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Koepf, Wolfram

On close-to-convex functions and linearly accessible domains. (English)

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The author proves the following interesting lemma: Let $|y| = |x_k| = 1$, $\mu_k > 0$, for $k = 1, ..., n$, and $\sum_{k=1}^{n} \mu_k = 1$.
Then the functions p_n defined by $p_n = \sum_{k=1}^{n} \mu_k (1 + yx_k z)/(1 - x_k z)$, $|z| < 1$, have a representation of the form

$$p_n(z) = \prod_{k=1}^{n} (1 - y_k z)/(1 - x_k z), \quad \textit{where} \quad |x_k| = |y_k| = 1, \quad k = 1, ..., n,$$

and $\arg x_1 < \arg y_1 < \arg x_2 < \arg y_2 < ... < \arg x_n < \arg y_n < \arg x_1 + 2\pi$. Using this lemma and the Schwarz-Christoffel formula he gives a nice geometrical proof of the equivalence of the close-to-convexity of order β of a univalent function f and the accessibility of f({ z: $|z| < 1$}) of the same order.

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Keywords : close-to-convexity

Classification:

- 30C45 Special classes of univalent and multivalent functions
- 30C20 Conformal mappings of special domains