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Masjed-Jamei, Mohammad (IR-KNTUS); **Koepf, Wolfram** (D-UKSL)
On incomplete symmetric orthogonal polynomials of Jacobi type. (English summary)
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In this contribution, the authors introduce an "incomplete" sequence $(\Phi_k)_{k\in\Lambda}$ of polynomials with $\deg \Phi_k = k$, orthogonal with respect to the weight function $|x|^{2a}(1-x^{2m})^b$ in the interval $[-1,1]$, where $a = m-1+\frac{q}{2}$, $b=-1-\frac{p+q}{2m}$, and m is a fixed nonnegative integer number. Here $\Lambda=\Lambda_1\cup\Lambda_2$, where

$$\Lambda_1=\{2(s+mn); n\in\mathbb{N}\} \text{ and } \Lambda_2=\{2(r+mn)+1; n\in\mathbb{N}\}.$$

Such a family of polynomials is obtained from the shifted Jacobi polynomials with parameters $(\frac{q+4s-1}{2m}, b)$ (respectively, $(\frac{q+4r+1}{2m}, b)$) using a change in the variable $x\mapsto x^{2m}$ and the multiplication by x^{2s} (respectively, x^{2r+1}).

A second-order linear differential equation with polynomial coefficients (generalized Sturm-Liouville problem) that these polynomials satisfy is deduced. Its square norm with respect to the weight function is explicitly given. Finally, these results are analyzed when (i) $m=a=2$ and $b=1/2$ and (ii) $m=1$. This last situation yields an extension of generalized ultraspherical polynomials.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.