```
> restart;
> read "REtoqDE.mpl";
```

Package "q-Hypergeometric Summation", Maple V-2019

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Package "Hypergeometric Summation", Maple V - Maple 2019

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Example 2

```
> jnsum := 2^n*(a*d-b*c)^n*pochhammer(-p+1-I*q*(1/2), n)*hyperterm(
  [-n, n+1-2*p], [-p+1-I*q*(1/2)], I*(a^2+c^2)*(x-(I*(a*d-b*c)-a*b-
  c*d)/(a^2+c^2))/(2*(a*d-b*c)), j)/I^n;
```

$$jnsum := \frac{1}{\mathrm{pochhammer}\left(-p+1-\frac{1}{2}\,\mathrm{I}\,q, j\right) j!\,\mathrm{I}^n}\left(2^n\,(a\,d-b\,c)^n\,\mathrm{pochhammer}\left(-p+1-\frac{1}{2}\,\mathrm{I}\,q, n\right)\mathrm{pochhammer}(-n, j)\,\mathrm{pochhammer}(n+1-2\,p, j)\left(\frac{\mathrm{I}\,(a^2+c^2)\left(x-\frac{\mathrm{I}\,(a\,d-b\,c)-a\,b-c\,d}{a^2+c^2}\right)}{2\,a\,d-2\,b\,c}\right)^j\right) \tag{2}$$

```
> Rjn := sumrecursion(jnsum, j, S(n), recursion = up);
```

$$Rjn := (n+1-p)\,(n+2-2\,p)\,S(n+2)+(2\,n+3-2\,p)\,\big(2\,a^2\,x\,n^2-4\,a^2\,x\,p\,n+2\,a^2\,p^2\,x+2\,c^2\,x\,n^2-4\,c^2\,x\,p\,n+2\,c^2\,p^2\,x+6\,a^2\,x\,n-6\,a^2\,x\,p+2\,a\,b\,n^2-4\,a\,b\,p\,n+2\,a\,b\,p^2-a\,d\,p\,q+b\,c\,p\,q+6\,c^2\,x\,n-6\,c^2\,x\,p+2\,c\,d\,n^2-4\,c\,d\,p\,n+2\,c\,d\,p^2+4\,a^2\,x+6\,a\,b\,n-6\,a\,b\,p+4\,c^2\,x+6\,c\,d\,n-6\,c\,d\,p+4\,a\,b+4\,c\,d\big)\,S(n+1)-(n+1)\,(-p+2+n)\,\big(4\,n^2-8\,n\,p+4\,p^2+q^2+8\,n-8\,p+4\big)\,(a\,d-b\,c)^2\,S(n)=0 \tag{3}$$

```
> REtoJacobi(Rjn, S(n), x);
```

"Warning, parameters have the values", $\left\{\left\{aJ=-\frac{1}{2}\,\mathrm{I}\,q-p, bJ=\frac{1}{2}\,\mathrm{I}\,q-p, f=-\frac{\mathrm{I}\,(a^2+c^2)}{a\,d-b\,c}, g=-\frac{\mathrm{I}\,(a\,b+c\,d)}{a\,d-b\,c}\right\}, \left\{aJ=\frac{1}{2}\,\mathrm{I}\,q-p, bJ=-\frac{1}{2}\,\mathrm{I}\,q-p, f=\frac{\mathrm{I}\,(a^2+c^2)}{a\,d-b\,c}, g=\frac{\mathrm{I}\,(a\,b+c\,d)}{a\,d-b\,c}\right\}\right\}$

"Warning, several solutions found"

$$\Big[\Big[\sigma(x)=a^2\,x^2+c^2\,x^2+2\,a\,b\,x+2\,c\,d\,x+b^2+d^2, \tau(x)=-2\,a^2\,p\,x-2\,c^2\,p\,x+2\,a^2\,x-2\,a\,b\,p+a\,d\,q-b\,c\,q+2\,c^2\,x-2\,c\,d\,p+2\,a\,b+2\,c\,d, \lambda_n=-n\,(a^2+c^2)\,(n+1 \tag{4}$$

$-2p), S(n,x) = P_n\left(-\frac{1}{2}\,\mathrm{I}\,q - p, \frac{1}{2}\,\mathrm{I}\,q - p, -\frac{\mathrm{I}\,(a^2+c^2)\,x}{a\,d-b\,c} - \frac{\mathrm{I}\,(a\,b+c\,d)}{a\,d-b\,c}\right), w(x)$
$= \left(a^2x^2+c^2x^2+2\,a\,b\,x+2\,c\,d\,x+b^2+d^2\right)^{-p} \mathrm{e}^{\arctan\left(\frac{a^2x+c^2x+ab+cd}{ad-bc}\right)q}, \frac{k_{n+1}}{k_n} =$
$-\frac{2\,\mathrm{I}\,(a\,d-b\,c)\,(2\,n+1-2\,p)\,(n+1-p)}{n+1-2\,p}, \mathrm{I} = \left[-\frac{b+\mathrm{I}\,d}{a+\mathrm{I}\,c}, -\frac{\mathrm{I}\,d-b}{\mathrm{I}\,c-a}\right]\Bigg]\Bigg], \Bigg[\sigma(x) = a^2x^2$
$+c^2x^2+2\,a\,b\,x+2\,c\,d\,x+b^2+d^2, \tau(x) = -2\,a^2\,p\,x-2\,c^2\,p\,x+2\,a^2\,x-2\,a\,b\,p+a\,d\,q$
$-b\,c\,q+2\,c^2\,x-2\,c\,d\,p+2\,a\,b+2\,c\,d, \lambda_n = -n\,(a^2+c^2)\,(n+1-2\,p), S(n,x)$
$= P_n\left(\frac{1}{2}\,\mathrm{I}\,q-p, -\frac{1}{2}\,\mathrm{I}\,q-p, \frac{\mathrm{I}\,(a^2+c^2)\,x}{a\,d-b\,c} + \frac{\mathrm{I}\,(a\,b+c\,d)}{a\,d-b\,c}\right), w(x) = \left(a^2x^2+c^2x^2\right.$
$\left.+2\,a\,b\,x+2\,c\,d\,x+b^2+d^2\right)^{-p} \mathrm{e}^{\arctan\left(\frac{a^2x+c^2x+ab+cd}{ad-bc}\right)q}, \frac{k_{n+1}}{k_n}$
$= \frac{2\,\mathrm{I}\,(a\,d-b\,c)\,(2\,n+1-2\,p)\,(n+1-p)}{n+1-2\,p}, \mathrm{I} = \left[-\frac{\mathrm{I}\,d-b}{\mathrm{I}\,c-a}, -\frac{b+\mathrm{I}\,d}{a+\mathrm{I}\,c}\right]\Bigg]\Bigg]$

Example 4

```
> qhsum := subs([alpha = 2*beta, beta = 1], qphihyperterm([q^(-n),
  alpha*beta*q^(n+1), x], [alpha*q, q^(-N)], q, q, j));
```

$$qhsum := \frac{qpochhammer(q^{-n}, q, j)\; qpochhammer(2\,\beta\,q^{n+1}, q, j)\; qpochhammer(x, q, j)\; q^j}{qpochhammer(2\,\beta\,q, q, j)\; qpochhammer(q^{-N}, q, j)\; qpochhammer(q, q, j)} \qquad \textbf{(5)}$$

```
> RE := qsumrecursion(qhsum, q, j, S(n), recursion = up);
```

$$RE := \left(2\,q^{n+2}\beta-1\right)^2\left(-q^{n+1}+q^N\right)\left(2\,q^{2n+2}\beta-1\right)S(n+2) - \left(2\,q^{2n+3}\beta - 1\right)\left(4\,q^{N+4n+6}x\,\beta^2 - 8\,q^{N+3n+5}\beta^2 + 4\,q^{N+2n+4}\beta^2 - 2\,q^{N+2n+4}x\,\beta - 4\,q^{3n+4}\beta^2 + 4\,q^{N+2n+3}\beta^2 + 2\,q^{N+2n+4}\beta - 2\,q^{3n+4}\beta + 2\,q^{N+2n+3}\beta - 2\,q^{N+2n+2}x\,\beta + 4\,q^{2n+3}\beta - 4\,q^{N+n+2}\beta + 4\,q^{2n+2}\beta - 2\,\beta\,q^{n+1} + x\,q^N - q^{n+1}\right) S(n+1) + 2\left(2\,q^{N+n+2}\beta-1\right)q^{n+1}\left(2\,q^{2n+4}\beta-1\right)\beta\left(q^{n+1}-1\right)^2 S(n) = 0 \qquad \textbf{(6)}$$

```
> REtoqde(RE, S(n), x, q);
```

"Warning, parameters have the values", $\{\{aB = 1, bB = 2\,\beta, cB = 2\,q^{1+N}\beta, f = q^{1+N}, g = 0\}, \{aB = q^{-1-N}, bB = 2\,q^{1+N}\beta, cB = 2\,\beta, f = 1, g = 0\}, \{aB = 2\,\beta, bB = 1, cB = q^{-1-N}, f = 1, g = 0\}, \{aB = 2\,q^{1+N}\beta, bB = q^{-1-N}, cB = 1, f = q^{1+N}, g = 0\}\}$

"Warning, several solutions found"

"Warning, parameters have the values", $\{\{N=N, aB=0, bB=bB, \beta=0, f=q^N, g=0, q=q, q^N=q^N\}, \{N=N, aB=2\beta, bB=1, \beta=\beta, f=q^N, g=0, q=0, q^N=q^N\}\}$

"Warning, parameters have the values", $\{\{N=N, \beta=0, f=f, g=g, q=0, u=u, q^N=0\}, \{N=N, \beta=\beta, f=f, g=g, q=q, u=0, q^N=0\}\}$

"Warning, parameters have the values", $\{\{N=N, aB=0, \beta=0, f=f, g=0, q=q, q^N=f\}, \{N=N, aB=0, \beta=\beta, f=f, g=g, q=0, q^N=q^N\}\}$

"Warning, parameters have the values", $\{\{N=N, aB=0, \beta=0, f=q^N, g=0, q=q, q^N=q^N\}, \{N=N, aB=aB, \beta=0, f=q^N aB+q^N, g=0, q=0, q^N=q^N\}\}$

"Warning, parameters have the values", $\{\{N=N, bB=bB, \beta=0, cB=0, f=f, g=g, q=0, q^N=q^N\}, \{N=N, bB=bB, \beta=0, cB=cB, f=f, g=g, q=0, q^N=0\}\}$

"Warning, parameters have the values", $\{\{\beta=0, f=f, g=0, pB=0, q=q, u=u, v=f\,u\}, \{\beta=\beta, f=f, g=g, pB=0, q=0, u=u, v=v\}, \{\beta=\beta, f=f, g=g, pB=pB, q=q, u=0, v=0\}\}$

"Warning, parameters have the values", $\left\{\left\{aH=1, bH=2\beta, f=q\,q^N, g=0, q^{NH}=\frac{1}{2\beta q^2 q^N}\right\}, \left\{aH=\frac{1}{q^N q}, bH=2q^N\beta q, f=1, g=0, q^{NH}=\frac{1}{2\beta q}\right\}, \{aH=2\beta, bH=1, f=1, g=0, q^{NH}=q^N\}, \left\{aH=2q^N\beta q, bH=\frac{1}{q^N q}, f=q\,q^N, g=0, q^{NH}=\frac{1}{q}\right\}\right\}$

"Warning, several solutions found"

$$\left[\left[\text{"Has a solution as Big q-Jacobi"}, \left[\left[\sigma(x)=-(2\beta q-x)\,(x q^N-1)\,(q^N)^3 q^2, \tau(x)=\frac{(2q^N\beta q^2 x-2q^N\beta q^2+2q^N\beta q-x q^N-2\beta q+1)\,q^N}{q-1}, \lambda_{q,n}=-\frac{(q^n-1)\,(2\beta q^n q-1)}{q^n\,(q-1)^2\,q}, S(n,x)=P_n(1, 2\beta, 2q^{1+N}\beta, q^{1+N}x, q), \frac{\rho(q\,x)}{\rho(x)}=\frac{1}{(q^N)^2 q^3\,(2\beta-x)\,(q\,q^N x-1)}\Big(2\,(q^N)^3\beta q^3 x-(q^N)^3 q^2 x^2-2q^3\,(q^N)^2\beta-2q^N\beta q^2 x^2+(q^N)^2 q^2 x+2q^N\beta q^2 x-2q^N\beta q x+q^N x^2+2\beta q x-x\Big), \frac{k_{n+1}}{k_n}=\right.\right.\right.\right. \tag{7}$$

$$-\frac{\left(2\,(q^n)^2\,q\,\beta-1\right)\left(2\,(q^n)^2\,\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^n\,q-1\right)^2\,(q^n-q^N)\,q^{1+N}}, \mathrm{I}=\left[2\,\beta\,q,\ \frac{q}{q^{1+N}}\right]\Bigg], \Bigg[\sigma(x)=$$

$$-\frac{(2\,\beta\,q-x)\,(x\,q^N-1)}{q\,q^N}, \tau(x)=\frac{2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1}{q\,(q-1)\,q^N},$$

$$\lambda_{q,\,n}=-\frac{(q^n-1)\,(2\,\beta\,q^n\,q-1)}{(q-1)^2\,q^n}, S(n,x)=P_n\left(q^{-1-N},2\,q^{1+N}\beta,2\,\beta,x,q\right), \frac{\rho(q\,x)}{\rho(x)}=$$

$$-\frac{2\,(x-1)\,\beta}{2\,\beta-x}, \frac{k_{n+1}}{k_n}=-\frac{\left(2\,(q^n)^2\,q\,\beta-1\right)\left(2\,(q^n)^2\,\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^n\,q-1\right)^2\,(q^n-q^N)}, \mathrm{I}=\left[2\,\beta\,q,\right.$$

$$\left.q^{-1-N}\,q\right]\Bigg], \Bigg[\sigma(x)=-\frac{(2\,\beta\,q-x)\,(x\,q^N-1)}{q\,q^N}, \tau(x)$$

$$=\frac{2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1}{q\,(q-1)\,q^N}, \lambda_{q,\,n}=$$

$$-\frac{(q^n-1)\,(2\,\beta\,q^n\,q-1)}{(q-1)^2\,q^n}, S(n,x)=P_n\left(2\,\beta,1,q^{-1-N},x,q\right), \frac{\rho(q\,x)}{\rho(x)}=-\frac{2\,(x-1)\,\beta}{2\,\beta-x},$$

$$\frac{k_{n+1}}{k_n}=-\frac{\left(2\,(q^n)^2\,q\,\beta-1\right)\left(2\,(q^n)^2\,\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^n\,q-1\right)^2\,(q^n-q^N)}, \mathrm{I}=\left[q^{-1-N}\,q,2\,\beta\,q\right]\Bigg], \Bigg[\sigma(x)=$$

$$-(2\,\beta\,q-x)\,(x\,q^N-1)\,(q^N)^3\,q^2, \tau(x)$$

$$=\frac{\left(2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1\right)q^N}{q-1}, \lambda_{q,\,n}=$$

$$-\frac{(q^n-1)\,(2\,\beta\,q^n\,q-1)}{q^n\,(q-1)^2\,q}, S(n,x)=P_n\left(2\,q^{1+N}\beta,q^{-1-N},1,q^{1+N}x,q\right), \frac{\rho(q\,x)}{\rho(x)}$$

$$=\frac{1}{(q^N)^2\,q^3\,(2\,\beta-x)\,(q\,q^N\,x-1)}\left(2\,(q^N)^3\,\beta\,q^3\,x-(q^N)^3\,q^2\,x^2-2\,q^3\,(q^N)^2\,\beta\right.$$

$$-2 q^N \beta q^2 x^2 + (q^N)^2 q^2 x + 2 q^N \beta q^2 x - 2 q^N \beta q x + q^N x^2 + 2 \beta q x - x\Big), \frac{k_{n+1}}{k_n} =$$

$$-\frac{\left(2 (q^n)^2 q \beta - 1\right)\left(2 (q^n)^2 \beta q^2 - 1\right) q^N}{(2 \beta q^n q - 1)^2 (q^n - q^N) q^{1+N}}, \mathrm{I} = \left[\frac{q}{q^{1+N}}, 2 \beta q\right]\Big]\Big]\Big],$$

$$\Bigg[\text{"Has a solution as q-Hahn"}, \Bigg[\Bigg[\sigma(x) = -(2 \beta q - x)(x q^N - 1)(q^N)^3 \beta q^3, \tau(x)$$

$$= \frac{(2 q^N \beta q^2 x - 2 q^N \beta q^2 + 2 q^N \beta q - x q^N - 2 \beta q + 1) q^N \beta q}{q - 1}, \lambda_{q,n} =$$

$$-\frac{\beta (q^n - 1)(2 \beta q^n q - 1)}{(q-1)^2 q^n}, S(n, x) = Q_n\left(1, 2\beta, -\frac{N \ln(q) - \ln\left(\frac{1}{2 \beta q^2}\right)}{\ln(q)}, q q^N x, q\right),$$

$$\frac{\rho(q x)}{\rho(x)} = \frac{1}{(q^N)^2 q^3 (2\beta - x)(q q^N x - 1)} \Big(2 (q^N)^3 \beta q^3 x - (q^N)^3 q^2 x^2$$

$$-2 q^3 (q^N)^2 \beta - 2 q^N \beta q^2 x^2 + (q^N)^2 q^2 x + 2 q^N \beta q^2 x - 2 q^N \beta q x + q^N x^2 + 2 \beta q x$$

$$-x\Big), \frac{k_{n+1}}{k_n} = -\frac{\left(2 (q^n)^2 q \beta - 1\right)\left(2 (q^n)^2 \beta q^2 - 1\right)}{(2 \beta q^n q - 1)^2 (q^n - q^N) q}, \mathrm{I} = \Bigg[0, \frac{1}{q^N q}, \frac{2}{q^N q}, \text{"..."},$$

$$-\frac{N \ln(q) - \ln\left(\frac{1}{2 \beta q^2}\right)}{\ln(q) q q^N}\Bigg]\Bigg], \Bigg[\sigma(x) = -\frac{(2 \beta q - x)(x q^N - 1) \beta}{q q^N}, \tau(x)$$

$$= \frac{(2 q^N \beta q^2 x - 2 q^N \beta q^2 + 2 q^N \beta q - x q^N - 2 \beta q + 1) \beta}{q q^N (q - 1)}, \lambda_{q,n} =$$

$$-\frac{\beta (q^n - 1)(2 \beta q^n q - 1)}{(q-1)^2 q^n}, S(n, x) = Q_n\left(\frac{1}{q^N q}, 2 q^N \beta q, \frac{\ln\left(\frac{1}{2 \beta q}\right)}{\ln(q)}, x, q\right), \frac{\rho(q x)}{\rho(x)}$$

$$= -\frac{2 (x - 1) \beta}{2 \beta - x}, \frac{k_{n+1}}{k_n} = -\frac{\left(2 (q^n)^2 q \beta - 1\right)\left(2 (q^n)^2 \beta q^2 - 1\right) q^N}{(2 \beta q^n q - 1)^2 (q^n - q^N)}, \mathrm{I} = \Big[0, 1, 2, \text{"..."},$$

$$\frac{\ln\left(\frac{1}{2\beta q}\right)}{\ln(q)}\Bigg]\Bigg]\Bigg], \Bigg[\sigma(x) = -\frac{(2\beta q - x)(x q^N - 1)}{q q^N}, \tau(x)$$

$$= \frac{2 q^N \beta q^2 x - 2 q^N \beta q^2 + 2 q^N \beta q - x q^N - 2\beta q + 1}{q (q-1) q^N}, \lambda_{q,n} =$$

$$-\frac{(q^n - 1)(2\beta q^n q - 1)}{(q-1)^2 q^n}, S(n, x) = Q_n(2\beta, 1, N, x, q), \frac{\rho(q x)}{\rho(x)} = -\frac{2 (x-1)\beta}{2\beta - x},$$

$$\frac{k_{n+1}}{k_n} = -\frac{\left(2 (q^n)^2 q \beta - 1\right)\left(2 (q^n)^2 \beta q^2 - 1\right) q^N}{(2\beta q^n q - 1)^2 (q^n - q^N)}, \mathrm{I} = [0, 1, 2, \text{"..."}, N]\Bigg], \Bigg[\sigma(x) =$$

$$-(2\beta q - x)(x q^N - 1)(q^N)^3 q^2, \tau(x)$$

$$= \frac{(2 q^N \beta q^2 x - 2 q^N \beta q^2 + 2 q^N \beta q - x q^N - 2\beta q + 1) q^N}{q-1}, \lambda_{q,n} =$$

$$-\frac{(q^n - 1)(2\beta q^n q - 1)}{q^n (q-1)^2 q}, S(n, x) = Q_n\left(2 q^N \beta q, \frac{1}{q^N q}, -1, q q^N x, q\right), \frac{\rho(q x)}{\rho(x)}$$

$$= \frac{1}{(q^N)^2 q^3 (2\beta - x)(q q^N x - 1)}\Big(2 (q^N)^3 \beta q^3 x - (q^N)^3 q^2 x^2 - 2 q^3 (q^N)^2 \beta$$

$$- 2 q^N \beta q^2 x^2 + (q^N)^2 q^2 x + 2 q^N \beta q^2 x - 2 q^N \beta q x + q^N x^2 + 2\beta q x - x\Big), \frac{k_{n+1}}{k_n} =$$

$$-\frac{\left(2 (q^n)^2 q \beta - 1\right)\left(2 (q^n)^2 \beta q^2 - 1\right)}{(2\beta q^n q - 1)^2 (q^n - q^N) q}, \mathrm{I} = \left[0, \frac{1}{q^N q}, \frac{2}{q^N q}, \text{"..."}, -\frac{1}{q^N q}\right]\Bigg]\Bigg]\Bigg]\Bigg]$$

Meixner

```
> msum := proc (b, c, x, j) hyperterm([-n, -x], [beta], 1-1/c, j)
  end proc;
```

$$msum := \mathbf{proc}(b, c, x, j)\ hyperterm([-n, -x], [\mathrm{beta}], 1 - 1/c, j)\ \mathbf{end\ proc} \quad \textbf{(8)}$$

```
> RM1 := sumrecursion(msum(b, c, x, j), j, S(n), recursion = up);
```

$$RM1 := c\,(\beta + n + 1)\,S(n+2) - (\beta c + c n + x c + c + n - x + 1)\,S(n+1) + (n + 1)\,S(n) = 0 \quad \textbf{(9)}$$

```
> RM2 := sumrecursion(msum(beta, 1/c, -x-beta, j), j, S(n),
  recursion = up);
```

$$RM2 := (\beta + n + 1)\,S(n+2) - (\beta c + c n + x c + c + n - x + 1)\,S(n+1) + (n + 1)\,c\,S(n) = 0 \quad \textbf{(10)}$$

```
> `recursion/compare`(RM1, RM2, S(n));
```

Recursions are NOT identical! **(11)**

```
> msum(beta, 1/c, -x-beta, n);
```

$$\frac{\text{pochhammer}(-n, n)\,\text{pochhammer}(x+\beta, n)\,(1-c)^n}{\text{pochhammer}(\beta, n)\,n!} \tag{12}$$

```
> denumM := msum(beta, 1/c, -x-beta, n)/pochhammer(x+beta, n);
```

$$denumM := \frac{\text{pochhammer}(-n, n)\,(1-c)^n}{\text{pochhammer}(\beta, n)\,n!} \tag{13}$$

```
> msum(b, c, x, n);
```

$$\frac{\text{pochhammer}(-n, n)\,\text{pochhammer}(-x, n)\left(1-\frac{1}{c}\right)^n}{\text{pochhammer}(\beta, n)\,n!} \tag{14}$$

```
> numM := msum(b, c, x, n)*(-1)^n/pochhammer(-x, n);
```

$$numM := \frac{\text{pochhammer}(-n, n)\left(1-\frac{1}{c}\right)^n(-1)^n}{\text{pochhammer}(\beta, n)\,n!} \tag{15}$$

```
> CM := denumM/numM;
```

$$CM := \frac{(1-c)^n}{\left(1-\frac{1}{c}\right)^n(-1)^n} \tag{16}$$

```
> NRM1 := sumrecursion(CM*msum(b, c, x, j), j, S(n), recursion =
  up);
```

$$NRM1 := (\beta+n+1)\,S(n+2) - (\beta c + c n + x c + c + n - x + 1)\,S(n+1) + (n+1)\,c\,S(n) = 0 \tag{17}$$

```
> `recursion/compare`(NRM1, RM2, S(n));
```

Recursions are identical. (18)

```
> simplify([seq(CM*add(msum(b, c, x, j), j = 0 .. n), n = 0 .. 3)-
  seq(add(msum(beta, 1/c, -x-beta, j), j = 0 .. n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \tag{19}$$

Krawtchouk

```
> ksum := proc (p, N, x, j) hyperterm([-n, -x], [-N], 1/p, j) end
  proc;
```

$$ksum := \textbf{proc}(p, N, x, j)\ hyperterm([-n, -x], [-N], 1/p, j)\ \textbf{end proc} \tag{20}$$

```
> RK1 := sumrecursion(ksum(p, N, x, j), j, S(n), recursion = up);
```

$$RK1 := -p\,(-n-1+N)\,S(n+2) + (N p - 2 n p + n - 2 p - x + 1)\,S(n+1) + (n+1)\,(-1+p)\,S(n) = 0 \tag{21}$$

```
> RK2 := sumrecursion(ksum(1-p, N, -x+N, j), j, S(n), recursion =
  up);
```

$$RK2 := -(-1+p)\,(-n-1+N)\,S(n+2) + (N p - 2 n p + n - 2 p - x + 1)\,S(n+1) + (n+1)\,p\,S(n) = 0 \tag{22}$$

```
> `recursion/compare`(RK1, RK2, S(n));
```

Recursions are NOT identical! (23)

```
> ksum(p, N, x, n);
```

$$\frac{\text{pochhammer}(-n,n)\,\text{pochhammer}(-x,n)\left(\frac{1}{p}\right)^n}{\text{pochhammer}(-N,n)\,n!} \quad \textbf{(24)}$$

```
> denumK := ksum(p, N, x, n)*(-1)^n/pochhammer(-x, n);
```

$$denumK := \frac{\text{pochhammer}(-n,n)\left(\frac{1}{p}\right)^n(-1)^n}{\text{pochhammer}(-N,n)\,n!} \quad \textbf{(25)}$$

```
> ksum(1-p, N, -x+N, n);
```

$$\frac{\text{pochhammer}(-n,n)\,\text{pochhammer}(x-N,n)\left(\frac{1}{1-p}\right)^n}{\text{pochhammer}(-N,n)\,n!} \quad \textbf{(26)}$$

```
> numK := ksum(1-p, N, -x+N, n)/pochhammer(x-N, n);
```

$$numK := \frac{\text{pochhammer}(-n,n)\left(\frac{1}{1-p}\right)^n}{\text{pochhammer}(-N,n)\,n!} \quad \textbf{(27)}$$

```
> CK := numK/denumK;
```

$$CK := \frac{\left(\frac{1}{1-p}\right)^n}{\left(\frac{1}{p}\right)^n(-1)^n} \quad \textbf{(28)}$$

```
> NRK1 := sumrecursion(CK*ksum(p, N, x, j), j, S(n), recursion =
  up);
```

$$NRK1 := -(-1+p)(-n-1+N)\,S(n+2)+(N\,p-2\,n\,p+n-2\,p-x+1)\,S(n+1) + (n+1)\,p\,S(n)=0 \quad \textbf{(29)}$$

```
> `recursion/compare`(NRK1, RK2, S(n));
```

Recursions are identical. **(30)**

```
> simplify([seq(CK*add(ksum(p, N, x, j), j = 0 .. n), n = 0 .. 3)-
  seq(add(ksum(1-p, N, -x+N, j), j = 0 .. n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \quad \textbf{(31)}$$

Hahn

```
> hsum := proc (alpha, beta, N, x, j) hyperterm([-n, n+alpha+
  beta+1, -x], [alpha+1, -N], 1, j) end proc;
```

hsum := **proc**(alpha, beta, N, x, j) **(32)**

hyperterm([$-n$, n + alpha + beta + 1, $-x$], [alpha + 1, $-N$], 1, j)

end proc

```
> RH1 := sumrecursion(hsum(alpha, beta, N, x, j), j, S(n),
  recursion = up);
```

$$RH1 := (\alpha+2+n)(2\,n+\alpha+\beta+2)(n+2+\alpha+\beta)(-n-1+N)\,S(n+2)-(2\,n+3+\alpha+\beta)\big(N\,\alpha^2+N\,\alpha\,\beta+2\,N\,\alpha\,n+2\,N\,\beta\,n+2\,N\,n^2-\alpha^2\,n-\alpha^2\,x-2\,\alpha\,\beta\,x-\alpha\,n^2-4\,\alpha\,n\,x+\beta^2\,n-\beta^2\,x+\beta\,n^2-4\,\beta\,n\,x-4\,n^2\,x+3\,\alpha\,N+3\,N\,\beta+6\,N\,n-\alpha^2-3\,n\,\alpha-6\,\alpha\,x+\beta^2+3\,n\,\beta-6\,\beta\,x-12\,x\,n+4\,N-2\,\alpha+2\,\beta-8\,x\big)\,S(n+1)+(n+1)(\beta+n+1)(2\,n+4+\alpha+\beta)(n+\alpha+\beta+2+N)\,S(n)=0 \quad \textbf{(33)}$$

```
> RH2 := sumrecursion(hsum(beta, alpha, N, -x+N, j), j, S(n),
  recursion = up);
```

$$RH2 := (\beta+2+n)\,(2\,n+\alpha+\beta+2)\,(n+2+\alpha+\beta)\,(-n-1+N)\,S(n+2)+(2\,n+3+\alpha+\beta)\,\big(N\,\alpha^2+N\,\alpha\,\beta+2\,N\,\alpha\,n+2\,N\,\beta\,n+2\,N\,n^2-\alpha^2\,n-\alpha^2\,x-2\,\alpha\,\beta\,x-\alpha\,n^2-4\,\alpha\,n\,x+\beta^2\,n-\beta^2\,x+\beta\,n^2-4\,\beta\,n\,x-4\,n^2\,x+3\,\alpha\,N+3\,N\,\beta+6\,N\,n-\alpha^2-3\,n\,\alpha-6\,\alpha\,x+\beta^2+3\,n\,\beta-6\,\beta\,x-12\,x\,n+4\,N-2\,\alpha+2\,\beta-8\,x\big)\,S(n+1)+(n+1)\,(\alpha+1+n)\,(2\,n+4+\alpha+\beta)\,(n+\alpha+\beta+2+N)\,S(n)=0 \quad (34)$$

```
> `recursion/compare`(RH1, RH2, S(n));
```

Recursions are NOT identical! (35)

```
> hsum(alpha, beta, N, x, n);
```

$$\frac{\mathrm{pochhammer}(-n,n)\,\mathrm{pochhammer}(n+\alpha+\beta+1,n)\,\mathrm{pochhammer}(-x,n)}{\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(-N,n)\,n!} \quad (36)$$

```
> denumH := hsum(alpha, beta, N, x, n)*(-1)^n/pochhammer(-x, n);
```

$$denumH := \frac{\mathrm{pochhammer}(-n,n)\,\mathrm{pochhammer}(n+\alpha+\beta+1,n)\,(-1)^n}{\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(-N,n)\,n!} \quad (37)$$

```
> hsum(beta, alpha, N, -x+N, n);
```

$$\frac{\mathrm{pochhammer}(-n,n)\,\mathrm{pochhammer}(n+\alpha+\beta+1,n)\,\mathrm{pochhammer}(x-N,n)}{\mathrm{pochhammer}(\beta+1,n)\,\mathrm{pochhammer}(-N,n)\,n!} \quad (38)$$

```
> numH21 := hsum(beta, alpha, N, -x+N, n)/pochhammer(x-N, n);
```

$$numH21 := \frac{\mathrm{pochhammer}(-n,n)\,\mathrm{pochhammer}(n+\alpha+\beta+1,n)}{\mathrm{pochhammer}(\beta+1,n)\,\mathrm{pochhammer}(-N,n)\,n!} \quad (39)$$

```
> CH21 := simplify(numH21/denumH);
```

$$CH21 := \frac{\mathrm{pochhammer}(\alpha+1,n)\,(-1)^{-n}}{\mathrm{pochhammer}(\beta+1,n)} \quad (40)$$

```
> NRH1 := sumrecursion(CH21*hsum(alpha, beta, N, x, j), j, S(n),
  recursion = up);
```

$$NRH1 := (\beta+2+n)\,(2\,n+\alpha+\beta+2)\,(n+2+\alpha+\beta)\,(-n-1+N)\,S(n+2)+(2\,n+3+\alpha+\beta)\,\big(N\,\alpha^2+N\,\alpha\,\beta+2\,N\,\alpha\,n+2\,N\,\beta\,n+2\,N\,n^2-\alpha^2\,n-\alpha^2\,x-2\,\alpha\,\beta\,x-\alpha\,n^2-4\,\alpha\,n\,x+\beta^2\,n-\beta^2\,x+\beta\,n^2-4\,\beta\,n\,x-4\,n^2\,x+3\,\alpha\,N+3\,N\,\beta+6\,N\,n-\alpha^2-3\,n\,\alpha-6\,\alpha\,x+\beta^2+3\,n\,\beta-6\,\beta\,x-12\,x\,n+4\,N-2\,\alpha+2\,\beta-8\,x\big)\,S(n+1)+(n+1)\,(\alpha+1+n)\,(2\,n+4+\alpha+\beta)\,(n+\alpha+\beta+2+N)\,S(n)=0 \quad (41)$$

```
> `recursion/compare`(NRH1, RH2, S(n));
```

Recursions are identical. (42)

```
> simplify([seq(CH21*add(hsum(alpha, beta, N, x, j), j = 0 .. n), n
  = 0 .. 3)-seq(add(hsum(beta, alpha, N, -x+N, j), j = 0 .. n), n =
  0 .. 3)]);
```

$$[0, 0, 0, 0] \quad (43)$$

Big q-Jacobi

```
> BJsummand := proc (a, b, c, x, q, j) qphihyperterm([q^(-n), a*b*
```

```
  q^(n+1), x], [a*q, c*q], q, q, j) end proc;
```
$BJsummand := \mathbf{proc}(a, b, c, x, q, j)$ **(44)**

$qphihyperterm([q^{\wedge}(-n), a*b*q^{\wedge}(n+1), x], [a*q, c*q], q, q, j)$

end proc

```
> RB1 := qsumrecursion(BJsummand(a, b, c, x, q, j), q, j, S(n),
  recursion = up);
```
$$RB1 := -\left(q^{n+2} c-1\right)\left(q^{2 n+2} a b-1\right)\left(q^{n+2} a-1\right)\left(q^{n+2} a b-1\right) S(n+2) - \left(q^{2 n+3} a b-1\right)\left(q^{4 n+6} x a^2 b^2-q^{3 n+5} a^2 b^2-q^{3 n+5} c a^2 b-q^{3 n+5} a^2 b - q^{3 n+5} c a b+q^{2 n+4} a^2 b+q^{2 n+4} c a b-q^{2 n+4} x a b+q^{2 n+3} a^2 b+q^{2 n+3} c a b + q^{2 n+4} a b+q^{2 n+4} c a+q^{2 n+3} a b-q^{2 n+2} x a b+q^{2 n+3} c a-q^{n+2} a b - q^{n+2} c a-q^{n+2} a-q^{n+2} c+x\right) S(n+1)+\left(q^{n+1} b-1\right) q^{n+2} a\left(a b q^{n+1} - c\right)\left(q^{2 n+4} a b-1\right)\left(q^{n+1}-1\right) S(n)=0 \quad (45)$$

```
> RB2 := qsumrecursion(BJsummand(b, a, a*b/c, b*x/c, q, j), q, j, S
  (n), recursion = up);
```
$$RB2 := -\left(q^{2 n+2} a b-1\right)\left(q^{n+2} a b-c\right)\left(q^{n+2} a b-1\right)\left(q^{n+2} b-1\right) S(n+2) - \left(q^{2 n+3} a b-1\right)\left(q^{4 n+6} x a^2 b^2-q^{3 n+5} a^2 b^2-q^{3 n+5} c a^2 b-q^{3 n+5} a^2 b - q^{3 n+5} c a b+q^{2 n+4} a^2 b+q^{2 n+4} c a b-q^{2 n+4} x a b+q^{2 n+3} a^2 b+q^{2 n+3} c a b + q^{2 n+4} a b+q^{2 n+4} c a+q^{2 n+3} a b-q^{2 n+2} x a b+q^{2 n+3} c a-q^{n+2} a b - q^{n+2} c a-q^{n+2} a-q^{n+2} c+x\right) b S(n+1)+\left(q^{n+1} c-1\right) q^{n+2}\left(q^{n+1} a - 1\right) a\left(q^{2 n+4} a b-1\right)\left(q^{n+1}-1\right) b^2 S(n)=0 \quad (46)$$

```
> RB3 := qsumrecursion(BJsummand(c, a*b/c, a, x, q, j), q, j, S(n),
  recursion = up);
```
$$RB3 := -\left(q^{n+2} c-1\right)\left(q^{2 n+2} a b-1\right)\left(q^{n+2} a-1\right)\left(q^{n+2} a b-1\right) S(n+2) - \left(q^{2 n+3} a b-1\right)\left(q^{4 n+6} x a^2 b^2-q^{3 n+5} a^2 b^2-q^{3 n+5} c a^2 b-q^{3 n+5} a^2 b - q^{3 n+5} c a b+q^{2 n+4} a^2 b+q^{2 n+4} c a b-q^{2 n+4} x a b+q^{2 n+3} a^2 b+q^{2 n+3} c a b + q^{2 n+4} a b+q^{2 n+4} c a+q^{2 n+3} a b-q^{2 n+2} x a b+q^{2 n+3} c a-q^{n+2} a b - q^{n+2} c a-q^{n+2} a-q^{n+2} c+x\right) S(n+1)+\left(q^{n+1} b-1\right) q^{n+2} a\left(a b q^{n+1} - c\right)\left(q^{2 n+4} a b-1\right)\left(q^{n+1}-1\right) S(n)=0 \quad (47)$$

```
> RB4 := qsumrecursion(BJsummand(a*b/c, c, b, b*x/c, q, j), q, j, S
  (n), recursion = up);
```
$$RB4 := -\left(q^{2 n+2} a b-1\right)\left(q^{n+2} a b-c\right)\left(q^{n+2} a b-1\right)\left(q^{n+2} b-1\right) S(n+2) - \left(q^{2 n+3} a b-1\right)\left(q^{4 n+6} x a^2 b^2-q^{3 n+5} a^2 b^2-q^{3 n+5} c a^2 b-q^{3 n+5} a^2 b - q^{3 n+5} c a b+q^{2 n+4} a^2 b+q^{2 n+4} c a b-q^{2 n+4} x a b+q^{2 n+3} a^2 b+q^{2 n+3} c a b + q^{2 n+4} a b+q^{2 n+4} c a+q^{2 n+3} a b-q^{2 n+2} x a b+q^{2 n+3} c a-q^{n+2} a b - q^{n+2} c a-q^{n+2} a-q^{n+2} c+x\right) b S(n+1)+\left(q^{n+1} c-1\right) q^{n+2}\left(q^{n+1} a - 1\right) a\left(q^{2 n+4} a b-1\right)\left(q^{n+1}-1\right) b^2 S(n)=0 \quad (48)$$

```
>
> `recursion/compare`(RB1, simplify(RB2), S(n));
```
Recursions are NOT identical! **(49)**

```
> `recursion/compare`(RB1, simplify(RB3), S(n));
```

$$\textit{Recursions are identical.} \tag{50}$$

```
> `recursion/compare`(RB1, simplify(RB4), S(n));
```

$$\textit{Recursions are NOT identical!} \tag{51}$$

```
> `recursion/compare`(RB2, simplify(RB3), S(n));
```

$$\textit{Recursions are NOT identical!} \tag{52}$$

```
> `recursion/compare`(RB2, simplify(RB4), S(n));
```

$$\textit{Recursions are identical.} \tag{53}$$

```
> `recursion/compare`(RB3, simplify(RB4), S(n));
```

$$\textit{Recursions are NOT identical!} \tag{54}$$

Obtaining Cn

```
> BJsummand(a, b, c, x, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(a\, b\, q^{n+1}, q, n)\ qpochhammer(x, q, n)\ q^{n}}{qpochhammer(a\, q, q, n)\ qpochhammer(c\, q, q, n)\ qpochhammer(q, q, n)} \tag{55}$$

```
> BJsummand(b, a, a*b/c, b*x/c, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(a\, b\, q^{n+1}, q, n)\ qpochhammer\left(\frac{b\, x}{c}, q, n\right)\ q^{n}}{qpochhammer(b\, q, q, n)\ qpochhammer\left(\frac{a\, b\, q}{c}, q, n\right)\ qpochhammer(q, q, n)} \tag{56}$$

```
> num21 := BJsummand(b, a, a*b/c, b*x/c, q, n)*(-1)^n*q^binomial(n,
  2)*(b/c)^n/qpochhammer(b*x/c, q, n);
```

$$num21 := \tag{57}$$

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(a\, b\, q^{n+1}, q, n)\ q^{n}\ (-1)^{n}\ q^{\mathrm{binomial}(n,\, 2)} \left(\frac{b}{c}\right)^{n}}{qpochhammer(b\, q, q, n)\ qpochhammer\left(\frac{a\, b\, q}{c}, q, n\right)\ qpochhammer(q, q, n)}$$

```
> denumbj := BJsummand(a, b, c, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$denumbj := \frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(a\, b\, q^{n+1}, q, n)\ q^{n}\ (-1)^{n}\ q^{\mathrm{binomial}(n,\, 2)}}{qpochhammer(a\, q, q, n)\ qpochhammer(c\, q, q, n)\ qpochhammer(q, q, n)} \tag{58}$$

```
> C21 := simplify(num21/denumbj);
```

$$C21 := \frac{\left(\frac{b}{c}\right)^{n} qpochhammer(a\, q, q, n)\ qpochhammer(c\, q, q, n)}{qpochhammer(b\, q, q, n)\ qpochhammer\left(\frac{a\, b\, q}{c}, q, n\right)} \tag{59}$$

Checking initial conditions

```
> qsimpcomb([seq(C21*add(BJsummand(a, b, c, x, q, j), j = 0 .. n),
  n = 0 .. 3)-seq(add(BJsummand(b, a, a*b/c, b*x/c, q, j), j = 0 ..
  n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \tag{60}$$

Comparing recurrence equations with new relation

```
> NRB1 := qsumrecursion(C21*BJsummand(a, b, c, x, q, j), q, j, S
  (n), recursion = up);
```

$$\begin{aligned} NRB1 := &-(q^{2n+2} a\, b - 1)\,(q^{n+2} a\, b - c)\,(q^{n+2} a\, b - 1)\,(q^{n+2} b - 1)\, S(n+2) \\ &- (q^{2n+3} a\, b - 1)\,(q^{4n+6} x\, a^{2} b^{2} - q^{3n+5} a^{2} b^{2} - q^{3n+5} c\, a^{2} b - q^{3n+5} a^{2} b \\ &- q^{3n+5} c\, a\, b + q^{2n+4} a^{2} b + q^{2n+4} c\, a\, b - q^{2n+4} x\, a\, b + q^{2n+3} a^{2} b + q^{2n+3} c\, a\, b \end{aligned} \tag{61}$$

$$+q^{2n+4}ab+q^{2n+4}ca+q^{2n+3}ab-q^{2n+2}xab+q^{2n+3}ca-q^{n+2}ab-q^{n+2}ca-q^{n+2}a-q^{n+2}c+x\big)\,b\,S(n+1)+\big(q^{n+1}c-1\big)\,q^{n+2}\,\big(q^{n+1}a-1\big)\,a\,\big(q^{2n+4}ab-1\big)\,\big(q^{n+1}-1\big)\,b^2\,S(n)=0$$

```
> `recursion/compare`(NRB1, simplify(RB2), S(n));
```

Recursions are identical. **(62)**

```
> BJsummand(c, a*b/c, a, x, q, n);
```

$$\frac{qpochhammer(q^{-n},q,n)\;qpochhammer(a\,b\,q^{n+1},q,n)\;qpochhammer(x,q,n)\;q^n}{qpochhammer(a\,q,q,n)\;qpochhammer(c\,q,q,n)\;qpochhammer(q,q,n)} \tag{63}$$

```
> num31 := BJsummand(c, a*b/c, a, x, q, n)*(-1)^n*q^binomial(n, 2)
/qpochhammer(x, q, n);
```

$$num31 := \frac{qpochhammer(q^{-n},q,n)\;qpochhammer(a\,b\,q^{n+1},q,n)\;q^n\;(-1)^n\;q^{\mathrm{binomial}(n,\,2)}}{qpochhammer(a\,q,q,n)\;qpochhammer(c\,q,q,n)\;qpochhammer(q,q,n)} \tag{64}$$

```
> C31 := num31/denumbj;
```

$$C31 := 1 \tag{65}$$

```
> BJsummand(a*b/c, c, b, b*x/c, q, n);
```

$$\frac{qpochhammer(q^{-n},q,n)\;qpochhammer(a\,b\,q^{n+1},q,n)\;qpochhammer\left(\frac{b\,x}{c},q,n\right)\;q^n}{qpochhammer(b\,q,q,n)\;qpochhammer\left(\frac{a\,b\,q}{c},q,n\right)\;qpochhammer(q,q,n)} \tag{66}$$

```
> num41 := BJsummand(a*b/c, c, b, b*x/c, q, n)*(-1)^n*q^binomial(n,
2)*(b/c)^n/qpochhammer(b*x/c, q, n);
```

$$num41 := \frac{qpochhammer(q^{-n},q,n)\;qpochhammer(a\,b\,q^{n+1},q,n)\;q^n\;(-1)^n\;q^{\mathrm{binomial}(n,\,2)}\left(\frac{b}{c}\right)^n}{qpochhammer(b\,q,q,n)\;qpochhammer\left(\frac{a\,b\,q}{c},q,n\right)\;qpochhammer(q,q,n)} \tag{67}$$

```
> C41 := num41/denumbj;
```

$$C41 := \frac{\left(\frac{b}{c}\right)^n qpochhammer(a\,q,q,n)\;qpochhammer(c\,q,q,n)}{qpochhammer(b\,q,q,n)\;qpochhammer\left(\frac{a\,b\,q}{c},q,n\right)} \tag{68}$$

```
> C21/C41;
```

$$1 \tag{69}$$

```
> NRB3 := qsumrecursion(C21*BJsummand(c, a*b/c, a, x, q, j), q, j,
S(n), recursion = up);
```

$$NRB3 := -\big(q^{2n+2}ab-1\big)\big(q^{n+2}ab-c\big)\big(q^{n+2}ab-1\big)\big(q^{n+2}b-1\big)S(n+2)-\big(q^{2n+3}ab-1\big)\big(q^{4n+6}xa^2b^2-q^{3n+5}a^2b^2-q^{3n+5}ca^2b-q^{3n+5}a^2b-q^{3n+5}cab+q^{2n+4}a^2b+q^{2n+4}cab-q^{2n+4}xab+q^{2n+3}a^2b+q^{2n+3}cab+q^{2n+4}ab+q^{2n+4}ca+q^{2n+3}ab-q^{2n+2}xab+q^{2n+3}ca-q^{n+2}ab-q^{n+2}ca-q^{n+2}a-q^{n+2}c+x\big)\,b\,S(n+1)+\big(q^{n+1}c-1\big)\,q^{n+2}\,\big(q^{n+1}a-1\big)\,a\,\big(q^{2n+4}ab-1\big)\big(q^{n+1}-1\big)\,b^2\,S(n)=0 \tag{70}$$

```
> `recursion/compare`(RB2, simplify(NRB3), S(n));
```

Recursions are identical. **(71)**

Little q-Jacobi

Liitle q-Jacobi to q-Krawtchouk

```
> LJsummand := proc (a, b, x, q, j) qphihyperterm([q^(-n), a*b*q^(n+1)], [a*q], q, q*x, j) end proc;
```

$LJsummand := \mathbf{proc}(a, b, x, q, j)$ **(72)**

$qphihyperterm([q\hat{}(-n), a*b*q\hat{}(n+1)], [a*q], q, q*x, j)$

end proc

```
> RL := qsumrecursion(LJsummand(a, b, x, q, j), q, j, S(n), recursion = up);
```

$$RL := q^{n+1}\left(q^{2n+2}ab-1\right)\left(q^{n+2}a-1\right)\left(q^{n+2}ab-1\right)S(n+2)+\left(q^{2n+3}ab-1\right)\left(q^{4n+6}xa^2b^2-q^{3n+4}a^2b-q^{3n+4}ab-q^{2n+4}xab+q^{2n+3}ab-q^{2n+2}xab+q^{2n+2}ab+q^{2n+3}a+q^{2n+2}a-q^{n+1}a-q^{n+1}+x\right)S(n+1)+\left(q^{n+1}b-1\right)q^{n+1}a\left(q^{2n+4}ab-1\right)\left(q^{n+1}-1\right)S(n)=0 \quad \textbf{(73)}$$

```
> qksum := proc (p, N, x, q, j) qphihyperterm([q^(-n), x, -p*q^n], [q^(-N), 0], q, q, j) end proc;
```

$qksum := \mathbf{proc}(p, N, x, q, j)$ **(74)**

$qphihyperterm([q\hat{}(-n), x, -p*q\hat{}n], [q\hat{}(-N), 0], q, q, j)$

end proc

```
> RK := qsumrecursion(qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b, q, j), q, j, S(n), recursion = up);
```

$$RK := \left(q^{2n+2}ab-1\right)\left(q^{n+1}-q^{\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}}\right)\left(q^{n+2}ab-1\right)S(n+2)-q\left(q^{2n+3}ab-1\right)\left(q^{4n+\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+6}xa^2b^3-q^{3n+\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+4}a^2b^2-q^{\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+2n+4}xab^2-q^{\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+2n+2}xab^2-q^{3n+3}ab+q^{\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+2n+3}ab+q^{\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+2n+2}ab+q^{2n+2}ab+q^{2n+1}ab-q^{n+1+\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}}ab+q^{\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}}xb-q^n\right)S(n+1)+q^{2n+2}\left(q^{n+\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}+2}ab-1\right)a\left(q^{2n+4}ab-1\right)\left(q^{n+1}-1\right)bS(n)=0 \quad \textbf{(75)}$$

```
> `recursion/compare`(RL, simplify(RK), S(n));
```

Recursions are NOT identical! **(76)**

```
> qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer(b\,x\,q, q, n)\, qpochhammer\left(a\,b\,q^n\,q, q, n\right)\, q^n}{qpochhammer\left(q^{-\frac{\ln\left(\frac{1}{bq}\right)}{\ln(q)}}, q, n\right)\, qpochhammer(q, q, n)} \quad \textbf{(77)}$$

```
> denumLK := qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b, q, n)*(-1)^n*(b*q)^n*q^binomial(n, 2)/qpochhammer(q*x*b, q, n);
```

$denumLK :=$ **(78)**

$$\frac{qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(a\, b\, q^{n}\, q, q, n\right)\, q^{n}\, (-1)^{n}\, (b\, q)^{n}\, q^{\mathrm{binomial}(n,\, 2)}}{qpochhammer\left(q^{-\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}}, q, n\right)\, qpochhammer(q, q, n)}$$

```
> LJsummand(a, b, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(a\, b\, q^{n+1}, q, n\right)\, (q\, x)^{n}}{qpochhammer(a\, q, q, n)\, qpochhammer(q, q, n)} \quad (79)$$

```
> numLB := LJsummand(a, b, x, q, n)*q^n/(q*x)^n;
```

$$numLB := \frac{qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(a\, b\, q^{n+1}, q, n\right)\, q^{n}}{qpochhammer(a\, q, q, n)\, qpochhammer(q, q, n)} \quad (80)$$

```
>
> CLK := simplify(numLB/denumLK);
```

$$CLK := \frac{qpochhammer(b\, q, q, n)\, (-1)^{-n}\, (b\, q)^{-n}\, q^{-\mathrm{binomial}(n,\, 2)}}{qpochhammer(a\, q, q, n)} \quad (81)$$

```
> NRK := qsumrecursion(CLK*qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b,
  q, j), q, j, S(n), recursion = up);
```

$$\begin{aligned} NRK := {} & q^{n+1}\, \left(q^{n+1}\, a-1\right)\, \left(q^{2n+2}\, a\, b-1\right)\, \left(q^{n+1}-q^{\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}}\right)\, \left(q^{n+2}\, a-1\right)\, \left(q^{n+2}\, a\, b\right. \\ & \left.-1\right)\, b\, S(n+2)+\left(q^{2n+3}\, a\, b-1\right)\, \left(q^{4n+\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+6}\, x\, a^{2}\, b^{3}-q^{3n+\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+4}\, a^{2}\, b^{2}\right. \\ & -q^{\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+2n+4}\, x\, a\, b^{2}-q^{\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+2n+2}\, x\, a\, b^{2}-q^{3n+3}\, a\, b+q^{\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+2n+3}\, a\, b \\ & +q^{\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+2n+2}\, a\, b+q^{2n+2}\, a\, b+q^{2n+1}\, a\, b-q^{n+1+\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}}\, a\, b+q^{\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}}\, x\, b \\ & \left.-q^{n}\right)\, \left(q^{n+1}\, a-1\right)\, \left(q^{n+2}\, b-1\right)\, S(n+1)+\left(q^{n+1}\, b-1\right)\, q^{n}\, \left(q^{n+\frac{\ln\left(\frac{1}{b\, q}\right)}{\ln(q)}+2}\, a\, b\right. \\ & \left.-1\right)\, a\, \left(q^{2n+4}\, a\, b-1\right)\, \left(q^{n+1}-1\right)\, \left(q^{n+2}\, b-1\right)\, S(n)=0 \end{aligned} \quad (82)$$

```
> `recursion/compare`(RL, simplify(NRK), S(n));
```

Recursions are identical. (83)

```
>
> simplify(qsimpcomb([seq(CLK*add(qksum(-a*b*q, ln(1/(b*q))/ln(q),
  q*x*b, q, j), j = 0 .. n), n = 0 .. 5)-seq(add(LJsummand(a, b, x,
  q, j), j = 0 .. n), n = 0 .. 5)]));
```

$$[0, 0, 0, 0, 0, 0] \quad (84)$$

q-Laguerre

```
> qlsum := proc (a, x, q, j) qpochhammer(q^(a+1), q, n)*
  qphihyperterm([q^(-n)], [q^(a+1)], q, -q^(n+a+1)*x, j)
  /qpochhammer(q, q, n) end proc;
```

$qlsum :=$ **proc**(a, x, q, j) (85)

$qpochhammer(q^{\wedge}(a+1), q, n) * qphihyperterm([q^{\wedge}(-n)], [q^{\wedge}(a+1)], q, -q^{\wedge}(n+a+1) * x, j) / qpochhammer(q, q, n)$

end proc

```
> Rll := qsumrecursion(qlsum(a, x, q, j), q, j, S(n), recursion = up);
```

$$Rll := \left(q^{n+2}-1\right) S(n+2) + \left(-q^{a+2n+3} x - q^{n+2+a} - q^{n+2} + q + 1\right) S(n+1) + q\left(q^{n+a+1}-1\right) S(n) = 0 \quad (86)$$

```
> qmsum := proc (b, c, x, q, j) qphihyperterm([q^(-n), x], [b*q], q, -q^(n+1)/c, j) end proc;
```

$qmsum :=$ **proc**(b, c, x, q, j) (87)

$qphihyperterm([q^{\wedge}(-n), x], [b*q], q, -q^{\wedge}(n+1)/c, j)$

end proc

```
> RM := qsumrecursion(qmsum(0, -q^(-a), -x, q, j), q, j, S(n), recursion = up);
```

$$RM := S(n+2) + \left(q^{a+2n+3} x + q^{n+2+a} + q^{n+2} - q - 1\right) S(n+1) + q\left(q^{n+a+1}-1\right)\left(q^{n+1}-1\right) S(n) = 0 \quad (88)$$

```
> `recursion/compare`(Rll, simplify(RM), S(n));
```

Recursions are NOT identical! (89)

```
> qlsum(a, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right)\left(-q^{n+a+1} x\right)^{n}(-1)^{n} q^{\frac{1}{2} n(n-1)}}{qpochhammer(q, q, n)^{2}} \quad (90)$$

```
> numqL := qlsum(a, x, q, n)*(-q^(n+a+1))^n/(-q^(n+a+1)*x)^n;
```

$$numqL := \frac{qpochhammer\left(q^{-n}, q, n\right)(-1)^{n} q^{\frac{1}{2} n(n-1)}\left(-q^{n+a+1}\right)^{n}}{qpochhammer(q, q, n)^{2}} \quad (91)$$

```
> qmsum(0, -q^(-a), -x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer(-x, q, n)\left(\frac{q^{n+1}}{q^{-a}}\right)^{n}}{qpochhammer(q, q, n)} \quad (92)$$

```
> denumqLM := qmsum(0, -q^(-a), -x, q, n)*q^binomial(n, 2)/qpochhammer(-x, q, n);
```

$$denumqLM := \frac{qpochhammer\left(q^{-n}, q, n\right)\left(\frac{q^{n+1}}{q^{-a}}\right)^{n} q^{\mathrm{binomial}(n,2)}}{qpochhammer(q, q, n)} \quad (93)$$

```
> CqLM := qsimpcomb(numqL/denumqLM);
```

$$CqLM := \frac{1}{qpochhammer(q, q, n)} \quad (94)$$

```
> NRM := qsumrecursion(CqLM*qmsum(0, -q^(-a), -x, q, j), q, j, S(n), recursion = up);
```

$$NRM := \left(q^{n+2}-1\right) S(n+2) + \left(-q^{a+2n+3} x - q^{n+2+a} - q^{n+2} + q + 1\right) S(n+1) + q\left(q^{n+a+1}-1\right) S(n) = 0 \quad (95)$$

```
> `recursion/compare`(Rll, simplify(NRM), S(n));
```

$$\textit{Recursions are identical.} \tag{96}$$

```
> qsimpcomb([seq(CqLM*add(qmsum(0, -q^(-a), -x, q, j), j = 0 .. n),
  n = 0 .. 5)-seq(add(qlsum(a, x, q, j), j = 0 .. n), n = 0 .. 5)])
  ;
```

$$[0, 0, 0, 0, 0, 0] \tag{97}$$

Al-Salam Carlitz I

```
> A1summand := proc (a, x, q, j) (-a)^n*q^binomial(n, 2)*
  qphihyperterm([q^(-n), 1/x], [0], q, q*x/a, j) end proc;
```

$A1summand := \mathbf{proc}(a, x, q, j)$ (98)

$(-a)\text{^}n * q\text{^}\mathrm{binomial}(n, 2) * qphihyperterm([q\text{^}(-n), 1/x], [0], q, q*x/a, j)$

end proc

```
> RA1 := qsumrecursion(A1summand(a, x, q, j), q, j, S(n), recursion
  = up);
```

$$RA1 := S(n+2) + \left(q^{n+1} a + q^{n+1} - x\right) S(n+1) + q^n a \left(q^{n+1} - 1\right) S(n) = 0 \tag{99}$$

```
> RA2 := qsumrecursion(A1summand(1/a, x/a, q, j), q, j, S(n),
  recursion = up);
```

$$RA2 := a\, S(n+2) + \left(q^{n+1} a + q^{n+1} - x\right) S(n+1) + q^n \left(q^{n+1} - 1\right) S(n) = 0 \tag{100}$$

```
> `recursion/compare`(RA1, simplify(RA2), S(n));
```

$$\textit{Recursions are NOT identical!} \tag{101}$$

```
> A1summand(a, x, q, n);
```

$$\frac{(-a)^n q^{\mathrm{binomial}(n,2)}\, qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(\frac{1}{x}, q, n\right) \left(\frac{q\,x}{a}\right)^n}{qpochhammer(q, q, n)} \tag{102}$$

```
> denuma1 := simplify(A1summand(a, x, q, n)*(q/a)^n/(qpochhammer
  (1/x, q, n)*(x*q/a)^n));
```

$$denuma1 := \frac{(-a)^n q^{\mathrm{binomial}(n,2)}\, qpochhammer\left(q^{-n}, q, n\right) \left(\frac{q}{a}\right)^n}{qpochhammer(q, q, n)} \tag{103}$$

```
> A1summand(1/a, x/a, q, n);
```

$$\frac{\left(-\frac{1}{a}\right)^n q^{\mathrm{binomial}(n,2)}\, qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(\frac{a}{x}, q, n\right) (q\,x)^n}{qpochhammer(q, q, n)} \tag{104}$$

```
> numa1 := simplify(A1summand(1/a, x/a, q, n)*q^n/(qpochhammer(a/x,
  q, n)*(q*x)^n));
```

$$numa1 := \frac{\left(-\frac{1}{a}\right)^n q^{\mathrm{binomial}(n,2)+n}\, qpochhammer\left(q^{-n}, q, n\right)}{qpochhammer(q, q, n)} \tag{105}$$

```
> CA1 := qsimpcomb(numa1/denuma1);
```

$$CA1 := \frac{1}{a^n} \tag{106}$$

```
> qsimpcomb([seq(CA1*add(A1summand(a, x, q, j), j = 0 .. n), n = 0
  .. 3)-seq(add(A1summand(1/a, x/a, q, j), j = 0 .. n), n = 0 .. 3)
  ]);
```

$$[0, 0, 0, 0] \tag{107}$$

```
> NRA1 := qsumrecursion(CA1*A1summand(a, x, q, j), q, j, S(n),
  recursion = up);
```

$$NRA1 := a\,S(n+2) + \left(q^{n+1}\,a + q^{n+1} - x\right) S(n+1) + q^n \left(q^{n+1} - 1\right) S(n) = 0 \quad (108)$$

```
> `recursion/compare`(NRA1, simplify(RA2), S(n));
```

Recursions are identical. **(109)**

Discrete q-Hermite I

```
> qsumrecursion(A1summand(-1, -x, q, j), q, j, S(n), recursion =
  up);
```

$$-S(n+2) - x\,S(n+1) + q^n \left(q^{n+1} - 1\right) S(n) = 0 \quad (110)$$

```
> qsumrecursion(A1summand(-1, x, q, j), q, j, S(n), recursion = up)
  ;
```

$$-S(n+2) + x\,S(n+1) + q^n \left(q^{n+1} - 1\right) S(n) = 0 \quad (111)$$

```
> `recursion/compare`(qsumrecursion(A1summand(-1, -x, q, j), q, j,
  S(n), recursion = up), qsumrecursion(A1summand(-1, x, q, j), q,
  j, S(n), recursion = up), S(n));
```

Recursions are NOT identical! **(112)**

```
> A1summand(-1, -x, q, n);
```

$$\frac{q^{\mathrm{binomial}(n,\,2)}\,\mathit{qpochhammer}\left(q^{-n}, q, n\right)\,\mathit{qpochhammer}\left(-\frac{1}{x}, q, n\right)\,(q\,x)^n}{\mathit{qpochhammer}(q, q, n)} \quad (113)$$

```
> numd1 := A1summand(-1, -x, q, n)*q^n/(qpochhammer(-1/x, q, n)*(q*
  x)^n);
```

$$\mathit{numd1} := \frac{q^{\mathrm{binomial}(n,\,2)}\,\mathit{qpochhammer}\left(q^{-n}, q, n\right)\,q^n}{\mathit{qpochhammer}(q, q, n)} \quad (114)$$

```
> A1summand(-1, x, q, n);
```

$$\frac{q^{\mathrm{binomial}(n,\,2)}\,\mathit{qpochhammer}\left(q^{-n}, q, n\right)\,\mathit{qpochhammer}\left(\frac{1}{x}, q, n\right)\,(-q\,x)^n}{\mathit{qpochhammer}(q, q, n)} \quad (115)$$

```
> denumd1 := A1summand(-1, x, q, n)*(-1)^n*q^n/(qpochhammer(1/x, q,
  n)*(-q*x)^n);
```

$$\mathit{denumd1} := \frac{q^{\mathrm{binomial}(n,\,2)}\,\mathit{qpochhammer}\left(q^{-n}, q, n\right)\,(-1)^n\,q^n}{\mathit{qpochhammer}(q, q, n)} \quad (116)$$

```
> Cd1 := simplify(numd1/denumd1);
```

$$\mathit{Cd1} := (-1)^{-n} \quad (117)$$

```
> `recursion/compare`(qsumrecursion(Cd1*A1summand(-1, -x, q, j), q,
  j, S(n), recursion = up), qsumrecursion(A1summand(-1, x, q, j),
  q, j, S(n), recursion = up), S(n));
```

Recursions are identical. **(118)**

```
> qsimpcomb([seq(Cd1*add(A1summand(-1, -x, q, j), j = 0 .. n), n =
  0 .. 3)-seq(add(A1summand(-1, x, q, j), j = 0 .. n), n = 0 .. 3)]
  );
```

$$[0, 0, 0, 0] \quad (119)$$

Al-Salam Carlitz II

```
> A2summand := proc (a, x, q, j) (-a)^n*q^(-binomial(n, 2))*
  qphihyperterm([q^(-n), x], [], q, q^n/a, j) end proc;
```

A2summand := **proc**(a, x, q, j) **(120)**

$(-a)$^$n*q$^$(-\mathrm{binomial}(n,2))*qphihyperterm([q$^$(-n),x],[\,],q,q$^$n/a,j)$

end proc

```
> RA21 := qsumrecursion(CA1*A2summand(a, x, q, j), q, j, S(n),
  recursion = up);
```

$$RA21 := -q^{2n+1}\,a\,S(n+2)+q^{n}\left(q^{n+1}x-a-1\right)S(n+1)+\left(q^{n+1}-1\right)S(n)=0 \quad (121)$$

```
> RA22 := qsumrecursion(A2summand(1/a, x/a, q, j), q, j, S(n),
  recursion = up);
```

$$RA22 := -q^{2n+1}\,a\,S(n+2)+q^{n}\left(q^{n+1}x-a-1\right)S(n+1)+\left(q^{n+1}-1\right)S(n)=0 \quad (122)$$

```
> `recursion/compare`(RA21, simplify(RA22), S(n));
```

Recursions are identical. **(123)**

```
> A2summand(a, x, q, n);
```

$$\frac{(-a)^{n}\,q^{-\mathrm{binomial}(n,2)}\,qpochhammer\left(q^{-n},q,n\right)\,qpochhammer(x,q,n)\left(\frac{q^{n}}{a}\right)^{n}}{(-1)^{n}\,q^{\frac{1}{2}n(n-1)}\,qpochhammer(q,q,n)} \quad (124)$$

```
> denuma2 := simplify(A2summand(a, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n));
```

$$denuma2 := \frac{(-a)^{n}\,q^{-\frac{1}{2}n(n-1)}\,qpochhammer\left(q^{-n},q,n\right)\left(\frac{q^{n}}{a}\right)^{n}}{qpochhammer(q,q,n)} \quad (125)$$

```
> A2summand(1/a, x/a, q, n);
```

$$\frac{\left(-\frac{1}{a}\right)^{n}q^{-\mathrm{binomial}(n,2)}\,qpochhammer\left(q^{-n},q,n\right)\,qpochhammer\left(\frac{x}{a},q,n\right)\left(a\,q^{n}\right)^{n}}{(-1)^{n}\,q^{\frac{1}{2}n(n-1)}\,qpochhammer(q,q,n)} \quad (126)$$

```
> numa2 := simplify(A2summand(1/a, x/a, q, n)*(-1)^n*(1/a)^n*
  q^binomial(n, 2)/qpochhammer(x/a, q, n));
```

$$numa2 := \frac{\left(-\frac{1}{a}\right)^{n}q^{-\frac{1}{2}n(n-1)}\,qpochhammer\left(q^{-n},q,n\right)\left(a\,q^{n}\right)^{n}\left(\frac{1}{a}\right)^{n}}{qpochhammer(q,q,n)} \quad (127)$$

```
> CA2 := qsimpcomb(numa2/denuma2);
```

$$CA2 := \frac{1}{a^{n}} \quad (128)$$

```
> `recursion/compare`(qsumrecursion(Cd1*A2summand(-1, -I*x, q, j),
  q, j, S(n), recursion = up), qsumrecursion(A2summand(-1, I*x, q,
  j), q, j, S(n), recursion = up), S(n));
```

Recursions are identical. **(129)**

```
> qsimpcomb([seq(CA2*add(A2summand(a, x, q, j), j = 0 .. n), n = 0
  .. 3)-seq(add(A2summand(1/a, x/a, q, j), j = 0 .. n), n = 0 .. 3)
  ]);
```

$$[0,0,0,0] \quad (130)$$

Discrete q-Hermite II

```
> qsimpcomb([seq(Cd1*add(A2summand(-1, -I*x, q, j), j = 0 .. n), n
  = 0 .. 3)-seq(add(A2summand(-1, I*x, q, j), j = 0 .. n), n = 0 ..
  3)]);
```

$$[0, 0, 0, 0] \tag{131}$$

q-Meixner

```
> qMsummand := proc (b, c, x, q, j) qphihyperterm([q^(-n), x], [b*q], q, -q^(n+1)/c, j) end proc;
```

$qMsummand := \mathbf{proc}(b, c, x, q, j)$ **(132)**

$\quad qphihyperterm([q\string^(-n), x], [b * q], q, -q\string^(n+1)/c, j)$

end proc

```
> RM1 := qsumrecursion(qMsummand(b, c, x, q, j), q, j, S(n), recursion = up);
```

$$RM1 := c\left(q^{n+2}b-1\right)S(n+2)+\left(-q^{2n+3}x-q^{n+2}bc-q^{n+2}c+q^{n+2}+cq+c\right)S(n+1)+q\left(q^{n+1}+c\right)\left(q^{n+1}-1\right)S(n)=0 \tag{133}$$

```
> RM2 := qsumrecursion(qMsummand(-1/c, -1/b, -x/(b*c), q, j), q, j, S(n), recursion = up);
```

$$RM2 := \left(q^{n+2}+c\right)S(n+2)+\left(q^{2n+3}x+q^{n+2}bc+q^{n+2}c-q^{n+2}-cq-c\right)S(n+1)+\left(q^{n+1}b-1\right)qc\left(q^{n+1}-1\right)S(n)=0 \tag{134}$$

```
> `recursion/compare`(RM1, simplify(RM2), S(n));
```

Recursions are NOT identical! **(135)**

```
> qMsummand(b, c, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right)qpochhammer(x, q, n)\left(-\frac{q^{n+1}}{c}\right)^{n}}{qpochhammer(bq, q, n)\,qpochhammer(q, q, n)} \tag{136}$$

```
> denumqm := qMsummand(b, c, x, q, n)*(-1)^n*q^binomial(n, 2)/qpochhammer(x, q, n);
```

$$denumqm := \frac{qpochhammer\left(q^{-n}, q, n\right)\left(-\frac{q^{n+1}}{c}\right)^{n}(-1)^{n}q^{\mathrm{binomial}(n,\,2)}}{qpochhammer(bq, q, n)\,qpochhammer(q, q, n)} \tag{137}$$

```
> qMsummand(-1/c, -1/b, -x/(b*c), q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right)qpochhammer\left(-\frac{x}{bc}, q, n\right)\left(q^{n+1}b\right)^{n}}{qpochhammer\left(-\frac{q}{c}, q, n\right)qpochhammer(q, q, n)} \tag{138}$$

```
> numqm := qMsummand(-1/c, -1/b, -x/(b*c), q, n)*(-1)^n*(-1/(b*c))^n*q^binomial(n, 2)/qpochhammer(-x/(b*c), q, n);
```

$$numqm := \frac{qpochhammer\left(q^{-n}, q, n\right)\left(q^{n+1}b\right)^{n}(-1)^{n}\left(-\frac{1}{bc}\right)^{n}q^{\mathrm{binomial}(n,\,2)}}{qpochhammer\left(-\frac{q}{c}, q, n\right)qpochhammer(q, q, n)} \tag{139}$$

```
> CqM := simplify(numqm/denumqm);
```

$$CqM := \frac{\left(q^{n+1}b\right)^{n}\left(-\frac{1}{bc}\right)^{n}\left(-\frac{q^{n+1}}{c}\right)^{-n}qpochhammer(bq, q, n)}{qpochhammer\left(-\frac{q}{c}, q, n\right)} \tag{140}$$

```
> qsimpcomb((b*q^(n+1))^n*(-1/(b*c))^n*(-q^(n+1)/c)^(-n));
```

$$1 \quad \textbf{(141)}$$

```
> NRM1 := qsumrecursion(CqM*qMsummand(b, c, x, q, j), q, j, S(n),
  recursion = up);
```

$$NRM1 := \left(q^{n+2}+c\right) S(n+2) + \left(q^{2n+3} x + q^{n+2} b c + q^{n+2} c - q^{n+2} - c q - c\right) S(n+1) + \left(q^{n+1} b - 1\right) q c \left(q^{n+1} - 1\right) S(n) = 0 \quad \textbf{(142)}$$

```
> `recursion/compare`(NRM1, simplify(RM2), S(n));
```

Recursions are identical. **(143)**

```
> qsimpcomb([seq(CqM*add(qMsummand(b, c, x, q, j), j = 0 .. n), n =
  0 .. 3)-seq(add(qMsummand(-1/c, -1/b, -x/(b*c), q, j), j = 0 ..
  n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \quad \textbf{(144)}$$

q-Krawtchouk

Checking Big q-Jacobi and q-Krawtchouck

```
> qksum := proc (p, N, x, q, j) qphihyperterm([q^(-n), x, -p*q^n],
  [q^(-N), 0], q, q, j) end proc;
```

qksum := **proc**(p, N, x, q, j) **(145)**

  $qphihyperterm([q\wedge(-n), x, -p * q\wedge n], [q\wedge(-N), 0], q, q, j)$

end proc

```
> RK := qsumrecursion(qksum(p, N, x, q, j), q, j, S(n), recursion =
  up);
```

$$RK := \left(p q^{n+1}+1\right) \left(q^{2n+1} p+1\right) \left(-q^{n+1}+q^{N}\right) S(n+2) - \left(q^{N+4n+4} x p^2 - q^{N+3n+3} p^2 + q^{N+2n+3} x p - q^{N+2n+3} p + q^{3n+3} p - q^{N+2n+2} p + q^{N+2n+1} x p - q^{2n+2} p + q^{N+n+1} p - q^{2n+1} p + x q^{N} - q^{n+1}\right) \left(q^{2n+2} p+1\right) S(n+1) + q^{2n+1} \left(q^{2n+3} p+1\right) p \left(q^{n+1}-1\right) \left(q^{N+n+1} p+1\right) S(n) = 0 \quad \textbf{(146)}$$

```
> qksum(p, N, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer(x, q, n) qpochhammer\left(-p q^{n}, q, n\right) q^{n}}{qpochhammer\left(q^{-N}, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(147)}$$

```
> denumqK := qksum(p, N, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$denumqK := \frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(-p q^{n}, q, n\right) q^{n} (-1)^{n} q^{\mathrm{binomial}(n,\,2)}}{qpochhammer\left(q^{-N}, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(148)}$$

```
> BJsummand(q^(-N-1), -q^N*p, 0, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(-q^{-1-N} q^{N} p q^{n+1}, q, n\right) qpochhammer(x, q, n) q^{n}}{qpochhammer\left(q^{-1-N} q, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(149)}$$

```
> numBK := BJsummand(q^(-N-1), -q^N*p, 0, x, q, n)*(-1)^n*
  q^binomial(n, 2)/qpochhammer(x, q, n);
```

$$numBK := \frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(-q^{-1-N} q^{N} p q^{n+1}, q, n\right) q^{n} (-1)^{n} q^{\mathrm{binomial}(n,\,2)}}{qpochhammer\left(q^{-1-N} q, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(150)}$$

```
> CBK := simplify(numBK/denumqK);
```

$$CBK := 1 \quad \textbf{(151)}$$

```
> RBK := qsumrecursion(BJsummand(q^(-N-1), -q^N*p, 0, x, q, j), q,
```

```
j, S(n), recursion = up);
```

$$RBK := \left(p\,q^{n+1}+1\right)\left(q^{2n+1}p+1\right)\left(-q^{n+1}+q^{N}\right)S(n+2)-\left(q^{N+4n+4}x\,p^{2}-q^{N+3n+3}p^{2}+q^{N+2n+3}x\,p-q^{N+2n+3}p+q^{3n+3}p-q^{N+2n+2}p+q^{N+2n+1}x\,p-q^{2n+2}p+q^{N+n+1}p-q^{2n+1}p+x\,q^{N}-q^{n+1}\right)\left(q^{2n+2}p+1\right)S(n+1)+q^{2n+1}\left(q^{2n+3}p+1\right)p\left(q^{n+1}-1\right)\left(q^{N+n+1}p+1\right)S(n)=0 \tag{152}$$

```
> `recursion/compare`(RBK, RK, S(n));
```

Recursions are identical. (153)

```
> simplify([seq(add(BJsummand(q^(-N-1), -q^N*p, 0, x, q, j), j = 0
  .. n), n = 0 .. 5)-seq(add(qksum(p, N, x, q, j), j = 0 .. n), n =
  0 .. 5)]);
```

$$[0, 0, 0, 0, 0, 0] \tag{154}$$

Little q-Jacobi and q-Krawtchouk

```
> RLK := qsumrecursion(LJsummand(-q^N*p, q^(-N-1), x*q^N, q, j), q,
  j, S(n), recursion = up);
```

$$RLK := q^{n+1}\left(p\,q^{n+1}+1\right)\left(q^{2n+1}p+1\right)\left(q^{N+n+2}p+1\right)S(n+2)+\left(q^{N+4n+4}x\,p^{2}-q^{N+3n+3}p^{2}+q^{N+2n+3}x\,p-q^{N+2n+3}p+q^{3n+3}p-q^{N+2n+2}p+q^{N+2n+1}x\,p-q^{2n+2}p+q^{N+n+1}p-q^{2n+1}p+x\,q^{N}-q^{n+1}\right)\left(q^{2n+2}p+1\right)S(n+1)+q^{n+1}\left(q^{2n+3}p+1\right)\left(q^{N}-q^{n}\right)p\left(q^{n+1}-1\right)S(n)=0 \tag{155}$$

```
> `recursion/compare`(RLK, RK, S(n));
```

Recursions are NOT identical! (156)

```
> LJsummand(-q^N*p, q^(-N-1), x*q^N, q, n);
```

$$\frac{\mathit{qpochhammer}\left(q^{-n},q,n\right)\mathit{qpochhammer}\left(-q^{-1-N}q^{N}p\,q^{n+1},q,n\right)\left(q\,x\,q^{N}\right)^{n}}{\mathit{qpochhammer}\left(-q^{N}p\,q,q,n\right)\mathit{qpochhammer}\left(q,q,n\right)} \tag{157}$$

```
> numLK := LJsummand(-q^N*p, q^(-N-1), x*q^N, q, n)*(q^(1+N))^n/(q*
  x*q^N)^n;
```

$$\mathit{numLK} := \frac{\mathit{qpochhammer}\left(q^{-n},q,n\right)\mathit{qpochhammer}\left(-q^{-1-N}q^{N}p\,q^{n+1},q,n\right)\left(q^{1+N}\right)^{n}}{\mathit{qpochhammer}\left(-q^{N}p\,q,q,n\right)\mathit{qpochhammer}\left(q,q,n\right)} \tag{158}$$

```
> CKL := simplify(numLK/denumqK);
```

$$\mathit{CKL} := \frac{(-1)^{-n}\left(q^{1+N}\right)^{n}\mathit{qpochhammer}\left(q^{-N},q,n\right)q^{-n-\mathrm{binomial}(n,\,2)}}{\mathit{qpochhammer}\left(-q^{1+N}p,q,n\right)} \tag{159}$$

```
> NRK := qsumrecursion((-1)^n*(q^N)^n*qpochhammer(q^(-N), q, n)*
  qksum(p, N, x, q, j)/(qpochhammer(-p*q^(1+N), q, n)*q^binomial(n,
  2)), q, j, S(n), recursion = up);
```

$$NRK := -q^{n+1}\left(p\,q^{n+1}+1\right)\left(q^{2n+1}p+1\right)\left(q^{N+n+2}p+1\right)S(n+2)-\left(q^{N+4n+4}x\,p^{2}-q^{N+3n+3}p^{2}+q^{N+2n+3}x\,p-q^{N+2n+3}p+q^{3n+3}p-q^{N+2n+2}p+q^{N+2n+1}x\,p-q^{2n+2}p+q^{N+n+1}p-q^{2n+1}p+x\,q^{N}-q^{n+1}\right)\left(q^{2n+2}p+1\right)S(n+1)+q^{n+1}\left(q^{2n+3}p+1\right)\left(-q^{N}+q^{n}\right)p\left(q^{n+1}-1\right)S(n)=0 \tag{160}$$

```
> `recursion/compare`(RLK, NRK, S(n));
```

Recursions are identical. (161)

```
> simplify([seq(CKL*add(qksum(p, N, x, q, j), j = 0 .. n), n = 0 ..
  3)-seq(add(LJsummand(-q^N*p, q^(-N-1), x*q^N, q, j), j = 0 .. n),
  n = 0 .. 3)]);
```

(162)

$$[0, 0, 0, 0] \tag{162}$$

q-Hahn

```
> qhahnsummand := proc (alpha, beta, N, x, q, k) qphihyperterm([q^(-n), alpha*beta*q^(n+1), x], [alpha*q, q^(-N)], q, q, k) end proc;
```

$$qhahnsummand := \mathbf{proc}(\text{alpha}, \text{beta}, N, x, q, k) \tag{163}$$
$$qphihyperterm([q\hat{}(-n), \text{alpha}*\text{beta}*q\hat{}(n+1), x], [\text{alpha}*q, q\hat{}(-N)], q, q, k)$$
end proc

```
> RE1 := qsumrecursion(qhahnsummand(alpha, beta, N, x, q, k), q, k, S(n), recursion = up);
```

$$RE1 := -\left(q^{n+2}\alpha-1\right)\left(q^{2n+2}\beta\alpha-1\right)\left(q^{n+2}\beta\alpha-1\right)\left(q^{n+1}-q^{N}\right)S(n+2) - \left(q^{2n+3}\beta\alpha-1\right)\left(q^{N+4n+6}x\beta^2\alpha^2-q^{N+3n+5}\beta^2\alpha^2-q^{N+3n+5}\beta\alpha^2-q^{3n+4}\beta\alpha^2 + q^{N+2n+4}\beta\alpha^2-q^{N+2n+4}x\beta\alpha-q^{3n+4}\beta\alpha+q^{N+2n+3}\beta\alpha^2+q^{N+2n+4}\beta\alpha + q^{N+2n+3}\beta\alpha-q^{N+2n+2}x\beta\alpha+q^{2n+3}\beta\alpha+q^{2n+2}\beta\alpha+q^{2n+3}\alpha-q^{N+n+2}\beta\alpha + q^{2n+2}\alpha-q^{N+n+2}\alpha-q^{n+1}\alpha-q^{n+1}+x q^{N}\right)S(n+1)+\left(q^{N+n+2}\beta\alpha - 1\right)q^{n+1}\left(q^{2n+4}\beta\alpha-1\right)\left(q^{n+1}-1\right)\left(\beta q^{n+1}-1\right)\alpha S(n)=0 \tag{164}$$

```
> RE2 := qsumrecursion(qhahnsummand(beta, alpha, -(ln(q)*N-ln(1/(alpha*beta*q^2)))/ln(q), q^N*beta*q*x, q, k), q, k, S(n), recursion = up);
```

$$RE2 := \left(q^{n+2}\beta-1\right)\left(q^{2n+2}\beta\alpha-1\right)\left(q^{n+2}\beta\alpha-1\right)\left(q^{N+n+1}-q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}}\right)S(n+2) + q\left(q^{4n+N+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+6}x\beta^3\alpha^2-q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+N+2n+4}x\beta^2\alpha - q^{3n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+4}\beta^2\alpha^2-q^{3n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+4}\beta^2\alpha-q^{N+3n+3}\beta^2\alpha - q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+N+2n+2}x\beta^2\alpha-q^{N+3n+3}\beta\alpha+q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+3}\beta^2\alpha + q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+2}\beta^2\alpha+q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+3}\beta\alpha+q^{N+2n+2}\beta\alpha + q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+2}\beta\alpha+q^{N+2n+1}\beta\alpha+q^{N+2n+2}\beta+q^{N+2n+1}\beta - q^{n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+1}\beta\alpha+q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+N}x\beta-q^{n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+1}\beta-q^{n+N}\beta-q^{n+N}\right) \tag{165}$$

$$\left(q^{2n+3}\beta\alpha-1\right)S(n+1)+\left(q^{n+1}\alpha-1\right)q^{n+1}\left(q^{2n+4}\beta\alpha-1\right)\beta\left(-q^{n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+2}\beta\alpha+q^N\right)\left(q^{n+1}-1\right)S(n)=0$$

```
> RE2 := simplify(RE2);
```

$$RE2 := S(n+2)\left(q^{5n+N+7}\alpha^2\beta^3-q^{N+3n+5}\alpha\beta^2-q^{4n+N+5}\alpha^2\beta^2+q^{N+2n+3}\beta\alpha-q^{4n+4}\beta^2\alpha+q^{2n+2}\beta+q^{3n+2}\alpha\beta-q^n-q^{4n+N+5}\beta^2\alpha+q^{N+2n+3}\beta+q^{N+3n+3}\beta\alpha-q^{N+n+1}+\beta q^{3n+2}-\frac{q^n}{\alpha}-q^{2n}+\frac{1}{\alpha\beta q^2}\right)+q\,S(n+1)\left(-q^{N+2n+2}\beta\alpha-q^{N+2n+1}\beta\alpha+q^{4n+N+5}\alpha^2\beta^2+q^{4n+N+5}\beta^2\alpha-q^{N+4n+4}\alpha\beta^2x+q^{N+2n+1}x\beta-q^{5n+N+6}\alpha^2\beta^3-q^{5n+N+6}\alpha^2\beta^2+q^{N+4n+4}\alpha^2\beta^2+q^{N+4n+4}\beta^2\alpha+q^{2n+N}\beta x+q^{n-1}+q^{n+N}+q^{N+6n+7}\alpha^2\beta^3x-q^{4n+N+5}x\beta^2\alpha-q^{4n+N+3}\beta^2x\alpha+q^{N+2n+2}x\beta-q^{5n+5}\alpha^2\beta^2-\beta^2q^{5n+5}\alpha+q^{4n+3}\beta^2\alpha+q^{4n+4}\alpha\beta+q^{4n+3}\alpha\beta-q^{2n+1}\beta-q^{2n}\beta-\frac{q^{N-2}x}{\alpha}+q^{4n+4}\beta^2\alpha+\frac{q^{n-1}}{\alpha}+q^{n+N}\beta-q^{N+2n+2}\beta-q^{N+2n+1}\beta-q^{2n}-q^{2n+1}\right)+\beta S(n)\left(q^{5n+N+7}\alpha^2\beta-q^{N+4n+6}\alpha\beta-q^{N+3n+3}\alpha+q^{N+2n+2}-q^{N+4n+6}\alpha^2\beta+q^{N+3n+5}\alpha\beta+q^{N+2n+2}\alpha-q^{N+n+1}-q^{6n+7}\alpha^2\beta+q^{5n+6}\alpha\beta+q^{4n+3}\alpha-q^{3n+2}+q^{5n+6}\alpha^2\beta-q^{4n+5}\alpha\beta-q^{3n+2}\alpha+q^{2n+1}\right)=0 \quad \textbf{(166)}$$

```
> `recursion/compare`(RE1, RE2, S(n));
```

Recursions are NOT identical! **(167)**

```
> RE3 := qsumrecursion(qhahnsummand(1/(q*q^N), alpha*beta*q*q^N, ln
  (1/(alpha*q))/ln(q), x, q, k), q, k, S(n), recursion = up);
```

$$RE3 := \left(q^{2n+2}\beta\alpha-1\right)\left(-q^{n+1}+q^N\right)\left(q^{n+2}\beta\alpha-1\right)\left(q^{n+1}-q^{\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}}\right)S(n+2) \quad \textbf{(168)}$$

$$- \left(q^{4n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+6} x \beta^2 \alpha^2 - q^{3n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+5} \beta^2 \alpha^2 \right.$$
$$- q^{2n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+4} x \beta \alpha - q^{3n+N+4} \beta \alpha + q^{2n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+4} \beta \alpha$$
$$- q^{3n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+4} \beta \alpha + q^{2n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+3} \beta \alpha - q^{2n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2} x \beta \alpha$$
$$+ q^{N+2n+3} \beta \alpha - q^{3n+3} \beta \alpha + q^{2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+3} \beta \alpha + q^{N+2n+2} \beta \alpha$$
$$- q^{n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2} \beta \alpha + q^{2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2} \beta \alpha + q^{2n+2} - q^{N+n+1} + q^{\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+N} x$$
$$\left. + q^{2n+1} - q^{n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+1} - q^n \right) \left(q^{2n+3} \beta \alpha - 1\right) S(n+1) + \left(q^{N+n+2} \beta \alpha\right.$$
$$\left. - 1\right) q^n \left(q^{n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2} \beta \alpha - 1 \right) \left(q^{2n+4} \beta \alpha - 1\right) \left(q^{n+1} - 1\right) S(n) = 0$$

```
> RE3 := simplify(RE3);
```

$$RE3 := S(n+2) \left(-q^{5n+6} \alpha^2 \beta^2 + q^{4n+N+5} \alpha^2 \beta^2 + q^{3n+4} \beta \alpha - q^{N+2n+3} \beta \alpha \right. \tag{169}$$
$$+ q^{4n+4} \alpha \beta - q^{N+3n+3} \beta \alpha - q^{2n+2} + q^{N+n+1} + q^{4n+4} \beta^2 \alpha - q^{N+3n+3} \beta^2 \alpha$$
$$\left. - q^{2n+2} \beta + q^{N+n+1} \beta - \beta q^{3n+2} + q^{N+2n+1} \beta + \frac{q^n}{\alpha} - \frac{q^{-1+N}}{\alpha} \right) - S(n+1) \Big($$
$$-q^{N+2n+3} \beta \alpha - q^{N+2n+2} \beta \alpha - q^{5n+N+7} \alpha^2 \beta^3 + q^{4n+N+5} \alpha^2 \beta^2 + q^{4n+N+5} \beta^2 \alpha$$
$$- q^{N+4n+4} \alpha \beta^2 x + q^{N+2n+1} x \beta - q^{5n+6} \alpha^2 \beta^2 + q^{N+n+1} \beta - \frac{q^{-1+N} x}{\alpha}$$
$$+ q^{N+6n+8} x \beta^3 \alpha^2 - q^{N+4n+6} x \beta^2 \alpha - q^{4n+N+5} x \beta^2 \alpha + q^{N+2n+3} x \beta$$
$$- q^{N+2n+3} \beta - q^{2n+2} \beta + q^{N+2n+2} x \beta + q^{4n+5} \alpha \beta - q^{5n+N+7} \alpha^2 \beta^2$$
$$+ q^{N+4n+6} \beta^2 \alpha + q^{N+4n+6} \alpha^2 \beta^2 + \frac{q^n}{\alpha} - q^{2n+2} + q^{4n+4} \alpha \beta - q^{2n+1} \beta$$
$$+ q^{4n+4} \beta^2 \alpha + q^{N+n+1} - q^{N+2n+2} \beta + q^n - q^{5n+6} \beta^2 \alpha + q^{4n+5} \alpha \beta^2 - q^{2n+1} \Big)$$
$$+ S(n) \left(q^{N+6n+8} \beta^3 \alpha^2 - q^{5n+6} \beta^2 \alpha - q^{N+4n+4} \beta^2 \alpha + \beta q^{3n+2} - q^{5n+N+7} \alpha^2 \beta^2 \right.$$

$$+q^{4n+5}\alpha\beta+q^{N+3n+3}\beta\alpha-q^{2n+1}-q^{5n+N+7}\alpha^2\beta^3+q^{4n+5}\alpha\beta^2+q^{N+3n+3}\beta^2\alpha-q^{2n+1}\beta+q^{N+4n+6}\alpha^2\beta^2-q^{3n+4}\beta\alpha-q^{N+2n+2}\beta\alpha+q^n\Big)=0$$

```
> `recursion/compare`(RE1, RE3, S(n));
```

Recursions are identical. **(170)**

```
> `recursion/compare`(RE2, RE3, S(n));
```

Recursions are NOT identical! **(171)**

```
> RE4 := qsumrecursion(qhahnsummand(alpha*beta*q*q^N, 1/(q*q^N), ln
  (1/(beta*q))/ln(q), q^N*beta*q*x, q, k), q, k, S(n), recursion =
  up);
```

$$RE4:=-\left(q^{N+n+3}\alpha\beta-1\right)\left(q^{2n+2}\beta\alpha-1\right)\left(q^{n+2}\beta\alpha-1\right)\left(q^{n+1}-q^{\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}}\right)S(n+2)-q\Bigg(q^{4n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+6}x\beta^3\alpha^2-q^{3n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+5}\beta^2\alpha^2-q^{3n+N+4}\beta^2\alpha^2-q^{3n+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+4}\beta^2\alpha^2+q^{2n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+4}\beta^2\alpha^2-q^{2n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+4}x\beta^2\alpha+q^{2n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+3}\beta^2\alpha^2-q^{2n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+2}x\beta^2\alpha-q^{3n+3}\beta\alpha+q^{N+2n+3}\beta\alpha+q^{\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+2n+3}\beta\alpha+q^{N+2n+2}\beta\alpha+q^{\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+2n+2}\beta\alpha-q^{n+N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+2}\beta\alpha+q^{2n+2}\beta\alpha+q^{2n+1}\beta\alpha-q^{N+n+1}\beta\alpha-q^{n+1+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}}\beta\alpha+q^{N+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}}x\beta-q^n\Bigg)\left(q^{2n+3}\beta\alpha-1\right)S(n+1)+q^{n+2}\left(-q^N+q^n\right)\left(q^{2n+4}\beta\alpha-1\right)\beta\left(q^{n+1}-1\right)\left(q^{n+\frac{\ln\left(\frac{1}{\beta q}\right)}{\ln(q)}+2}\beta\alpha-1\right)\alpha S(n)=0 \quad \textbf{(172)}$$

```
> RE4 := simplify(RE4);
```

$$RE4:=-S(n+2)\Big(q^{5n+N+8}\alpha^3\beta^3-q^{3n+N+6}\alpha^2\beta^2-q^{4n+5}\alpha^2\beta^2+q^{2n+3}\beta\alpha-q^{N+4n+6}\alpha^2\beta^2+q^{N+2n+4}\beta\alpha+q^{3n+3}\beta\alpha-q^{n+1}-q^{N+4n+6}\alpha^3\beta^2+q^{N+2n+4}\beta\alpha^2+q^{3n+3}\alpha^2\beta-q^{n+1}\alpha+q^{3n+N+4}\beta\alpha^2-q^{N+n+2}\alpha-q^{2n+1}\alpha+\frac{1}{\beta q}\Big)-q\,S(n+1)\Big(-q^{5n+6}\alpha^3\beta^2+q^{N+n+1}\alpha-q^{N+2n+3}\beta\alpha^2-q^{N+2n+3}\beta\alpha-q^{2n+2}\beta\alpha-q^{N+2n+2}\beta\alpha+q^{4n+N+5}\alpha^2\beta^2-q^{5n+6}\alpha^2\beta^2-q^{N+4n+4}\alpha^2\beta^2x \quad \textbf{(173)}$$

$$-q^{4n+N+5}x\beta^2\alpha^2+q^n\alpha-q^{5n+N+7}\alpha^3\beta^3-q^{5n+N+7}\alpha^3\beta^2-q^{N+4n+6}x\beta^2\alpha^2$$
$$+q^{N+2n+2}x\beta\alpha+q^{N+4n+6}\alpha^2\beta^2-q^{2n+1}\alpha+q^{N+2n+3}x\beta\alpha+q^{4n+5}\alpha^2\beta$$
$$+q^{N+4n+6}\alpha^3\beta^2+q^{4n+5}\alpha^2\beta^2-q^{N+2n+2}\beta\alpha^2-q^{2n+1}\beta\alpha-q^{2n+2}\alpha$$
$$+q^{4n+4}\alpha^2\beta+q^{4n+4}\beta^2\alpha^2+q^{4n+N+5}\alpha^3\beta^2+q^{N+2n+1}x\beta\alpha+q^{N+6n+8}x\beta^3\alpha^3$$
$$+q^{N+n+1}\beta\alpha-q^{-1+N}x+q^n\big)+\beta\alpha S(n)\big(-q^{5n+N+8}\alpha^2\beta+q^{3n+N+4}\alpha$$
$$+q^{6n+8}\alpha^2\beta-q^{4n+4}\alpha+q^{4n+N+7}\alpha\beta-q^{N+2n+3}-q^{5n+7}\alpha\beta+q^{3n+3}$$
$$+q^{4n+N+7}\alpha^2\beta-q^{N+2n+3}\alpha-q^{5n+7}\alpha^2\beta+q^{3n+3}\alpha-q^{3n+N+6}\alpha\beta+q^{N+n+2}$$
$$+q^{4n+6}\alpha\beta-q^{2n+2}\big)=0$$

```
> `recursion/compare`(RE1, RE4, S(n));
```

Recursions are NOT identical! **(174)**

```
> `recursion/compare`(RE2, RE4, S(n));
```

Recursions are identical. **(175)**

```
> `recursion/compare`(RE3, RE4, S(n));
```

Recursions are NOT identical! **(176)**

Obtaining the relations

```
> qhahnsummand(alpha, beta, N, x, q, n);
```

$$\frac{\mathit{qpochhammer}(q^{-n},q,n)\,\mathit{qpochhammer}(\alpha\beta q^{n+1},q,n)\,\mathit{qpochhammer}(x,q,n)\,q^n}{\mathit{qpochhammer}(\alpha q,q,n)\,\mathit{qpochhammer}(q^{-N},q,n)\,\mathit{qpochhammer}(q,q,n)} \quad \textbf{(177)}$$

```
> denumh := qhahnsummand(alpha, beta, N, x, q, n)*(-1)^n*q^binomial
  (n, 2)/qpochhammer(x, q, n);
```

$$\mathit{denumh} := \frac{\mathit{qpochhammer}(q^{-n},q,n)\,\mathit{qpochhammer}(\alpha\beta q^{n+1},q,n)\,q^n\,(-1)^n\,q^{\mathrm{binomial}(n,\,2)}}{\mathit{qpochhammer}(\alpha q,q,n)\,\mathit{qpochhammer}(q^{-N},q,n)\,\mathit{qpochhammer}(q,q,n)} \quad \textbf{(178)}$$

```
> simplify(qhahnsummand(beta, alpha, -(ln(q)*N-ln(1/(alpha*beta*
  q^2)))/ln(q), q^N*beta*q*x, q, n));
```

$$\frac{\mathit{qpochhammer}(q^{-n},q,n)\,\mathit{qpochhammer}(\alpha\beta q^{n+1},q,n)\,\mathit{qpochhammer}(q^{1+N}x\beta,q,n)\,q^n}{\mathit{qpochhammer}(\beta q,q,n)\,\mathit{qpochhammer}(q^{N+2}\beta\alpha,q,n)\,\mathit{qpochhammer}(q,q,n)} \quad \textbf{(179)}$$

```
> numh21 := simplify(simplify(qhahnsummand(beta, alpha, -(ln(q)*N-
  ln(1/(alpha*beta*q^2)))/ln(q), q^N*beta*q*x, q, n))*(-1)^n*
  beta^n*q^((1/2)*(2*N+n+1)*n)/qpochhammer(q^(1+N)*beta*x, q, n));
```

$$\mathit{numh21} := \frac{(-1)^n\,\mathit{qpochhammer}(q^{-n},q,n)\,\mathit{qpochhammer}(\alpha\beta q^{n+1},q,n)\,q^{\frac{1}{2}n(3+2N+n)}\,\beta^n}{\mathit{qpochhammer}(\beta q,q,n)\,\mathit{qpochhammer}(q^{N+2}\beta\alpha,q,n)\,\mathit{qpochhammer}(q,q,n)} \quad \textbf{(180)}$$

```
> Ch21 := simplify(numh21/denumh);
```

$$\mathit{Ch21} := \frac{q^{Nn+\frac{1}{2}n^2+\frac{1}{2}n-\mathrm{binomial}(n,\,2)}\,\beta^n\,\mathit{qpochhammer}(\alpha q,q,n)\,\mathit{qpochhammer}(q^{-N},q,n)}{\mathit{qpochhammer}(\beta q,q,n)\,\mathit{qpochhammer}(q^{N+2}\beta\alpha,q,n)} \quad \textbf{(181)}$$

```
> simplify([seq(Ch21*add(qhahnsummand(alpha, beta, N, x, q, j), j =
  0 .. n), n = 0 .. 1)-seq(add(qhahnsummand(beta, alpha, -(ln(q)*N-
  ln(1/(alpha*beta*q^2)))/ln(q), q^N*beta*q*x, q, j), j = 0 .. n),
  n = 0 .. 1)]);
```

$$[0, 0] \quad \textbf{(182)}$$

```
> NRE1 := qsumrecursion(Ch21*qhahnsummand(alpha, beta, N, x, q, k),
  q, k, S(n), recursion = up);
```

$$NRE1 := -\left(q^{n+2}\beta-1\right)\left(q^{N+n+3}\alpha\beta-1\right)\left(q^{2n+2}\beta\alpha-1\right)\left(q^{n+2}\beta\alpha-1\right)S(n+2) - q\,\beta\left(q^{2n+3}\beta\alpha-1\right)\left(q^{N+4n+6}x\beta^2\alpha^2 - q^{N+3n+5}\beta^2\alpha^2 - q^{N+3n+5}\beta\alpha^2 - q^{3n+4}\beta\alpha^2 + q^{N+2n+4}\beta\alpha^2 - q^{N+2n+4}x\beta\alpha - q^{3n+4}\beta\alpha + q^{N+2n+3}\beta\alpha^2 + q^{N+2n+4}\beta\alpha + q^{N+2n+3}\beta\alpha - q^{N+2n+2}x\beta\alpha + q^{2n+3}\beta\alpha + q^{2n+2}\beta\alpha + q^{2n+3}\alpha - q^{N+n+2}\beta\alpha + q^{2n+2}\alpha - q^{N+n+2}\alpha - q^{n+1}\alpha - q^{n+1} + x\,q^N\right)S(n+1) + \left(q^{n+1}\alpha-1\right)q^{n+3}\left(-q^N+q^n\right)\left(q^{2n+4}\beta\alpha-1\right)\beta^2\left(q^{n+1}-1\right)\alpha\,S(n) = 0 \quad \textbf{(183)}$$

```
> `recursion/compare`(NRE1, RE2, S(n));
```

Recursions are identical. **(184)**

```
> simplify(qhahnsummand(1/(q*q^N), alpha*beta*q*q^N, ln(1/(alpha*q)
  )/ln(q), x, q, n));
```

$$\frac{\mathrm{qpochhammer}\left(q^{-n}, q, n\right)\mathrm{qpochhammer}\left(\alpha\beta q^{n+1}, q, n\right)\mathrm{qpochhammer}(x, q, n)\,q^n}{\mathrm{qpochhammer}(\alpha q, q, n)\,\mathrm{qpochhammer}\left(q^{-N}, q, n\right)\mathrm{qpochhammer}(q, q, n)} \quad \textbf{(185)}$$

```
> numh31 := simplify(qhahnsummand(1/(q*q^N), alpha*beta*q*q^N, ln
  (1/(alpha*q))/ln(q), x, q, n))*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$numh31 := \frac{\mathrm{qpochhammer}\left(q^{-n}, q, n\right)\mathrm{qpochhammer}\left(\alpha\beta q^{n+1}, q, n\right)q^n\,(-1)^n\,q^{\mathrm{binomial}(n,\,2)}}{\mathrm{qpochhammer}(\alpha q, q, n)\,\mathrm{qpochhammer}\left(q^{-N}, q, n\right)\mathrm{qpochhammer}(q, q, n)} \quad \textbf{(186)}$$

```
> Ch31 := numh31/denumh;
```

$$Ch31 := 1 \quad \textbf{(187)}$$

Big q-Jacobi and q-Hahn

```
> qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, n);
```

$$\frac{\mathrm{qpochhammer}\left(q^{-n}, q, n\right)\mathrm{qpochhammer}\left(a\,b\,q^{n+1}, q, n\right)\mathrm{qpochhammer}(x, q, n)\,q^n}{\mathrm{qpochhammer}(a\,q, q, n)\,\mathrm{qpochhammer}\left(q^{-\frac{\ln\left(\frac{1}{c\,q}\right)}{\ln(q)}}, q, n\right)\mathrm{qpochhammer}(q, q, n)} \quad \textbf{(188)}$$

```
> denumBH := qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, n)*(-1)^n*
  q^binomial(n, 2)/qpochhammer(x, q, n);
```

$$denumBH := \frac{\mathrm{qpochhammer}\left(q^{-n}, q, n\right)\mathrm{qpochhammer}\left(a\,b\,q^{n+1}, q, n\right)q^n\,(-1)^n\,q^{\mathrm{binomial}(n,\,2)}}{\mathrm{qpochhammer}(a\,q, q, n)\,\mathrm{qpochhammer}\left(q^{-\frac{\ln\left(\frac{1}{c\,q}\right)}{\ln(q)}}, q, n\right)\mathrm{qpochhammer}(q, q, n)} \quad \textbf{(189)}$$

```
> numBH := BJsummand(a, b, c, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$numBH := \frac{\mathrm{qpochhammer}\left(q^{-n}, q, n\right)\mathrm{qpochhammer}\left(a\,b\,q^{n+1}, q, n\right)q^n\,(-1)^n\,q^{\mathrm{binomial}(n,\,2)}}{\mathrm{qpochhammer}(a\,q, q, n)\,\mathrm{qpochhammer}(c\,q, q, n)\,\mathrm{qpochhammer}(q, q, n)} \quad \textbf{(190)}$$

```
> CBH := simplify(numBH/denumBH);
```

$$CBH := 1 \quad \textbf{(191)}$$

```
> Rbh := qsumrecursion(qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q,
  j), q, j, S(n), recursion = up);
```

(192)

$$Rbh := -\left(q^{n+1} - q^{\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}}\right)\left(q^{2n+2}ab-1\right)\left(q^{n+2}a-1\right)\left(q^{n+2}ab-1\right)S(n+2) \tag{192}$$
$$-\left(q^{2n+3}ab-1\right)\left(q^{4n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+6}xa^2b^2 - q^{3n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+5}a^2b^2\right.$$
$$-q^{3n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+5}a^2b - q^{3n+4}a^2b + q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+4}a^2b - q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+4}xab$$
$$-q^{3n+4}ab + q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+3}a^2b + q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+4}ab + q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+3}ab$$
$$-q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2}xab + q^{2n+3}ab + q^{2n+2}ab + q^{2n+3}a - q^{n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2}ab$$
$$\left.+q^{2n+2}a - q^{n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2}a - q^{n+1}a - q^{n+1} + xq^{\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}}\right)S(n+1) + \left(q^{n+1}b\right.$$
$$\left.-1\right)\left(q^{n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2}ab-1\right)q^{n+1}a\left(q^{2n+4}ab-1\right)\left(q^{n+1}-1\right)S(n)=0$$

```
> `recursion/compare`(RB1, simplify(Rbh), S(n));
```

Recursions are identical. (193)

```
> simplify([seq(add(qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, j),
  j = 0 .. n), n = 0 .. 5)-seq(add(BJsummand(a, b, c, x, q, j), j =
  0 .. n), n = 0 .. 5)]);
```

$$[0, 0, 0, 0, 0, 0] \tag{194}$$

q-Hahn to Big q-Jacobi

```
> qhahnsummand(alpha, beta, N, x, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\, qpochhammer(\alpha\beta q^{n+1}, q, n)\, qpochhammer(x, q, n)\, q^n}{qpochhammer(\alpha q, q, n)\, qpochhammer(q^{-N}, q, n)\, qpochhammer(q, q, n)} \tag{195}$$

```
> numHB := qhahnsummand(alpha, beta, N, x, q, n)*(-1)^n*q^binomial
  (n, 2)/qpochhammer(x, q, n);
```

$$numHB := \frac{qpochhammer(q^{-n}, q, n)\, qpochhammer(\alpha\beta q^{n+1}, q, n)\, q^n(-1)^n q^{\mathrm{binomial}(n,2)}}{qpochhammer(\alpha q, q, n)\, qpochhammer(q^{-N}, q, n)\, qpochhammer(q, q, n)} \tag{196}$$

```
> BJsummand(alpha, beta, q^(-N-1), x, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\, qpochhammer(\alpha\beta q^{n+1}, q, n)\, qpochhammer(x, q, n)\, q^n}{qpochhammer(\alpha q, q, n)\, qpochhammer(q^{-1-N} q, q, n)\, qpochhammer(q, q, n)} \tag{197}$$

```
> denumHB := BJsummand(alpha, beta, q^(-N-1), x, q, n)*(-1)^n*
  q^binomial(n, 2)/qpochhammer(x, q, n);
```

$$denumHB := \frac{qpochhammer(q^{-n}, q, n)\, qpochhammer(\alpha\beta q^{n+1}, q, n)\, q^n(-1)^n q^{\mathrm{binomial}(n,2)}}{qpochhammer(\alpha q, q, n)\, qpochhammer(q^{-1-N} q, q, n)\, qpochhammer(q, q, n)} \tag{198}$$

```
> CHB := simplify(numHB/denumHB);
```

$$CHB := 1 \tag{199}$$

```
> RBH := qsumrecursion(BJsummand(alpha, beta, q^(-N-1), x, q, j),
```

```
q, j, S(n), recursion = up);
```

$$RBH := -\left(q^{n+2}\alpha-1\right)\left(q^{2n+2}\beta\alpha-1\right)\left(q^{n+2}\beta\alpha-1\right)\left(q^{n+1}-q^{N}\right)S(n+2) - \left(q^{2n+3}\beta\alpha-1\right)\left(q^{N+4n+6}x\beta^{2}\alpha^{2}-q^{N+3n+5}\beta^{2}\alpha^{2}-q^{N+3n+5}\beta\alpha^{2}-q^{3n+4}\beta\alpha^{2} + q^{N+2n+4}\beta\alpha^{2}-q^{N+2n+4}x\beta\alpha-q^{3n+4}\beta\alpha+q^{N+2n+3}\beta\alpha^{2}+q^{N+2n+4}\beta\alpha + q^{N+2n+3}\beta\alpha-q^{N+2n+2}x\beta\alpha+q^{2n+3}\beta\alpha+q^{2n+2}\beta\alpha+q^{2n+3}\alpha-q^{N+n+2}\beta\alpha + q^{2n+2}\alpha-q^{N+n+2}\alpha-q^{n+1}\alpha-q^{n+1}+x\,q^{N}\right)S(n+1)+\left(q^{N+n+2}\beta\alpha - 1\right)q^{n+1}\left(q^{2n+4}\beta\alpha-1\right)\left(q^{n+1}-1\right)\left(\beta q^{n+1}-1\right)\alpha S(n)=0 \tag{200}$$

```
> `recursion/compare`(RE1, simplify(RBH), S(n));
```

Recursions are identical. **(201)**

```
> simplify([seq(add(qhahnsummand(alpha, beta, N, x, q, j), j = 0 ..
  n), n = 0 .. 5)-seq(add(BJsummand(alpha, beta, q^(-N-1), x, q,
  j), j = 0 .. n), n = 0 .. 5)]);
```

$$[0, 0, 0, 0, 0, 0] \tag{202}$$

```
>
>
>
>
>
>
>
>
>
>
```