
MR2102896 (2005j:30028)
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Positivity and monotony properties of the de Branges functions. (English summary)
J. Comput. Appl. Math. **173** (2005), no. 2, 279–294.
30C50 (33C20 33C90)

The de Branges functions viz. $\tau_k^n(t)$ and $\Lambda_k^n(t)$ which arose in the famous de Branges conjecture and which are expressed in terms of hypergeometric functions by

$$\tau_k^n(t) = e^{-kt}\binom{n+k+1}{2k+1}\,{}_4F_3\left[\begin{matrix} k+1/2, n+k+2, k, k-n; \\ k+1, 2k+1, k+3/2; \end{matrix}\; e^{-t}\right]$$

and

$$\Lambda_k^n(t) = e^{-kt}\binom{n+k+1}{2k+1}\,{}_3F_2\left[\begin{matrix} k+1/2, n+k+2, k-n; \\ 2k+1, k+3/2; \end{matrix}\; e^{-t}\right],$$

satisfy the connecting relationship

$$\frac{d}{dt}\tau_k^n(t) = -k\Lambda_k^n(t).$$

The function $\Lambda_k^n(t)$ is attributed to L. Weinstein [Internat. Math. Res. Notices **1991**, no. 5, 61–64; MR1131432 (92m:30033)].

This interesting paper under review, in addition to lucidly giving relevant background details and valuable references in connection with the de Branges conjecture, derives various properties of the aforementioned functions which include recurrence relations and monotonicity and positivity properties. The positivity of the functions $\tau_k^n(t)$ and $\Lambda_k^n(t)$ leads the authors to consider the positivity properties of certain hypergeometric functions and to investigate the non-negativity of the difference of the de Branges and the Weinstein functions.

Reviewed by *R. K. Raina*