic of Bari), G. Criscuolo (Univ. of Napoli), B. Della Vecchia (Univ. of Roma, La Sapienza), G. Mastroianni (Univ. of Basilicata).

The full First Announcement, together with Preliminary Registration Form for participation and for giving a lecture, can be obtained from G. Mastroianni. The completed form should reach the following address as soon as possible.

Prof. Michele Campiti
Dipartimento di Matematica, Universita di Bari
Campus Universitario
Via Edoardo Orabona 4
70125 Bari, Italy
email: campiti@pascal.uniba.it

G. Mastroianni
(mastroianni@pzvx85.cisit.unibas.it)

13. MSRI Program on Combinatorics: Berkeley. Workshops October 14-18, 1996, and April 14-18, 1997

The *MSRI*, Berkeley organizes during 1996–97 a full-year program on Combinatorics. The program focuses on four main areas of approximately half a semester each. During each of these four periods there will be a one-week workshop. See WWW: http://www.msri.org/application/comb.html

Two of the areas may have some relevance for the members of our Activity Group:

13.1. Enumeration and partially ordered sets: September 1-October 25, 1996; Workshop: October 14-18, 1996

Combinatorial identities (especially computer proofs); conjectures on monotone triangles and plane partitions; enumeration and classification of tilings; combinatorial problems arising from statistical mechanics and knot theory; combinatorial properties of Kazhdan-Lusztig polynomials; q-analogues and quantum groups.

The workshop is being organized by Lynne Butler, Ira Gessel, Rodica Simion (chair), and Michelle Wachs. The program of the workshop revolves around enumerative and order-theoretic aspects in the study of combinatorial structures. Included among the workshop topics are:

- q-series
- Partitions
- Plane partitions
- Alternating sign matrices
- Enumerative aspects of group algebras and symmetric functions
- Combinatorics of orthogonal polynomials
- Computer algebra
- Combinatorial, topological, and algebraic aspects of the theory of partially ordered sets

For more information:
WWW: http://www.msri.org/sched/CombPosets.html
email: posets@msri.org

13.2. Symmetric functions and representation theory: March 17-May 30, 1997; Workshop: April 14-18, 1997

Macdonald's two-parameter symmetric functions and the corresponding two variable Kostka polynomial; immanent conjectures of Lieb, Goulden-Jackson, Stembridge, et al.; constant term identities and their connection with nilpotent Lie algebras and cyclic homology; random walks on groups and shuffling problems; internal products of symmetric functions; invariant theory.

The workshop is being organized by Curtis Greene (Chair), Sergey Fomin, Phil Hanlon, and Sheila Sundaram.

Scope: In recent years there have been many exciting developments in areas that link combinatorics (especially the theory of symmetric functions) with representation theory and algebraic geometry. This workshop will focus on current problems in these areas, emphasizing the interplay between algebraic and combinatorial methods. The program will include the following topics, as well as others:

- Schur functions and their generalizations
- Combinatorics and representations of finite Coxeter groups
- Quantum groups and Hecke algebras
- Centralizer algebras
- Homology representations

For more information:
WWW: http://www.msri.org/sched/CombSymfns.html
email: symfns@msri.org

Tom H. Koornwinder
(thk@fwi.uva.nl)

14. Workshop on Special Functions & Differential Equations: Madras, India, January 13-24, 1997

This is a Workshop in the field of Special Functions, Differential Equations and closely related topics, to be held at the *Institute of Mathematical Sciences* (I.M.Sc.), Madras during Jan. 13–24, 1997. Mini-series of lectures by experts will introduce the recent trends and developments in the topics listed. The objective of the Workshop is to bring together experts and active research students, to help strengthening of research activities at the universities and research institutions in this ever green area.

Topics include:

- Special Functions
- Differential Equations
- Orthogonal Polynomials (One/Several Variables)
- Group Theory and Special Functions
- Difference Equations
- q-Special Functions
- Quantum Groups and Special Functions
- Numerical Methods
- Algebraic/Symbolic Computer Packages

Plenary speakers include ([*] means: to be confirmed): R.P. Agarwal (India), B C. Berndt (U.S.A) [*], F. Calogero (Italy), H.D. Doebner (Germany) [*], R.F. Gustafson (U.S.A.) [*], E. Kalnins (New Zealand), T.H. Koornwinder (Netherlands), M. Lakshmanan (India), H.L. Manocha (India), S.C. Milne (U.S.A.) [*], M. Lohe (Australia), T.D. Palev (Bulgaria), C. Quesne (Belgium), A. Ronveaux (Belgium), T.S. Santhanam (U.S.A.), W. Van Assche (Belgium), G. Vanden Berghe (Belgium), J. Van der Jeugt

(Belgium), A. Verma (India), L. Vinet (Canada), M. Waldschmidt (France), P. Winternitz (Canada).

There is an international Advisory Committee and a local Organizing Committee.

Participants should be active research scientists, students, in the field of Special Functions, Differential Equations and related topics. Interested candidates should send by e-mail (or on plain paper) details giving their name, address, age, qualifications, present position and a resume of research work done. (Maximum number of participants: about 60).

Financial support for train travel, board and lodging will be provided to some of the participants. Registration Fee: US$ 100 (Rs. 200 for Indian participants)

Last date for receiving application: October 14, 1996. Further information: http://www.imsc.ernet.in/~wssf97/, or by sending a message to:

Prof. K. Srinivasa Rao
Workshop on Special Functions & Diff. Equations
The Institute of Mathematical Sciences (I.M.Sc.)
CIT Campus, Tharamani
Madras 600113, India
fax: +91-44-235 0586

K. Srinivasa Rao
(wssf97@imsc.ernet.in)

15. First ISAAC Conference: University of Delaware, Newark, Delaware, June 2-6, 1997

ISAAC is an abbreviation for *The International Society for Analysis, its Applications and Computation*. Analysis is understood here in the broadest sense, including differential equations, integral equations, functional analysis, and function theory. The first Congress of this newly constituted society will be held at the University of Delaware, between June 2 and June 6, 1997. The conference has its WWW page at http://www.math.udel.edu/isaac/conferen/congr97.htm.

One section of this conference will be devoted to *Orthogonal Polynomials*, organized by Wolfram Koepf. The emphasis of this section will be on the use of symbolic computation in connection with orthogonal polynomials and special functions. Symbolic computation has the potential to change the daily work of everybody who uses orthogonal polynomials or special functions in research or applications. It is the purpose of the proposed section to bring together developers of symbolic algorithms and implementations which are connected with orthogonal polynomials and special functions with users of computer algebra systems who need this type of software.

This is an early announcement encouraging people who are interested to participate in this section to get in touch with the organizer.

Wolfram Koepf
(koepf@zib-berlin.de)

16. Continued Fractions and Geometric Function Theory: Trondheim, Norway, June 24-28, 1997

Haakon Waadeland celebrates his 70th birthday on 20 May 1997. He is responsible for a long list of valuable contributions to the two fields of continued fractions and geometric function theory, and he is still very active. In recognition of his work we have decided to organize a conference in his honour. The conference will be held in Trondheim, Norway from 24 to 28 June 1997 under the title *Continued Fractions and Geometric Function Theory*. We hope to get together a group of people representing both fields, and we would very much appreciate if you would participate.

You are all welcome to present talks of 25 minutes and 5 minutes for questions. (If somebody has material suited for longer or shorter presentations, we may be able to arrange this.) We plan to publish proceedings from the conference.

The conference has its own world wide web page on http://www.matstat.unit.no/CFGT, where we will put new information when it is available. From this page you can also link to pages about Trondheim and about the university, NTNU.

If you want to receive the second announcement, please contact the conference address: confun@imf.unit.no.

Lisa Lorentzen
(lisa@imf.unit.no)

Books and Journals

1. Algebraic Structures and Operator Calculus II: Special Functions and Computer Science, Mathematics and its Applications 292
By Philip Feinsilver and René Schott

Kluwer Academic Publishers, Dordrecht, The Netherlands, July 1994, 160 pp., hardbound, ISBN 0-7923-2021-X, Dfl. 120.00/ US$ 70.00.

This is the second of three volumes which present, in an original way, some of the most important tools of applied mathematics in areas such as probability theory, operator calculus, representation theory, and special functions, used in solving problems in mathematics, physics and computer science.

This second volume—Special Funtions and Computer Science—presents some applications of special functions in computer science. It largely consists of adaptations of articles that have appeared in the literature, but here they are presented in a format made accessible for the non-expert by providing some context. The material on group representation and Young tableaux is introductory in nature. The algebraic approach of Chapter 2 is original to the

authors and has not appeared previously. Similarly, the material and approach based on Appell states, so formulated, is presented here for the first time. The solutions are tackled with the help of various analytical techniques, such as generating functions and probabilistic methods and insights appear regularly.

Contents: Preface · Introduction · Basic Data Structures · Data Structures and Orthogonal Polynomials · Applications of Bessel Functions and Lommel Polynomials · Fourier Transform on Finite Groups and Related Transforms · Young Tableaux and Combinatorial Enumeration in Parallel Processing · References · Index

Wolfram Koepf
(koepf@zib-berlin.de)

2. Quantum Group Symmetry and q-tensor Algebras
By L.C. Biedenharn and M.A. Lohe

World Scientific, 1995, x+293 pp.

The aim of the monograph is to develop and extend to quantum groups the symmetry techniques familiar from the application of classical groups to models in physics. The authors have taken a uniform approach to quantum groups based on the fundamental concept of a tensor operator.

The book contains some results about q-special functions in connection with quantum groups: in particular on q-analogues of Clebsch-Gordan coefficients and Racah coefficients, and about q-hypergeometric functions in connection with induced representations of the quantized universal enveloping algebra for $U(3)$.

Tom H. Koornwinder
(thk@fwi.uva.nl)

3. Special Functions: An Introduction to the Classical Special Functions of Mathematical Physics
By Nico M. Temme

Wiley, New York–Chichester–Brisbane–Toronto–Singapore, 1996, xii + 374 pp, ISBN 0-471-11313-1, US$ 54.95.

Contents:

1. Bernoulli, Euler and Stirling Numbers
2. Useful Methods and Techniques (Theorems from Analysis, Asymptotic Expansions of Integrals)
3. The Γ Function
4. Differential Equations (Separating the Wave Equation, DE's in the Complex Plane, Sturm's Comparison Theorem, Integrals as Solutions, Liouville Transformation)
5. Hypergeometric Functions (includes very brief introduction to q-functions)
6. Orthogonal Polynomials
7. Confluent Hypergeometric Functions (includes many special cases)
8. Legendre Functions
9. Bessel Functions
10. Separating the Wave Equation
11. Special Statistical Distribution Functions (Error functions, Incomplete Gamma Functions, Incomplete Beta Functions, Non-Central χ^2 Distribution, Incomplete Bessel Function)
12. Elliptic Integrals and Elliptic Functions
13. Numerical Aspects of Special Functions (mainly recurrence relations)

Bibliography · Notations and Symbols · Index

The author mentions that part of the material was collected from well-known books such as those by Hochstadt, Lebedev, Olver, Rainville, Szegő, and Whittaker & Watson as well as lecture notes by Lauwerier and Boersma. But there is much recent material, especially in the areas of asymptotic expansions and numerical aspects. About half of the approximately 200 references are dated 1975 and later.

The author states that the book "has been written with students of mathematics, physics and engineering in mind, and also researchers in these areas who meet special functions in their work, and for whom the results are too scattered in the general literature." Complex analysis (especially contour integration) would appear to be the main prerequisite. The book is clearly written, with good motivating examples and exercises. For example, the first chapter opens with a remarkable example of Borwein, Borwein and Dilcher (Amer. Math. Monthly 96 (1989), 681-687) which explains, using Euler numbers and Boole's summation formula, why the sum of 50 000 terms of the very slowly converging alternating series for $\pi/4$, though it gives an answer correct to only six digits, yet has many correct digits further out in its decimal expansion and, in fact, has nearly all its first 50 digits correct.

Martin Muldoon
(muldoon@mathstat.yorku.ca)

4. Index Transforms
By S.B. Yakubovich

World Scientific, Singapore–London–Hong Kong, 1996, 264 pp.

This book deals with the theory and some applications of integral transforms that involve integration with respect to an index or parameter of special functions of hypergeometric type as the kernel (index transforms). The basic index transforms are considered, such as the Kontorovich-Lebedev transform, the Mehler-Fock transform, the Olevskii transform, the Lebedev-Skalskaya transforms. The Lp theory of index transforms is discussed and new index transforms and convolution constructions are

demonstrated. For the first time, the essentially multidimensional Kontorovich-Lebedev transform is announced. The book is self-contained, and includes a list of symbols with definitions, author and subject indices, and an up-to-date bibliography.

S.B. Yakubovich
(root@nti.bsu.minsk.by)

5. Journal of Computational and Applied Mathematics
Volume Editor: Marcel G. de Bruin
Volume 65, Dec. 1995, North-Holland, Amsterdam.

The Proceedings of the International Conference on *Orthogonality, Moment Problems and Continued Fractions* (dedicated to Thomas Jan Stieltjes Jr.) and held October 31–November 4, 1994 at Delft University of Technology, The Netherlands, have now appeared in this special issue of the *Journal of Computational and Applied Mathematics*. We thank the volume editor, Marcel G. de Bruin, for supplying us with a LaTeX version of the table of contents, from which the text below has been prepared.

Contents

Martin Muldoon
(muldoon@mathstat.yorku.ca)

6. Electronic Journal of Combinatorics: Foata Festschrift
Volume Editors: J. Désarménien, A. Kerber, V. Strehl
Volume 3 (2), 1996.

A special issue of *The Electronic Journal of Combinatorics* appeared which is dedicated to Dominique Foata on the occasion of his 60th birthday:
http://ejc.math.gatech.edu:8080/Journal/Volume_3/festschrift.html

It contains 25 contributions, some of which are in the field of orthogonal polynomials or relevant for this field. I mention a few and I recommend the reader to inspect the contents for other titles:

R1: Daniel Barsky and Michel Carpentier: Polynômes de Jacobi généralisés et intégrales de Selberg (33pp)

R13: Doron Zeilberger: Proof of the alternating sign matrix conjecture (84pp)

R16: Shalosh B. Ekhad and J.E. Majewicz: A short WZ-style proof of Abel's identity (1p)

R17: Dominique Dumont and Armand Ramamonjisoa: Grammaire de Ramanujan et arbres de Cayley (18pp)

R19: Marko Petkovšek and Herbert S. Wilf: A high-tech proof of the Mills-Robbins-Rumsey determinant formula (3pp)

R20: Alun Morris and A. A. Abdel-Aziz: Schur Q-functions and spin characters of symmetric groups I (14pp)

R21: George E. Andrews: Pfaff's method (III): Comparison with the WZ method (18pp)

R24: A.M. Garsia and M. Haiman: Some natural bigraded S_n-modules (60pp) (related to Macdonald polynomials)

Tom H. Koornwinder
(thk@fwi.uva.nl)

Software Announcements

1. Askey-Wilson Computer Algebra Mini-Project
As already announced on p. 13 of the February Newsletter 6.2, René Swarttouw is working during the period January-June 1996 at RIACA, Eindhoven, The Netherlands on a project *A computer implementation of the Askey-Wilson scheme*. The purpose of the project is to make a start with bringing the report

R. Koekoek and R.F. Swarttouw
The Askey-scheme of hypergeometric orthogonal polynomials and its q-analogue
Report 94-05, Delft University of Technology, Faculty TWI, 1994

to the form of an interactive book, with facilities for symbolic manipulation of formulas.

Further information about this project is now available from WWW: http://www.can.nl/~renes/index.html

From there you can also download a preliminary version of the electronic book version. It is a dvi file named AW.dvi, which is equipped with hyperlinks. This file can be read by using the program xhdvi. Consult also the above WWW site for the way to obtain xhdvi. There is another version available which can be read via Netscape. It uses a dvi viewer created by Garth Dickie.

Swarttouw is also working on a package for calculating formulas for orthogonal polynomials belonging to the Askey-scheme by Maple. At the mentioned WWW site there is an online facility to do some of these computations, for which you do not need to have Maple on your own computer. The present demos use Maple procedures written by Wolfram Koepf.

Tom H. Koornwinder
(thk@fwi.uva.nl)

Problems and Solutions

Thus far 14 problems have been submitted five of which have been solved (#**1**, **4**, **6**, **7**, **10**), and one of which is new (#**14**).

Solutions of the problems #**11** and #**12** are presented in the current issue.

2. Is it true that

$$x^2 t^x \, {}_2F_1(x+1,\, x+1;\, 2;\, 1-t)$$

is a convex function of x whenever $-\infty < x < \infty$ and $0 < t < 1$?

Submitted by George Gasper, August 19, 1992.
(george@math.nwu.edu)

3. The following Toeplitz matrix arises in several applications. Define for $i \neq j$

$$A_{ij}(\alpha) = \frac{\sin \alpha\pi(i-j)}{\pi(i-j)},$$

and set $A_{ii} = \alpha$. Conjecture: the matrix

$$M = (I - A)^{-1}$$

has positive entries. A proof is known for $1/2 \leq \alpha < 1$. Can one extend this to $0 < \alpha < 1$?

Submitted by Alberto Grünbaum, November 3, 1992.
(grunbaum@math.berkeley.edu)

5. The result of Problem #**4** can be generalized to

$$\begin{aligned} S_m &= \sum_{n=0}^{\infty} \frac{(-1)^n (mn + 1/2)!}{\sqrt{\pi}\,(mn+1)!} \\ &= \frac{1}{m} \sum_{k=0}^{m-1} \frac{\sin\left(5(2k+1)\pi/(4m) + \pi/4\right)}{[2\sin\left((2k+1)\pi/(2m)\right)]^{1/2}} \end{aligned}$$

valid for integral $m \geq 2$.

Submitted by J. Boersma and P.J. de Doelder, July 12, 1993.
(wstanal@win.tue.nl)

8. Can the real and imaginary parts of a hypergeometric series of type ${}_pF_q$ with one complex parameter (either in the numerator or the denominator) be expressed by means of multiple hypergeometric series?

Submitted by Ernst D. Krupnikov, July 25, 1993.
(ernst@net.neic.nsk.su)

9. Prove or disprove: The functions

$$H_n(t) = (-1)^n\, F_n(nt)$$

that are defined in terms of the *Bateman functions*

$$\begin{aligned} F_n(t) &= e^{-t}\left(L_n(2t) - L_{n-1}(2t)\right) \\ &= -e^{-t}\,\frac{2t}{n}\,L_{n-1}^{(1)}(2t) \\ &= (-1)^n \frac{2}{\pi}\int_0^{\pi/2} \cos\left(t\,\tan\theta - 2\,n\,\theta\right) d\theta\ , \end{aligned}$$

$L_n^{(\alpha)}(t)$ denoting the *generalized Laguerre polynomials*, have the property that $H_n(t_0)$ is strictly decreasing with increasing n at the point $t_0 = 2$. Note that this is not true for any $t_0 < 2$, but on the other hand at $t_0 = 2$ seems to be numerically evident. Note further that $H_n(t)$ satisfy the simple differential equation

$$t\,H_n''(t) = n^2\,(t-2)\,H_n(t)\ .$$

The differential equation demonstrates the importance of the point $t_0 = 2$.

These functions occur particularly in the study of nonvanishing analytic functions of the unit disk. For more details on these functions, see Koepf, W. and Schmersau, D.: Bounded nonvanishing functions and Bateman functions, Complex Variables **25** (1994), 237–259.

Submitted by Wolfram Koepf, February 10, 1995.
(koepf@zib-berlin.de)

11. A Bessel-Laplace Transform. For $p \geq 2$ show that

$$I = \int_0^\infty e^{-pt} J_0(t) K_0(at) dt = \frac{1}{2p}\mathbf{K}(k)$$

where

$$a = \sqrt{p^2 - 1 \pm p\sqrt{p^2-4}}$$

$$k = \frac{1}{p}\sqrt{\frac{1}{2}p^2 + 1 \mp \frac{1}{2}p\sqrt{p^2-4}}\ .$$

Submitted by Lawrence Glasser, Nov 9, 1995.
(laryg@sun.mcs.clarkson.edu)

12. Absolutely Monotonic Function. The functions $t_n(x)$ generated by

$$\frac{1}{(1-z)^2\,\left(1 - z + \sqrt{1-2xz+z^2}\right)^\alpha} = \sum_{n=0}^\infty t_n(x)\,z^n$$

are nonnegative for $\alpha \geq 0$, $x \in [-1,1]$. Can this be proved by elementary means?

Submitted by Dieter Schmersau, January 8, 1996.
(schmersau@math.fu-berlin.de)

13. Product of Chebyshev Polynomials. For any pair of positive even $n, m \in \mathbb{N}$ let

$$\begin{aligned} F^{(n,m)}(x) &= 2^n \prod_{k=0}^{n-1} \cosh\left(\frac{m}{2}\,\mathrm{arccosh}\left(x - \cos\frac{(2k+1)\pi}{n}\right)\right) \\ &= 2^n \prod_{k=0}^{n-1} T_{m/2}\left(x - \cos\left(\frac{(2k+1)\pi}{n}\right)\right)\ , \end{aligned}$$

where $T_m(x)$ denotes the Chebyshev polynomials of the first kind. These functions occur in statistical physics. They constitute polynomials in x

$$F^{(n,m)}(x) = \sum_{j=0}^{nm/4} A_j(n,m)\,x^{2j}$$

whose coefficients $A_j(n,m)$ are integers. Show the symmetry

$$F^{(n,m)}(x) = F^{(m,n)}(x)\ ,$$

and give a representation of the coefficients $A_j(n,m)$.

Submitted by Christian Hege, January 16, 1996.
(hege@zib-berlin.de)

14. A Trigonometric Integral. Evaluate the integral

$$\int_0^{\pi/4} \ln(\sin^{3/2}(x) + \cos^{3/2}(x))\,dx$$

which arose in a study [1] dealing with orthogonal polynomials.

[1] Joyce, G.S., private communication.

Submitted by I.J. Zucker, March 8, 1996.
(udap194@oak.cc.kcl.ac.uk)

Solution of Problem 11
A Bessel-Laplace Transform

by Lawrence Glasser
(laryg@sun.mcs.clarkson.edu)

By using the identity [1]

$$\int_0^\infty e^{-t} f(t) dt = \int_0^1 dy \int_0^\infty x dx J_0(x\sqrt{1-y^2}) f(xy)$$

and [2;(8.13.8)] one finds

$$\int_0^\infty e^{-t} J_0(at) K_0(ct) dt = \int_0^1 \frac{dy}{\sqrt{Ay^4 + 2By^2 + 1}}$$

with

$$A = (a^2 + c^2) + 2(a^2 - c^2) + 1,\ B = c^2 - a^2 - 1.$$

Hence,

$$\int_0^\infty e^{-pt} J_0(at) K_0(bt) dt =$$

$$\frac{1}{\sqrt{\alpha}} \left(\mathbf{K}(k) - \frac{1}{2} F\left(\cos^{-1}\left(\frac{\alpha - p^2}{\alpha + p^2} \right), k \right) \right)$$

with

$$\alpha = \sqrt{(a^2 + b^2)^2 + 2p^2(a^2 - b^2) + p^4},$$

$$k = \sqrt{\frac{\alpha + a^2 - b^2 + p^2}{2\alpha}}.$$

This is a complete elliptic integral when $\alpha = p^2$, e.g. Thus, for $(b^2 + 1)^2 = 2p^2(b^2 - 1)$,

$$\int_0^\infty e^{-pt} J_0(t) K_0(bt) dt = \frac{1}{2p} \mathbf{K}(\sqrt{1 - (b^2 - 1)/2p^2})$$

which is equivalent to the stated result.

References

[1] Glasser, M.L. and Kowalenko, V., to be published.

[2] Erdélyi, A., Magnus, W., Oberhettinger, F. and Tricomi, F.G.: *Tables of Integral Transforms, Vol. 2.* McGraw-Hill, New York, 1954.

Solution of Problem 12
Absolutely Monotonic Function

by Dieter Schmersau
(schmersau@math.fu-berlin.de)

To deduce the inequalities $t_n(x) \geq 0$, we remark that the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ have the generating function

$$\sum_{n=0}^\infty P_n^{(\alpha,\beta)}(x)\, z^n = \frac{2^{\alpha+\beta}}{\sqrt{1-2xz+z^2}} \cdot \frac{1}{\left(1 - z + \sqrt{1-2xz+z^2}\right)^\alpha} \frac{1}{\left(1 + z + \sqrt{1-2xz+z^2}\right)^\beta}$$

(see e.g. [1], (22.9.1)), hence

$$\frac{1}{(1-z)^2 \left(1 - z + \sqrt{1 - 2xz + z^2}\right)^\alpha} =$$

$$\frac{1}{1-z} \frac{1}{\sqrt{1-2xz+z^2}} \frac{1}{\left(1-z+\sqrt{1-2xz+z^2}\right)^\alpha} \cdot \frac{\sqrt{1-2xz+z^2}}{1-z}$$

$$= \frac{1}{2^\alpha} \cdot \sum_{n=0}^\infty \sum_{j=0}^n P_j^{(\alpha,0)}(x)\, z^n \cdot \sum_{n=0}^\infty \sum_{j=0}^n C_j^{(-1/2)}(x)\, z^n ,$$

and the result follows from the positivity of the Jacobi polynomial sums

$$\sum_{j=0}^n P_j^{(\alpha,0)}(x) \geq 0 \qquad (\alpha > -2) \tag{1}$$

(see [2], Theorem 3), which is the so-called *Askey-Gasper inequality*, and the positivity of the Taylor coefficients (with respect to z) of the function

$$\frac{\sqrt{1-2xz+z^2}}{1-z}$$

(see [2], Theorem D).

References

[1] Abramowitz, M. and Stegun, I. A.: *Handbook of Mathematical Functions.* Dover Publ., New York, 1964.

[2] Askey, Richard and Gasper, George: Positive Jacobi polynomial sums II. Amer. J. Math. **98** (1976), 709–737.

The same proof was given by Dick Askey who wrote to the proposer

> I can prove the claim you made in problem 12. Take the generating function for the Jacobi polynomial with parameters α and zero, and multiply by $\sqrt{1 - 2xz + z^2}$. Then split $(1 - z)^{-2}$ into two equal parts. One of the factors is an absolutely monotonic function by an easy result of Askey and Pollard, which does not use anything difficult. The other is what is called the Askey-Gasper inequality (1). There are many proofs of this, and what one calls elementary depends on what you know. I can make a case that it is elementary, but most would not agree with me. Is this how you proved this result?

George Gasper wrote

> You could use the Fields and Wimp expansions in their Math. of Computation 15 (1961), pp. 390-395, paper as I did to derive Eq. (8.18) on p. 415 of my *Positivity and Special Functions* paper published in the book *Theory and Application of Special Functions*, ed. by R.A. Askey, Academic Press, 1975.

Indeed, the only open question remains whether or not the Askey-Gasper inequality is elementary or not.

Miscellaneous

1. Call for Questions

For several years William Connett and I have organized summer research groups designed to give a mathematical experience to undergraduates and high school students. A typical activity would be to use a computer algebra system to test conjectures about orthogonal polynomials or produce interesting graphics. We would be very grateful for any suggestions for projects.

Alan L. Schwartz
(schwartz@arch.umsl.edu)

2. Revising the 1991 Mathematics Subject Classification

The following was taken from Notices AMS, December 1995, p. 1547, see also
http://www.ams.org/committee/publications/msc-2000-let.html
I suggest that you send suggestions for revision of numbers 33 (Special functions), 42Cxx (Nontrigonometric Fourier analysis) and 44 (Integral transforms, operational calculus) also to the editors of OP-SF Net (thk@fwi.uva.nl and muldoon@mathstat.yorku.ca) or to the editor of the Newsletter (koepf@zib-berlin.de) in order that a summary of suggestions for this subject can be listed, and maybe discussed by the members of the Activity Group.

Note from Notices AMS:
Mathematics Subject Classification Scheme Revision
To the Mathematical Community:

The editors of Mathematical Reviews and Zentralblatt für Mathematik have initiated the process of revising the 1991 Mathematics Subject Classification, which is used by both journals as their classification system. The editors do not plan a radical revision of the present 1991 system, but it is clear that some changes will be needed in order to accommodate recent developments in mathematical research.

It will be necessary to have this revision completed by the end of 1998 so that it can begin to be used in Current Mathematical Publications in mid 1999, and in Mathematical Reviews and Zentralblatt für Mathematik beginning in 2000.

We hereby solicit comments and suggestions from the mathematical community to be considered in this revision process. These should be submitted by June, 1997. The preferred method of communication is by e-mail:
msc2000@ams.org or msc2000@zblmath.fiz-karlsruhe.de
(Comments and suggestions may also be sent to either one of us at the addresses given below.) We are eager that research mathematicians and scholars have input in this revision process as soon as possible.

R. Keith Dennis	Bernd Wegner
Executive Editor	Chefredakteur
Mathematical Reviews	Zentralblatt für Mathematik

Tom H. Koornwinder
(thk@fwi.uva.nl)

How to Contribute to the Newsletter

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preferably by email, and in LaTeX format. Other formats are also acceptable and can be submitted by email, regular mail or fax.

Deadline for submissions to be included in the October issue 1996 is September 15, 1996.

The Activity Group also sponsors an electronic news net, called the **OP-SF Net**, which is transmitted periodically by SIAM. The Net provides a rather fast turnaround compared to the Newsletter. To *receive* transmissions, just send your name and email address to poly-request@siam.org (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net *contributions* should be sent to poly@siam.org. Please note that submissions to OP-SF Net are automatically considered for publication in the Newsletter, and vice versa, unless the writer requests otherwise.

The Net is organized by Tom Koornwinder (thk@fwi.uva.nl) and Martin Muldoon (muldoon@mathstat.yorku.ca). Back issues of OP-SF Net can be obtained by anonymous ftp from `ftp.fwi.uva.nl`, in the directory

pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

or by WWW at the addresses

ftp://ftp.fwi.uva.nl/pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

http://www.math.ohio-state.edu/JAT

Martin Muldoon, moreover, manages our home page

http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/

on World Wide Web. Here you will find also a WWW version of the OP-SF Net. It currently covers the topics

- Conference Calendar
- Books, Conference Proceedings, etc.
- Compendia, tools, etc.
- Meeting Reports
- Projects
- Problems
- Personal, Obituaries, etc.
- Positions available
- Miscellaneous

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Activity Group: Addresses

The *SIAM Activity Group on Orthogonal Polynomials and Special Functions* consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We now have around 150 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/ is managed by Martin Muldoon (muldoon@mathstat.yorku.ca).