

# Orthogonal Polynomials and Special Functions

*SIAM Activity Group on Orthogonal Polynomials and Special Functions*

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Newsletter

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Published Quarterly

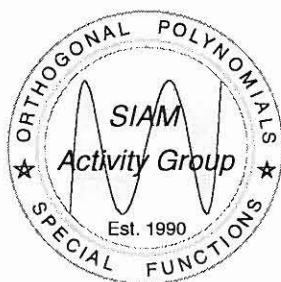
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Edition Typeset March 10, 1994



## The News

With this edition you should receive the annual list of members, plus a poster to hang outside your office. You may also want to check your own listing to see if it is accurate and complete. If not, let us know; or you can inform SIAM directly at [service@siam.org](mailto:service@siam.org), or in writing. We will continue to print address and e-mail updates in the Newsletter.

Mourad Ismail wishes to announce that The Journal of Computational and Applied Mathematics will publish a

special issue on  $q$ -series and related topics around 1996. The guest editors are Mourad Ismail and David Masson. Contributions are welcome. Please submit your paper to either editor.

N.M. Atakishiyev and S.K. Suslov have translated into Russian the book of George Gasper and Mizan Rahman, "*Basic Hypergeometric Series*". According to George, it has just been published by Mir Publishers in Moscow and, for the benefit of our Russian readers, it can be obtained in Russia at very low cost.

Hans Haubold's new e-mail at Technische Universität Wien is: [haubold@ekpvs2.dnet.tuwien.ac.at](mailto:haubold@ekpvs2.dnet.tuwien.ac.at). In an article he wrote for this edition you will find his new address.

The OP-SF Net has started, under Tom Koornwinder's direction, and there have already been two transmissions. The Net provides a fast turnaround as compared to the printed material. To receive the transmissions, just send your name and e-mail address to [poly-request@siam.org](mailto:poly-request@siam.org) (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net contributions should be sent to [poly@siam.org](mailto:poly@siam.org). Please tell your friends and colleagues about the OP-SF Net. Several items in this edition have been taken from the last Net transmission.

Martin Muldon and Charles Dunkl have been occupied with organizing our Minisymposium, which will take place during SIAM's annual meeting in San Diego, July 25–29, 1994. The title is "Special Functions and Asymptotics" and more details are given below.

In the meetings section of this issue you will find three new announcements, plus updated information on "Orthogonality, Moment Problems, and Continued Fractions", to be held at the Delft University of Technology in Holland during October 31–November 4, 1994.

(continues on p. 2)

===== SIAM Activity Group =====  
 on  
 Orthogonal Polynomials and Special Functions



Elected Officers

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MARTIN E. MULDOON, *Program Director*

TOM H. KOORNWINDER, *Secretary*

Appointed Officer

EUGENE TOMER, *Editor of the Newsletter*



THE PURPOSE of the Activity Group is  
 —to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.

**The News (continued)**

Part IV of "*Ramanujan's Notebooks*" (451 pages), by Bruce C. Berndt, is now available from Springer-Verlag. According to Springer over half the results are new and proofs are also given.

"*Generalized Fractional Calculus and Applications*" is the title of a book by Virginia Kiryakova of the Bulgarian Academy of Sciences. Among the topics listed in the Table of Contents are: generalized operators of fractional integration and differentiation, recent aspects of Erdélyi-Kober operators, hyper-Bessel differential and integral operators and equations, and applications to generalized hypergeometric functions.

You may contact Professor Kiryakova at the Institute of Mathematics, Bulgarian Academy of Sciences, P.O. Box 373, Sofia 1090, Bulgaria. email: virginia@bgearn.bitnet. For inquiries and orders from within the USA or Canada contact Beth Schacht, John Wiley & Sons, Inc., 605 Third Avenue, New York, NY 10158. Outside USA and Canada: Judy Higgins, Longman House, Burnt Mill, Harlow, Essex CM20 2JC, England; the Fax is (44 279) 623 862.

Hervé LeFerrand wishes to know whether something has been done on orthogonal polynomials with coefficients in a finite field. His e-mail is leferran@u-bourgogne.fr.

And finally, Andrew Van Tuyl is now connected to the Internet by way of the concise address: avt@clark.net.

Extra copies of the enclosed poster are available from the Editor.



S. Ramanujan—just before he sailed to England in 1914.  
 Photo: Ragami's Collections, Madras, South India.

**Stieltjes' Results on Meixner Polynomials**

by GALLIANO VALENT

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In his "Recherches sur les fractions continues", published in 1894–1895 [1], Stieltjes developed, in section 81 p.730–731, the continued fraction

$$\int_0^\infty \frac{d\Psi(u)}{z+u} = \int_0^\infty du \frac{(1-\lambda)^a e^{-zu}}{(e^{(1-\lambda)u} - \lambda)^a} \quad (1)$$

$$\int_0^\infty \frac{d\Psi(u)}{z+u} = \frac{1}{z + \frac{a}{1 + \frac{\lambda}{z + \frac{(a+1)}{1 + \frac{2\lambda}{z + \frac{(a+2)}{1 + \frac{3\lambda}{z + \dots}}}}}}}$$

It is assumed that  $a > 0$ ,  $\lambda > 0$  and  $\lambda \neq 1$ . Stieltjes gives one more parameter  $b$  which can be scaled out to 1 without any loss in generality.

Using modern terminology this result can be stated: the polynomials  $F_n(a, \lambda; x)$  with the three term recurrence

$$\begin{cases} (\lambda_n + \mu_n - x)F_n(x) = \mu_{n+1}F_{n+1}(x) + \lambda_{n-1}F_{n-1}(x) \\ F_{-1}(x) = 0, \quad F_0(x) = 1 \end{cases}$$

where

$$\lambda_n = n + a, \quad \mu_n = \lambda n$$

have for orthogonality measure  $\Psi$ .

From the recurrence relation of the Meixner polynomials [2, p.176] we have

$$\lambda^n n! F_n(a, \lambda; x) = \mathcal{M}_n \left( a, \frac{1}{\lambda}; \frac{x}{1-\lambda} \right).$$

Stieltjes expands the denominator of (1) in powers of  $e^{(1-\lambda)u}$  and integrates term by term to get for  $\lambda < 1$

$$\int_0^\infty \frac{d\Psi(u)}{z+u} = (1-\lambda)^a \sum_{n \geq 0} \frac{(a)_n}{n!} \frac{\lambda^n}{z + (n+a)(1-\lambda)}$$

and for  $\lambda > 1$

$$\int_0^\infty \frac{d\Psi(u)}{z+u} = (1 - (1/\lambda))^a \sum_{n \geq 0} \frac{(a)_n}{n!} \frac{(1/\lambda)^n}{z + n(\lambda - 1)}.$$

From these relations the orthogonality measure follows:

$$\Psi = (1-\lambda)^a \sum_{n \geq 0} \frac{(a)_n}{n!} \lambda^n \delta_{(n+a)(1-\lambda)}, \quad \lambda < 1$$

$$\Psi = (1 - (1/\lambda))^a \sum_{n \geq 0} \frac{(a)_n}{n!} \lambda^n \delta_{n(\lambda-1)}, \quad \lambda > 1.$$

In the limit  $\lambda \rightarrow 1$  Stieltjes gives

$$\int_0^\infty \frac{d\Psi(u)}{z+u} = \int_0^\infty du \frac{e^{-zu}}{(u+1)^a} = \frac{1}{\Gamma(a)} \int_0^\infty du \frac{u^{a-1} e^{-u}}{z+u}$$

in which we recognize the Laguerre orthogonality measure, in agreement with

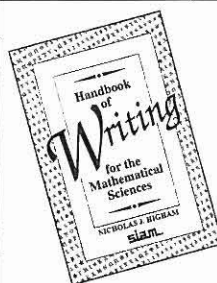
$$F_n(a, 1; x) = \frac{1}{n!} \lim_{\lambda \rightarrow 1} \mathcal{M}_n \left( a, \frac{1}{\lambda}; \frac{x}{1-\lambda} \right) = L_n^{(a-1)}(x).$$

So it is clear that Stieltjes had obtained the explicit form of the orthogonality measure for the Meixner polynomials before 1895. Indeed the results were published in 1895 but Stieltjes died on December 31, 1894, at the age of 38.

Observe that the generalized Laguerre  $L_n^a(x)$  polynomials are usually attributed to Laguerre, in his 1885 article [3] published ten years *before* Stieltjes' results. The Meixner polynomials are attributed to Meixner, in his 1934 article [4] which appeared forty years *after* Stieltjes' discovery.

#### References

- [1] *Stieltjes, Collected Papers*. GERRIT VAN DIJK, Editor, Springer-Verlag, Berlin, 1993.
- [2] T. S. CHIHARA. *An Introduction to Orthogonal Polynomials*. Gordon and Breach, New York, 1978.
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- [4] J. MEIXNER. J. London Math. Soc. **9** (1934), pp. 6-13.



## Handbook of Writing for the Mathematical Sciences

Nicholas J. Higham

"Higham's handbook contains a wealth of tips, rules, tables and checklists — all useful starting points for writers, editors and trainers and a boon to those working to tight schedules."

—Sheila McNab, *European Science Editing*, January, 1994; No.51.

"This book is a gem, and should be available in every math science department and library."

—Ingram Olkin, Department of Statistics, Stanford University.

This handy volume provides information on virtually every issue you will face when writing a technical paper or talk, from choosing the right journal to handling your references. You'll also get an overview of the entire publication process — invaluable for anyone hoping to publish in a technical journal.

This handbook provides much-needed advice on handling the basic ingredients of a research paper, like definitions, theorems, examples, and equations. In addition, appendices provide essential reference material, including summaries of LaTeX symbols and Emacs commands, addresses of mathematical societies, and a list of papers that have won expository writing prizes.

#### Contents

Preface; Chapter 1: *General Principles*; Chapter 2: *Writer's Tools and Recommended Reading*; Chapter 3: *Mathematical Writing*; Chapter 4: *English Usage*; Chapter 5: *When English is a Foreign Language*; Chapter 6: *Writing a Paper*; Chapter 7: *Revising a Draft*; Chapter 8: *Publishing a Paper*; Chapter 9: *Writing a Talk*; Chapter 10: *Computer Aids for Writing and Research*; Appendix A: *The Greek Alphabet*; Appendix B: *Summary of TeX and LaTeX Symbols*; Appendix C: *GNU Emacs—The Sixty+ Most Useful Commands*; Appendix D: *Mathematical Organizations*; Appendix E: *Winners of Prizes for Expository Writing*; Glossary; Bibliography; Index.

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## Special Functions of Mathematical Physics

A Contemporary Perspective  
by HANS J. HAUBOLD

United Nations Outer Space Office  
Vienna International Centre, A-1400 Vienna, Austria  
haubold@ekpvs2.dnet.tuwien.ac.at

The Meijer G-function, introduced into classical analysis by C.S. Meijer in 1936, is a natural generalization of the hypergeometric function. It appears in various places throughout applied mathematics, especially in statistical distribution problems; and it also appears in the physical sciences, for example in quantum field theory and cosmology [1], in particle physics [2], and in solid state physics [3]. The theory of G-functions is available in [4]. Most of the research work on G-functions has been in the area of series representations, certain properties such as asymptotic expansions, and integral representations and transforms. The geometrical background for the existence of relations among special functions is usually elucidated through their connection with the theory of group representations.

In order to make use of these functions in physics, one often needs computable representations for them. The classical mathematical technique for getting a computable series representation is to evaluate the G-function as a sum of residues by using the residue calculus. However, it may also be the case that the formula itself is of interest apart from the computational aspect.

In our recent work [5–7] we have employed this technique extensively to derive analytic representations of thermonuclear functions as they appear in nuclear astrophysics, to construct simple analytic models for the internal structure of solar-type stars belonging to stellar physics, and to obtain analytic solutions of differential equations governing the growth and decay of gravitational instabilities in the expanding Universe, as they are pursued in cosmology on the basis of Newton's theory of gravity. These achievements amount to modern applications of the theory of higher transcendental functions and their properties, as outlined in the classical treatises such as Whittaker and Watson's "A Course of Modern Analysis" [8]. At the same time we wanted to illustrate the quest for beauty and elegance in science so eloquently expressed, for example, by the astronomer S. Chandrasekhar [9].

We now briefly provide some details of our work.

a) One defines various thermonuclear functions by the following integrals:

$$\begin{aligned} I_1(z, \nu) &= \int_0^\infty y^\nu e^{-y} e^{-zy} y^{-\frac{1}{2}} dy \\ I_2(z, d, \nu) &= \int_0^d y^\nu e^{-y} e^{-zy} y^{-\frac{1}{2}} dy \\ I_3(z, t, \nu) &= \int_0^\infty y^\nu e^{-y} e^{-z(y+t)} y^{-\frac{1}{2}} dy \end{aligned}$$

$$I_4(z, \delta, b, \nu) = \int_0^\infty y^\nu e^{-y} e^{-by^\delta} e^{-zy} y^{-\frac{1}{2}} dy.$$

Using their Mellin-Barnes contour integral representations we have expressed these integrals in terms of Meijer's G-function, while further application of the residue calculus have lead us to series representations [5].

b) Solar-type stars are gas spheres in hydrostatic equilibrium. Their internal structure is given by a system of differential equations for mass and energy conservation, hydrostatic equilibrium, and energy transport. Assuming a non-linear two-parametric matter density distribution

$$\rho(r) = \rho_c [1 - (r/R)^\delta]^\gamma, \quad \delta > 0, \gamma > 0$$

and using the appropriate equations of state and energy generation rate, we have obtained mass, pressure, and temperature distributions throughout the nuclear powered gas sphere, and its luminosity in terms of Gauss' hypergeometric function and Kampé de Fériet's function [6].

c) The fundamental linear differential equation of fourth order governing gravitational instabilities, and covering a wide class of matter characterized by the adiabatic index  $\gamma_i$ , can be traced back to the differential equation satisfied by G-functions [7]:

$$\Delta^4 \Phi_i + \Delta^2 (b_i \Phi_i) - \frac{2}{3} (\Delta^2 \Phi_i + b_i \Phi_i) = -\frac{2}{3} \sum_{j=1}^m b_j \Omega_j \Phi_j$$

where  $i = 1, \dots, m$ , and  $\Delta = t d/dt$  is a time operator,  $\sum \Omega_j = 1$ ,  $\Phi_i = t^{-\alpha} \delta_i$ ,  $b_i = k_i^2 t^{\alpha_i} - \alpha^2$ ,  $\alpha_i = 2(2 - \eta - \gamma_i)$ ,  $\alpha = -(\frac{2\eta-1}{2})$ ,  $\eta$  and  $\gamma_i$  are constants.

Our program, in seeking to apply special functions to the above three problems, has evolved in the same spirit which special functions were originally introduced into mathematics; namely, to solve ordinary differential equations arising in physical problems while, instead of resorting to solutions in infinite series, to employ special functions and make use of their properties, thereby taking full advantage of contour integrals of the Mellin-Barnes type. This is likewise true for finding closed-form analytic representations of certain types of integrals as given above. The results emphasize once more that the hypergeometric function, as well as its generalizations, play a central role in mathematical analysis and theoretical physics.

Notwithstanding these developments one may encounter today the curious belief that the existence of computers makes the study of (perhaps even the use of) the special functions of mathematical physics somewhat obsolete. For differential equations can be solved numerically so that there is no need for closed-form representations of their solutions, and integrals can be evaluated numerically making the derivation of solutions in terms of special functions a mere academic exercise.

But the rich interplay between mathematics and physics, which has led to the field of mathematical physics, still continues today. As a result of developments in the theory of

analytic functions on the one hand, and electromagnetic field theory on the other, the nineteenth century was the golden age of special functions. There are two fundamental theories of physics in the twentieth century: general relativity and quantum field theory. Has the role of special functions really been replaced by the utilization of computing machines?

This brief note is a welcome opportunity to perhaps stimulate a discussion among members of the Activity Group about the relevance of modern generalized special function techniques in the study of problems in physics and astronomy, and the possible impact on these two fields.

#### References

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  - [2] S. KRASZNOVSZKY, "Analysis of the KW distribution using a generalization of the Meijer's G-function", *Mod. Phys. Letters*, **8** (1993), pp. 483-490.
  - [3] R.K. RAINA and H.M. SRIVASTAVA, "Evaluation of a certain class of Eulerian integrals", *J. Phys. A: Math. Gen.* **26** (1993), pp. 691-696.
  - [4] A.M. MATHAI, "A Handbook of Generalized Special Functions for Statistical and Physical Sciences", Clarendon Press, Oxford, 1993.
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  - [7] A.M. MATHAI and H.J. HAUBOLD, "Computational aspects of the gravitational instability problem for a multicomponent cosmological medium", McGill University Preprint, 1993.
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-



### The July Minisymposium in San Diego

SIAM's next annual meeting will be held in San Diego July 25–29, 1994, and we will sponsor a Minisymposium as we did last year. The title will be “Special Functions and Asymptotics”. This will take place during the morning and the afternoon of Friday, July 29. The co-organizers, Charles Dunkl and Martin Muldoon, have lined up a fair number of interesting talks. Here is the present setup as described recently by Martin:

Many problems arising in the natural sciences, engineering, combinatorics, statistics, etc., lead ultimately to approximating integrals or solving differential equations. The two topics of this minisymposium, special functions and asymptotics, describe basic solutions as well as techniques required to deal with such problems.

All but the simplest special functions are often best described by asymptotic information, typically as one or more of the variables approaches infinity; at the same time, the asymptotic behavior of more complicated functions is often best understood through the medium of known special functions. In this connection we should mention that Frank Olver's classic “*Asymptotics and Special Functions*”, Academic Press, 1974, stresses this duality. This minisymposium will explore a variety of current approaches to these classical subjects.

The speakers are:

Richard Askey, University of Wisconsin.

“Extensions of Hermite polynomials and other orthogonal polynomials”

Bruce Berndt, University of Illinois.

“Some asymptotic formulas of Ramanujan”

T. M. Dunster, San Diego State University.

“New uniform asymptotic approximations for Jacobi polynomials”

Ronald Evans, University of California at San Diego. “Multidimensional  $q$ -beta integrals”

George Gasper, Northwestern University.

“Applications of sums and integrals of squares of special functions”

Jeffrey S. Geronimo, Georgia Institute of Technology. “Asymptotics and spectral properties of orthogonal polynomials based on their recurrence coefficients”

Adri Olde Daalhuis, University of Maryland. “Hyperasymptotics”

André Ronveaux, FUNDP, Namur, Belgium.

“Heun's equations and the Schrödinger equation”

Renato Spigler, University of Padua, Italy.

“Discrete Liouville-Green approximations and orthogonal polynomial asymptotics”

Frank Stenger, University of Utah.

“Asymptotics methods complemented by numerical methods”

Audrey Terras, University of California at San Diego. “Comparison of special functions for finite and continuous symmetric spaces”

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### Meetings and Conferences

The Items #4, 5 and 6 are new here, as the others have already appeared in previous editions. Item #7 has been updated somewhat. At the SIAM annual meeting in July we will again sponsor our Minisymposium; you can read all about that earlier in this edition.

1. A session on the topic of special functions will be held during the March 25–26, 1994 meeting of the AMS in Manhattan, Kansas. Anyone who might be interested in presenting a paper at this meeting should please contact

Robert Gustafson  
Department of Mathematics  
Texas A&M University  
College Station, TX 77843,  
Tel: (409) 764-8933 and (409) 845-3950  
Fax: (409) 845-6028 rgustaf@math.tamu.edu

2. The 100th anniversary of T.J. Stieltjes' premature death will be commemorated in 1994-95. A number of activities are planned in Leiden, and also in Toulouse.

First of all, in April 1994 the Congress of Wiskundig Genootschap (The Dutch Mathematical Society) will be held in Leiden, where Richard Askey will give the Stieltjes Lecture. Contact:

Prof. Gerrit van Dijk  
Rijksuniversiteit Leiden  
Afdeling Wiskunde en Informatica  
P.O. Box 9512  
2300 RA Leiden, The Netherlands

Then in Toulouse there will be a colloquium in the Spring of 1995. The focus will be on continued fractions and moment problems, orthogonal polynomials, Laplace transforms, the Riemann hypothesis, and other topics. This will have a somewhat historical character. Contact:

Prof. J.-B. Hiriart-Urruty  
Groupe d'Histoire des Mathématiques  
de l'Université Paul Sabatier  
118, Route de Narbonne  
31062 Toulouse, France

3. The Thomas Jan Stieltjes Research Institute is a newly-organized inter-university mathematical research center located in the western part of The Netherlands. Within the framework of the Institute's program there will now be a period of concentration on representation theory and  $q$ -special functions.

Thus, between April and June, 1994, a number of guests are expected, for a longer or shorter time, and regular seminars will be held. The focus will be on Heckman-Opdam hypergeometric functions associated with root systems, and interpretations of Macdonald's orthogonal  $q$ -polynomials on quantum groups. The activity will take place both at the Universities of Leiden and Amsterdam.

The following have agreed to come for a period of two to eight weeks: Ivan V. Cherednik (Chapel Hill), Charles F. Dunkl (Charlottesville), Ian G. Macdonald (London), Masatoshi Noumi (Tokyo), Grigori I. Olshanski (Moscow). Further information can be obtained from Tom Koornwinder (thk@fwi.uva.nl) or from Eric Opdam (opdam@rubi.leidenuniv.nl).

4. The Joint Institute for Nuclear Research announces an International Workshop on "Finite Dimensional Integrable Systems" to be held in Dubna, Russia, July 18–21, 1994. The program includes invited talks as well as the shorter contributions to be followed by discussions.

Among the topics to be discussed will be classical and quantum integrable systems (with  $n$  degrees of freedom), supersymmetry aspects of integrable systems, separation of variables and special functions,  $q$ -special functions, and  $q$ -groups. About 60 scientists from various centers are planning to attend.

Registration for participants will be US\$ 250, and for non-participants it will be US\$ 150. This covers meals and lodging during the Workshop, the transportation to Dubna from Moscow and return, the social program, and a volume of abstracts. Payments will be accepted by the Organizing Committee in Dubna during the registration. A program for non-participants is also planned.

Authors should send their abstracts by e-mail to the Organizing Committee by April 30, 1994, and the abstracts should be in  $\LaTeX$ .

If you would like to participate, please apply before March 30, 1994 by Fax, Telex, or e-mail. Messages sent by e-mail, if not confirmed within a few days, should be sent again using Fax or Telex.

A.N. Sissakian is chairman of the organizing committee; George S. Pogosyan (Dubna) and Pavel Winternitz (Montreal) are vice-chairmen. Please apply to George Pogosyan, with a copy to Pavel Winternitz.

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For your visas, accommodations, and transportation please contact Elena Pankova at the above address in Dubna. Fax: (7 095) 975 23 81, Telex: 911621 dubna su, e-mail: pankova@cvxct0.jinr.dubna.su.

(continues on p. 8)

**Meetings and Conferences** (*continued*)

5. A Workshop on "Transform Methods and Special Functions" will be held in Sofia, Bulgaria, 12–17 August, 1994. The main subjects of the Workshop will be: Integral Transforms, Special Functions, Series Expansions, Fractional Calculus and Generalizations, Algebraical Analysis, Operational Calculus, Applications to Complex Analysis, Differential and Integral Equations.

Chairmen of the Organizing Committee are P. Rusev, I. Dimovski and S.L. Kalla. The Venue will be the resort town of Bankya, 20 km from Sofia.

Full board for the entire period will be approximately US\$ 150. This will include the lodging and three meals. The registration fee for participants is US\$ 100.

Since the rooms in Hotel Journalist should be booked long in advance, those who wish to participate are kindly asked to contact as soon as possible:

Prof. Virginia Kiryakova  
Institute of Mathematics  
Bulgarian Academy of Sciences  
Sofia 1090, Bulgaria  
Fax: (359 2) 752 078  
virginia@bgearn.bitnet

6. Patrick Van Fleet of Sam Houston State University in Huntsville, Texas has indicated that a conference called "Special Functions and Their Applications" may be held during August 25–27, 1994. This is contingent upon funding and the following have tentatively agreed to speak.

George Andrews, Penn State University  
Richard Askey, University of Wisconsin  
Bille C. Carlson, Iowa State University  
J. Dickey, University of Minnesota  
Robert Gustafson, Texas A&M University  
Edward Neuman, Southern Illinois University  
V. Retakh, Harvard University  
Roderick Wong, University of Manitoba

Graduate students are also encouraged to participate. Please contact

Patrick J. Van Fleet (mth\_pvf@shsu.edu) or  
Peter R. Massopust (mth\_prm@shsu.edu)  
Department of Mathematics  
Sam Houston State University  
Huntsville, TX 77341  
Tel. (409) 294-1493

7. Also in commemoration of Stieltjes there will be a conference with the title "Orthogonality, Moment Problems, and Continued Fractions" to be held at the Delft University of Technology, October 31–November 4, 1994.

Each of four days will feature a different aspect of the work of Stieltjes, from continued fractions, rational approximation, moment problems, orthogonal polynomials,

and asymptotics, to the properties of zeros and Gaussian quadrature. The format will consist of an invited lecture in the morning followed by the (short) communications.

The invited lectures are

C. Berg, "Indeterminate moment problems and the theory of entire functions"  
G. van Dijk, "Thomas Jan Stieltjes : mathematician by profession?"  
J.B. Hiriart-Urruty, "Differentiability and non-differentiability in mathematical problems"  
L. Lorentzen, "Continued fractions"  
F. Marcellan, "Differential-difference operators and orthogonal polynomials in Sobolev spaces"  
P. Nevai, "Generalized polynomials"  
F. Peherstorfer, "Stieltjes polynomials and Gauss-Kronrod quadrature"  
N.M. Temme, "Current problems in uniform asymptotic estimates of integrals"  
W. Van Assche, "The impact of Stieltjes' work on orthogonal polynomials"

A second announcement, including a registration form, has already gone out. This can be obtained by sending a letter to

TJS94, Mekelweg 4, kr. H4.11  
Department of Pure Mathematics  
Delft University of Technology  
P.O. Box 5031 2600 GA Delft  
The Netherlands

or by sending an e-mail to tjs94@twi.tudelft.nl.

If you want to contribute a short communication of about 20 minutes, please send an abstract (two A4 pages at most) to the above address before May 1, 1994.

The Proceedings of the conference will be published as a special issue of the Journal of Computational and Applied Mathematics. Manuscripts must be submitted in triplicate during, or before, the conference.

Conference fees are Dfl. 600 for participants, including the program and book of abstracts, a copy of the proceedings, all lunches and dinners, coffee, tea, plus an excursion.

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### Problems

Thus far eight problems have been submitted while three have been solved (#1, 4, 6). A printout of all the problems and the solutions is available from the Editor.

2. Is it true that

$$x^2 t^x {}_2F_1(x+1, x+1; 2; 1-t)$$

is a convex function of  $x$  whenever  $-\infty < x < \infty$  and  $0 < t < 1$ ?

Submitted by George Gasper, August 19, 1992.  
(g-gasper@nwu.edu)

3. The following Toeplitz matrix arises in several applications. Define for  $i \neq j$

$$A_{ij}(\alpha) = \frac{\sin \alpha \pi (i-j)}{\pi (i-j)},$$

and set  $A_{ii} = \alpha$ . Conjecture: the matrix

$$M = (I - A)^{-1}$$

has positive entries. A proof is known for  $0 < \alpha \leq 1/2$ . Can one extend this to  $0 < \alpha < 1$ ?

Submitted by Alberto Grünbaum, November 3, 1992.  
(grunbaum@math.berkeley.edu)

5. The result of Problem #4 can be generalized to

$$\begin{aligned} S_m &= \sum_{n=0}^{\infty} \frac{(-1)^n (mn + 1/2)!}{\sqrt{\pi} (mn + 1)!} \\ &= \frac{1}{m} \sum_{k=0}^{m-1} \frac{\sin(5(2k+1)\pi/(4m) + \pi/4)}{[2 \sin((2k+1)\pi/(2m))]^{1/2}} \end{aligned}$$

valid for integral  $m \geq 2$ .

Submitted by J. Boersma and P.J. de Doelder,  
July 12, 1993.  
(wstanal@win.tue.nl)

6. For nonnegative integral  $n$ , let

$$\phi_n(x) = P_n(1-2x) = {}_2F_1(-n, n+1; 1; x).$$

Evaluate the integral

$$\Delta_{m,n}(\alpha) = \int_0^1 \phi_n(x) \int_0^1 \frac{\phi_m(t)}{|x-t|^\alpha} dt dx, \quad (\alpha < 1)$$

as a rational function of  $\alpha$ .

Submitted by Barbara S. Bertram and Otto G. Ruehr,  
August 3, 1993.  
(bertram@math.mtu.edu otto@math.mtu.edu)

7. The incomplete Airy integral given by <sup>1</sup>

$$I_0(\sigma, \gamma; k) = \int_{\gamma}^{\infty} e^{jk(\sigma z + z^3/3)} dz \quad (1)$$

serves as a canonical integral for some sparsely explored diffraction phenomena involving the evaluation of high frequency EM fields <sup>2</sup> near terminated caustics and composite shadow boundaries. In equation (1),  $k$  is the wavenumber of the propagation medium and is assumed to be the large parameter. Both the parameters  $\sigma$  and  $\gamma$  are real.

The desired task is to derive a complete asymptotic expansion for  $I_0$  in inverse powers of  $k \rightarrow \infty$  for the case when the saddle points of the integrand satisfying

$$z^2 + \sigma = 0 \quad (2)$$

$$z_{1,2} = \pm(-\sigma)^{1/2} \quad (3)$$

are real and widely separated ( $\sigma \ll -1$ ). The asymptotic expansion should be of the form

$$I_0(\sigma, \gamma; k) \sim \sum_{n=0}^{\infty} k^{-n} f(\sigma, \gamma, n) \quad (4)$$

in which  $f(\sigma, \gamma, n)$  is expressed in terms of known and easily computed functions. The asymptotic expansion in (4) should also hold uniformly as the endpoint  $\gamma$  approaches, or coincides with, one of the saddle points.

Submitted by E.D. Constantinides and R.J. Marhefka,  
August 11, 1993.  
(evagoras@tiger.eng.ohio-state.edu)  
(rjm@tiger.eng.ohio-state.edu)

8. Can the real and imaginary parts of a hypergeometric series of type  ${}_pF_q$  with one complex parameter (either in the numerator or the denominator) be expressed by means of multiple hypergeometric series?

Submitted by Ernst D. Krupnikov, July 25, 1993.  
(ernst@net.neic.nsk.su)

Address: P.O. Box 300, Novosibirsk State University,  
Novosibirsk 90, 630090 Russian Federation.

<sup>1</sup> Electrical engineers use  $j$  for  $\sqrt{-1}$ , reserving  $i = v/r$  for current.

<sup>2</sup> See their brief article on electromagnetic (EM) diffraction in the Fall, 1993 issue of the Newsletter.

### Solutions to Problem #6

Three different solutions were received, including one from the proposers who indicate how the problem arose.

Solution to Problem #6

by P.J. DE DOELDER

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Through a change of variables  $u = 1 - 2x$ ,  $v = 1 - 2t$  we arrive at

$$\begin{aligned}\Delta_{m,n}(\alpha) &= 2^{\alpha-2} \int_{-1}^1 P_n(-u) du \int_{-1}^1 \frac{P_m(-v)}{|u-v|^\alpha} dv \\ &= (-1)^{m+n} \int_{-1}^1 P_n(u) du \int_{-1}^1 \frac{P_m(t)}{|t-x|^\alpha} dt \\ &= (-1)^{m+n} \Delta_{m,n}(\alpha)\end{aligned}\quad (1)$$

and this yields  $\Delta_{m,n}(\alpha) = 0$  for  $m+n$  odd.

For  $m+n$  even we calculate the integral

$$\begin{aligned}\int_{-1}^1 \frac{P_m(v)}{|v-u|^\alpha} dv &= \int_{-1}^u \frac{P_m(v)}{|v-u|^\alpha} dv + \int_u^1 \frac{P_m(v)}{|v-u|^\alpha} dv \\ &= J_1 + J_2.\end{aligned}$$

We substitute  $u-v = (1+u)s$  into  $J_1$  and  $v-u = (1-u)s$  into  $J_2$ . Since  $P_m(x) = {}_2F_1(-m, m+1; 1; \frac{1}{2}(1-x))$ , we get

$$\begin{aligned}J_1 &= (-1)^m \sum_{k=0}^m \frac{(-m)_k (m+1)_k}{(k!)^2} \left(\frac{1+u}{2}\right)^k \times \\ &\quad \times (1+u)^{1-\alpha} \int_0^1 (1-s)^k s^{-\alpha} ds \\ J_2 &= \sum_{k=0}^m \frac{(-m)_k (m+1)_k}{(k!)^2} \left(\frac{1-u}{2}\right)^k \times \\ &\quad \times (1-u)^{1-\alpha} \int_0^1 (1-s)^k s^{-\alpha} ds.\end{aligned}$$

Substituting these expressions for  $J_1$  and  $J_2$  into (1) we obtain the relation

$$\begin{aligned}\Delta_{m,n}(\alpha) &= 2^{\alpha-2} \sum_{k=0}^m \frac{(-m)_k (m+1)_k \Gamma(1-\alpha)}{2^k k! \Gamma(k+2-\alpha)} (-1)^m \times \\ &\quad \times \int_{-1}^1 ((1+u)^{k+1-\alpha} P_m(u) + (1-u)^{k+1-\alpha} P_n(u)) du.\end{aligned}\quad (2)$$

Replacing  $u$  by  $-u$  we find

$$\int_{-1}^1 P_n(u)(1+u)^{k+1-\alpha} du = \int_{-1}^1 P_n(-u)(1-u)^{k+1-\alpha} du$$

$$= (-1)^n \int_{-1}^1 P_n(u)(1-u)^{k+1-\alpha} du.$$

Next, from I.S. GRADSHTEYN and I.M. RYZHIK, *Table of Integrals*, 1965, p.797 we take the formula (7.127):

$$\int_{-1}^1 P_n(x)(1+x)^\sigma dx = \frac{2^{\sigma+1} \Gamma^2(\sigma+1)}{\Gamma(\sigma+n+2) \Gamma(1+\sigma-n)}.$$

Substituting this result into (2) it is obvious that

$$\begin{aligned}\Delta_{m,n}(\alpha) &= \sum_{k=0}^m \frac{(-m)_k (m+1)_k}{k!} ((-1)^m + (-1)^n) \times \\ &\quad \times \frac{\Gamma(1-\alpha) \Gamma(k+2-\alpha)}{\Gamma(k+3-\alpha+n) \Gamma(k+2-\alpha-n)}.\end{aligned}$$

Using Pochhammer symbols it is easy to show that

$$\begin{aligned}\Delta_{m,n}(\alpha) &= ((-1)^m + (-1)^n) \sum_{k=0}^m \frac{(-m)_k (m+1)_k}{k!} \times \\ &\quad \times \frac{\Gamma(1-\alpha) \Gamma(2-\alpha) (2-\alpha)_k}{\Gamma(3-\alpha) (3-\alpha)_n (3-\alpha+n)_k} \times \\ &\quad \times \frac{1}{\Gamma(2-\alpha) (2-\alpha-n)_k (2-\alpha)_n} \\ &= \frac{(\alpha-1)_n (-1)^n ((-1)^m + (-1)^n)}{(1-\alpha) (2-\alpha) (3-\alpha)_n} \times \\ &\quad \times \sum_{k=0}^m \frac{(-m)_k (m+1)_k (2-\alpha)_k}{(3-\alpha+n)_k (2-\alpha-n)_k k!} \\ &= \frac{2(\alpha-1)_n}{(1-\alpha) (2-\alpha) (3-\alpha)_n} \times \\ &\quad \times \sum_{k=0}^m \frac{(-m)_k (m+1)_k (2-\alpha)_k}{(3-\alpha+n)_k (2-\alpha-n)_k k!} \\ &= \frac{2(\alpha-1)_n}{(1-\alpha) (2-\alpha) (3-\alpha)_n} \times \\ &\quad \times {}_3F_2 \left( \begin{matrix} -m, & m+1, & 2-\alpha; & 1 \\ 3-\alpha+n, & 2-\alpha-n, \end{matrix} \right).\end{aligned}$$

The above  ${}_3F_2$  series can be summed by applying the Whipple summation formula:

$$\begin{aligned}{}_3F_2(a, 1-a, c; e, 1+2c-e; 1) &= \\ &= \frac{2^{1-2c} \pi \Gamma(e) \Gamma(1+2c-e)}{\Gamma(\frac{e+a}{2}) \Gamma(\frac{1+e-a}{2}) \Gamma(\frac{1+2c-e+a}{2}) \Gamma(\frac{2+2c-e-a}{2})}.\end{aligned}$$

[See for example L. SLATER, *Generalized Hypergeometric Functions*, 1966, page 54, formula (2.33.14)]. In our case  $a = -m$ ,  $c = 2 - \alpha$ ,  $d = 3 - \alpha + n$ , and  $e = 2 - \alpha - n$ . The condition  $c > 0$  is realized. We see that

$$\sum_{k=0}^m \frac{(-m)_k (m+1)_k (2-\alpha)_k}{(3+\alpha-n)_k (2-\alpha-n)_k k!} =$$

$$= \frac{2^{(2\alpha-3)} \pi \Gamma(2-\alpha-n) \Gamma(3-\alpha+n)}{\Gamma(1-\frac{\alpha}{2}-\frac{n+m}{2}) \Gamma(\frac{3}{2}-\frac{\alpha}{2}+\frac{m-n}{2})} \times \\ \times \frac{1}{\Gamma(\frac{3}{2}-\frac{\alpha}{2}+\frac{n-m}{2}) \Gamma(2-\frac{\alpha}{2}+\frac{m+n}{2})}.$$

From this result it appears that

$$\Delta_{m,n}(\alpha) = \frac{2^{2\alpha-2} \pi (\alpha-1)_n \Gamma(2-\alpha-n) \Gamma(3-\alpha+n)}{(1-\alpha)(2-\alpha)(3-\alpha)_n \Gamma(1-\frac{\alpha}{2}-\frac{n+m}{2})} \times \\ \times \frac{1}{\Gamma(\frac{3}{2}-\frac{\alpha}{2}+\frac{m-n}{2}) \Gamma(\frac{3}{2}-\frac{\alpha}{2}-\frac{m-n}{2}) \Gamma(2-\frac{\alpha}{2}+\frac{m+n}{2})}.$$

Using the relation  $(a)_{-n} = (-1)^n / (1-a)_n$  and the duplication formula for the  $\Gamma$ -function, we conclude that

$$\Delta_{m,n}(\alpha) = \frac{(\frac{\alpha}{2})_{(m+n)/2} (\frac{\alpha-1}{2})_{(m-n)/2}}{(1-\alpha)(\frac{3-\alpha}{2})_{(m-n)/2} (1-\frac{\alpha}{2})_{(m+n)/2}} \times \\ \times \frac{1}{(1-\frac{\alpha}{2}+\frac{m+n}{2})}$$

for  $m+n$  even. Now it is clear that  $\Delta_{m,n}(\alpha)$  is a rational function of  $\alpha$ .

*Remarks:*

1)

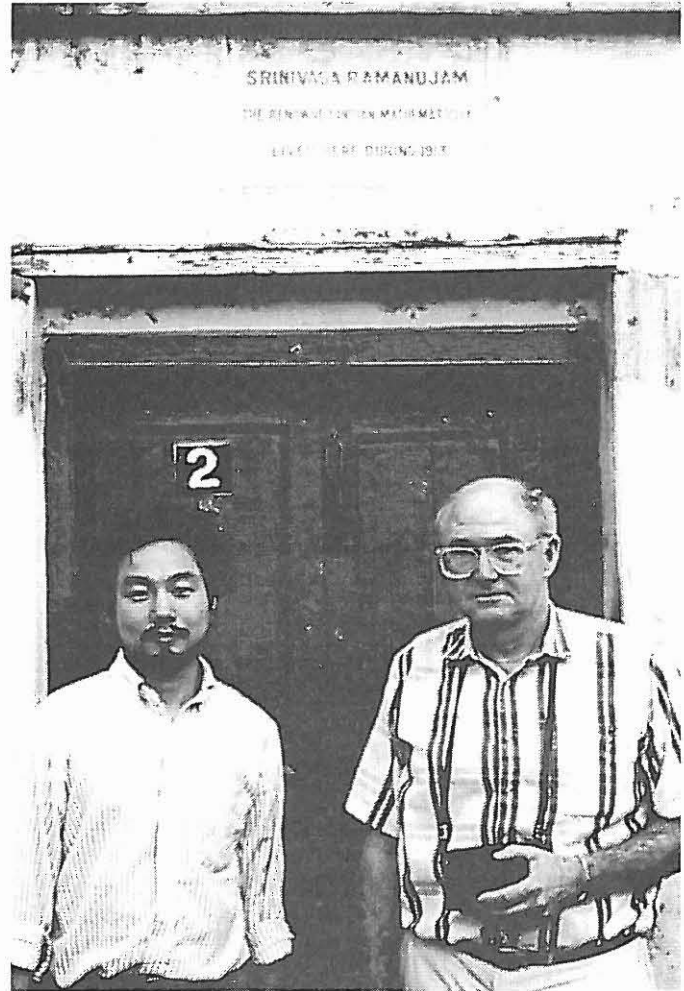
$$\Delta_{m,n}(0) = \begin{cases} 0 & \text{if } (m,n) \neq (0,0) \\ 1 & \text{if } (m,n) = (0,0). \end{cases}$$

2) The result is symmetric in  $m$  and  $n$  since

$$((\alpha-1)/2)_{(m-n)/2} = \frac{(-1)^{(m-n)/2}}{((3-\alpha)/2)_{(n-m)/2}}$$

and conversely.

(Received October 28, 1993)



Wei Cai and Alan Law in Madras. The inscription reads:

SRINIVASA RAMANUJAM  
THE RENOWNED INDIAN MATHEMATICIAN  
LIVED HERE DURING 1913.

Solution to Problem #6

by JOHANNES BOERSMA

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Rewrite the integral as

$$\Delta_{m,n}(\alpha) = 2^{\alpha-2} \int_{-1}^1 P_n(x) \int_{-1}^1 \frac{P_m(t)}{|x-t|^\alpha} dt dx.$$

Then by means of Parseval's formula and the convolution theorem for the Fourier Transform, the integral is reduced to

$$\begin{aligned} \Delta_{m,n}(\alpha) &= \frac{2^{\alpha-2}}{2\pi} \int_{-\infty}^{\infty} \mathcal{F}\{P_n(x) \chi_{[-1,1]}\} \times \\ &\quad \times \overline{\mathcal{F}\{P_m(x) \chi_{[-1,1]}\}} \overline{\mathcal{F}\{|x|^{-\alpha}\}} dy \quad (*) \end{aligned}$$

where  $\mathcal{F}$  denotes the Fourier transform, defined as

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{iyx} dx.$$

The necessary Fourier transforms in (\*) are taken from Reference [1, formulas 3.3(1) and 1.3(1)], namely

$$\begin{aligned} \mathcal{F}\{P_n(x) \chi_{[-1,1]}\} &= \int_{-1}^1 P_n(x) e^{iyx} dx \\ &= \sqrt{2\pi} i^n y^{-1/2} J_{n+\frac{1}{2}}(y) \end{aligned}$$

$$\begin{aligned} \mathcal{F}\{|x|^{-\alpha}\} &= 2 \int_0^{\infty} x^{-\alpha} \cos(yx) dx \\ &= 2 \sin(\alpha\pi/2) \Gamma(1-\alpha) |y|^{\alpha-1} \end{aligned}$$

where  $J_{n+\frac{1}{2}}(y)$  stands for the Bessel function of the first kind of order  $n + \frac{1}{2}$ .

It is easily recognized that the integrand in (\*) is even (odd) in  $y$  if  $m+n$  is even (odd). Hence  $\Delta_{m,n}(\alpha) = 0$  if  $m+n$  is odd, while for  $m+n$  even we have

$$\begin{aligned} \Delta_{m,n}(\alpha) &= 2^\alpha (-1)^{(n-m)/2} \sin(\alpha\pi/2) \Gamma(1-\alpha) \times \\ &\quad \times \int_0^{\infty} J_{m+\frac{1}{2}}(y) J_{n+\frac{1}{2}}(y) y^{\alpha-2} dy. \end{aligned}$$

The last integral above is known from Watson's book [2, form.13.41(2)] and we find

$$\begin{aligned} \Delta_{m,n}(\alpha) &= 2^{2\alpha-2} (-1)^{(n-m)/2} \sin(\alpha\pi/2) \times \\ &\quad \times \frac{\Gamma(1-\alpha) \Gamma(2-\alpha) \Gamma(\frac{\alpha+m+n}{2})}{\Gamma(\frac{3-\alpha-m+n}{2}) \Gamma(\frac{3-\alpha+m-n}{2}) \Gamma(\frac{4-\alpha+m+n}{2})} \end{aligned}$$

valid for  $\alpha < 1$  and  $m+n$  even.

By some standard manipulations of the  $\Gamma$ -functions,  $\Delta_{m,n}(\alpha)$  is expressible as a rational function of  $\alpha$ , namely

$$\Delta_{m,n}(\alpha) = \frac{2}{(1-\alpha)(2-\alpha)} \frac{(\frac{\alpha-1}{2})_{(m-n)/2} (\frac{\alpha}{2})_{(m+n)/2}}{(\frac{3-\alpha}{2})_{(m-n)/2} (\frac{4-\alpha}{2})_{(m-n)/2}}$$

valid for  $\alpha < 1$ ,  $m+n$  even, and  $m \geq n$ . Pochhammer's symbol,  $(a)_k$ , is defined here in the usual way:

$$(a)_0 = 1; \quad (a)_k = a(a+1) \cdots (a+k-1), \quad k = 1, 2, \dots$$

*Remark.* The integral

$$\Lambda_{m,n}(\alpha) = \int_0^1 \phi_n(x) \int_0^1 \frac{\text{sgn}(x-t)}{|x-t|^\alpha} \phi_m(t) dt dx$$

can be evaluated in the same manner, with the result

$$\begin{aligned} \Lambda_{m,n}(\alpha) &= 0 \quad \text{if } \alpha \leq 1, m+n \text{ even;} \\ \Lambda_{m,n}(\alpha) &= \frac{2}{(2-\alpha)(3-\alpha)} \times \\ &\quad \times \frac{(\frac{\alpha}{2})_{(m-n-1)/2} (\frac{\alpha+1}{2})_{(m+n-1)/2}}{(\frac{4-\alpha}{2})_{(m-n-1)/2} (\frac{5-\alpha}{2})_{(m+n-1)/2}} \end{aligned}$$

if  $\alpha \leq 1$ ,  $m+n$  odd, and  $m > n$ . Notice that these results remain valid for  $\alpha = 1$  provided the Cauchy principal value of the integral is taken.

#### References

- [1] A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER and F.G. TRICOMI. *Tables of Integral Transforms*, Vol.1, McGraw-Hill, New York, 1954.
- [2] G.N. WATSON. *A Treatise on the Theory of Bessel Functions*, Second Edition. Cambridge University Press, 1958.

(Received January 26, 1994)

## Solution to Problem #6

by BARBARA S. BERTRAM and OTTO G. RUEHR

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Define coefficients  $q_{n,j}$  by

$$\phi_n(x) = \sum_{j=0}^n q_{n,j} x^j.$$

Note that

$$\Delta_{m,n} = \Delta_{n,m} = (-1)^{n+m} \Delta_{m,n}$$

since  $\phi_n(1-x) = (-1)^n \phi_n(x)$ .To calculate  $\Delta_{m,n}$  we break the inner integral into two parts and use the symmetries described to find that

$$\Delta_{m,n} = (1 + (-1)^{n+m}) T_{m,n}$$

where

$$T_{m,n} = \sum_{j=0}^n q_{n,j} \sum_{s=0}^m q_{m,s} \times \\ \times \int_0^1 x^j \int_0^x \frac{t^s}{(x-t)^\alpha} dt dx.$$

The integral is easily evaluated and we have, with a change in the order of summation,

$$T_{m,n} = \sum_{s=0}^m \frac{\Gamma(1-\alpha) \Gamma(s+1) q_{m,s}}{\Gamma(s+2-\alpha)} \times \\ \times \sum_{j=0}^n \frac{q_{n,j}}{(j+s-\alpha+2)}.$$

The inner sum is proportional to a balanced, terminating  ${}_3F_2$  hypergeometric series with a unit argument. Applying Saalschütz's Theorem, we obtain

$$\sum_{j=0}^n \frac{q_{n,j}}{(j+s-\alpha+2)} = \\ = \frac{{}_3F_2(-n, n+1, s-\alpha+2; 1, s-\alpha+3; 1)}{s-\alpha+2} \\ = \frac{\Gamma(n+\alpha-s-1) \Gamma(s+2-\alpha)}{\Gamma(\alpha-s-1) \Gamma(n+s-\alpha+3)}.$$

Routine adjustments yield another terminating  ${}_3F_2$  with a unit argument. In this case Whipple's Theorem applies. With a liberal use of the Legendre duplication formula, we find that

$$T_{m,n} = \frac{\Gamma(1-\alpha) \Gamma(n+\alpha-1)}{\Gamma(n+3-\alpha) \Gamma(\alpha-1)} \times$$

$$\times {}_3F_2(-m, m+1, 2-\alpha; n-\alpha+3, 2-\alpha-n; 1)$$

$$= \frac{\Gamma(1-\alpha) \Gamma(n+\alpha-1) \Gamma(\frac{n-\alpha+3}{2})}{\Gamma(n+3-\alpha) \Gamma(\alpha-1) \Gamma(\frac{n-\alpha+3-m}{2})} \times \\ \times \frac{\Gamma(\frac{n-\alpha+4}{2}) \Gamma(\frac{2-\alpha-n}{2}) \Gamma(\frac{3-\alpha-n}{2})}{\Gamma(\frac{n-\alpha+4+m}{2}) \Gamma(\frac{2-\alpha-n-m}{2}) \Gamma(\frac{3-\alpha-n+m}{2})}.$$

The following identity, easily proved by induction, is useful in reducing our results to elementary form:

$$\frac{\Gamma(\frac{a}{2}) \Gamma(\frac{1+a}{2})}{\Gamma(\frac{a-m}{2}) \Gamma(\frac{a+m+1}{2})} = \prod_{i=0}^{[(m-1)/2]} \frac{a-m+2i}{a+m-1-2i}.$$

Using this identity we obtain

$$T_{m,n} = \frac{1}{(1-\alpha)(2-\alpha)} \prod_{k=1}^n \frac{\alpha+k-2}{k+2-\alpha} \times \\ \times \prod_{i=0}^{[(m-1)/2]} \frac{(n-m+3-\alpha+2i)(2-\alpha-n-m+2i)}{(n+m+2-\alpha-2i)(1-\alpha-n+m-2i)}.$$

With tedious, but elementary, algebra we simplify the above expression and obtain finally

$$\Delta_{m,n} = \frac{1 + (-1)^{n+m}}{(1-\alpha)(2-\alpha)} \times \\ \times \prod_{s=1}^{[(n-m)/2]} \frac{\alpha-3+2s}{1-\alpha+2s} \prod_{t=1}^{(n+m)/2} \frac{\alpha-2+2t}{2-\alpha+2t}.$$

Note once more that both  $m+n$  and  $n-m$  are even integers or the expression vanishes.

This problem arose in studying eigenvalues for the weakly singular integral operator

$$(Lf)(x) = \int_0^1 \frac{f(t)}{|x-t|^\alpha} dt.$$

Using the closed form given above greatly facilitated the numerical calculation of as many as 1500 eigenvalues for an appropriate discretization.

(Received 10 February, 1994)



**L<sup>A</sup>T<sub>E</sub>X, et cetera**

Please send your Newsletter contributions to the Editor

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etomer@netcom.com

The preferred mode is a L<sup>A</sup>T<sub>E</sub>X file sent by e-mail and this should be followed by a hard copy. At this stage of the technology a hard copy will be necessary if you have graphs or drawings. Here is the L<sup>A</sup>T<sub>E</sub>X *preamble* you will need when setting up your manuscript.

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If flawless L<sup>A</sup>T<sub>E</sub>X and e-mail are not suitable, then almost any other mode will do—provided the result is readable. A typewritten Fax will be fine.

Like most newsletters, this one relies on input from the members it is supposed to serve. Below is the publication schedule. Material received on or before a *Draft #1* date can be included in that particular edition. Any material received after such a date will have to be considered for a later edition.

<i>Edition</i>	<i>Draft #1</i>	<i>Mailing ≈</i>
Fall	August 10	September 1
Winter	November 10	December 1
Spring	February 10	March 1
Summer	May 10	June 1

With last minute items turnaround time can be short, so check over your material for accuracy and completeness before submitting it.

Since the membership embraces many languages, errors in an ümlaut, cedilla, or háček may be conspicuous if they occur in a name. Also, first names rather than first initials are preferred, in view of the wide geographical dispersion. For given only a first initial (with no address or affiliation), you cannot always prove the uniqueness of an individual even if you have every memberlist.

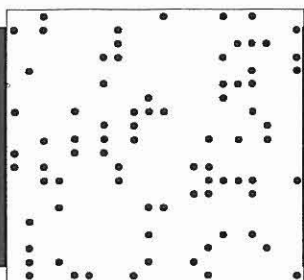
You may also send your contributions to the Chair of the Activity Group

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The **Newsletter** is a quarterly publication of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*. The Editorial Committee consists of Charles Dunkl, George Gasper, and Eugene Torner.

To join the group and thereby receive the Newsletter, please contact

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# CONTEMPORARY PROBLEMS IN STATISTICAL PHYSICS

EDITED BY **GEORGE H. WEISS**

This collection of independent articles describes some mathematical problems recently developed in statistical physics and theoretical chemistry. The book introduces and reviews current research on such topics as nonlinear systems and colored noise, stochastic resonance, percolation, the trapping problem in the theory of random walks, and diffusive models for chemical kinetics. Some of these topics have never before been presented in an expository book form.

Applied mathematicians will be introduced to some of the contemporary problems in statistical physics. In addition, a number of unsolved problems currently attracting intensive research efforts are described, and some of the techniques used in this research are outlined, along with principal results and outstanding questions.

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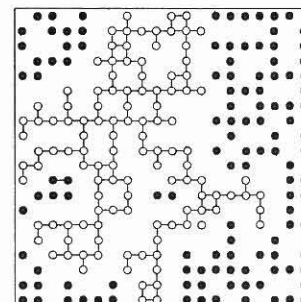
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*George H. Weiss* is Chief of the Physical Sciences Laboratory, Division of Computer Research and Technology at the National Institutes of Health, Bethesda, Maryland.

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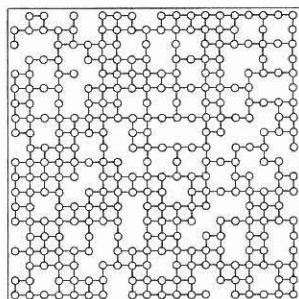
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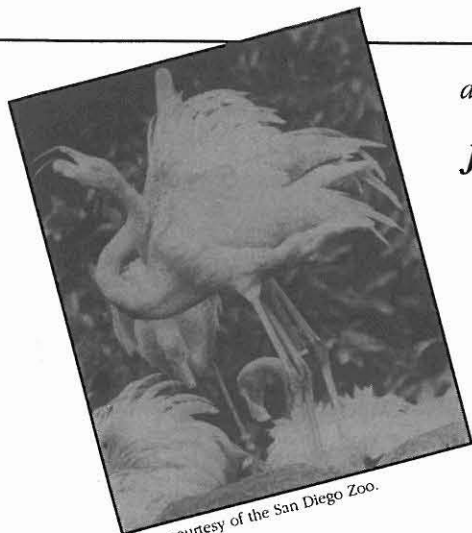


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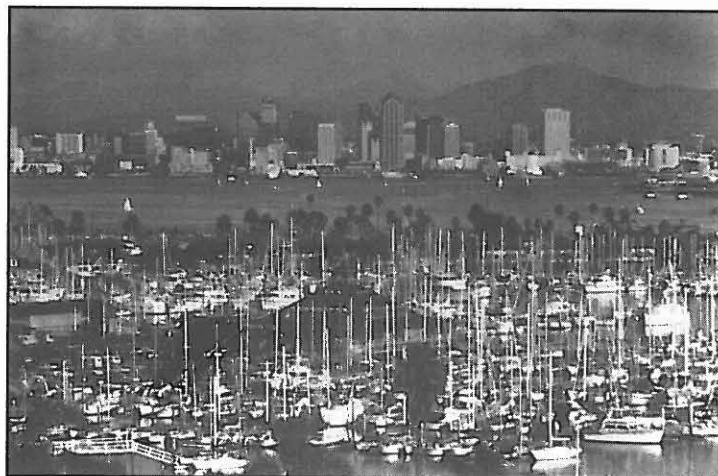


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