■ ganzzahlige Arithmetik

In[1]:= **50 !**

Out[1]= 30414093201713378043612608166064768844377641568960512000000000000

In[2]:= $\frac{\mathbf{50\,!}}{\mathbf{2^{100}}}$

Out[2]= $\frac{216105129892080882169214875191192738017616943359375}{9007199254740992}$

■ größte gemeinsame Teiler und Faktorisierung

In[3]:= **GCD[50 !, 2^{100}]**

Out[3]= 140737488355328

In[4]:= **FactorInteger[50 !]**

Out[4]= $\begin{pmatrix} 2 & 47 \\ 3 & 22 \\ 5 & 12 \\ 7 & 8 \\ 11 & 4 \\ 13 & 3 \\ 17 & 2 \\ 19 & 2 \\ 23 & 2 \\ 29 & 1 \\ 31 & 1 \\ 37 & 1 \\ 41 & 1 \\ 43 & 1 \\ 47 & 1 \end{pmatrix}$

In[5]:= $\mathbf{2^{67} - 1}$

Out[5]= 147573952589676412927

In[6]:= **FactorInteger[2^{67} - 1]**

Out[6]= $\begin{pmatrix} 193707721 & 1 \\ 761838257287 & 1 \end{pmatrix}$

In[7]:= $\mathbf{10^{51} - 1}$

Out[7]= 999999999999999999999999999999999999999999999999999

In[8]:= **FactorInteger[10^51 - 1]**

Out[8]=

3	3
37	1
613	1
210631	1
2071723	1
52986961	1
5363222357	1
13168164561429877	1

In[9]:= **p = Random[Integer, {1, 10^100}]**

Out[9]= 1129741995105994495352791673611836575134881309182020365904340120021977009786936979226686985088908·
752

In[10]:= **FactorInteger[p]**

Out[10]=

2
11
461
13924053380817325174432948057728216514677594522554973943802259416559566774143869296325761500307

In[11]:= **PrimeQ[p]**

Out[11]= False

■ Wie stellt man fest, dass Zahlen keine Primzahlen sind, ohne ihre Faktoren zu bestimmen?

In[12]:=
```
NextPrime[n_] := n + 1 /; PrimeQ[n + 1]
NextPrime[n_] := NextPrime[n + 1]
```

In[14]:= **NextPrime[10^100]**

Out[14]= 1000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000·
00267

■ Fermattest

In[15]:= **2^p**

General::ovfl : Overflow occurred in computation.

Out[15]= Overflow[]

In[16]:= **p**

Out[16]= 1129741995105994495352791673611836575134881309182020365904340120021977009786936979226686985088890·
8752

```
In[17]:= Mod[2^p, p]

General::ovfl : Overflow occurred in computation.

Out[17]= Overflow[]

In[18]:= PowerMod[2, p, p]

Out[18]= 9140151162587254617500423300143062904017596949725792932332964683506175340090511002269247291284964·
432
```

■ Schnelles Potenzieren

```
In[19]:= powermod[a_, 0, p_] := 1
         powermod[a_, n_, p_] := Mod[powermod[a, n/2, p]^2, p] /; EvenQ[n]
         powermod[a_, n_, p_] := Mod[a * powermod[a, n - 1, p], p] /; OddQ[n]
         $RecursionLimit = ∞;

In[23]:= powermod[2, p, p]

Out[23]= 9140151162587254617500423300143062904017596949725792932332964683506175340090511002269247291284964·
432

In[24]:= PowerMod[2, p, p]

Out[24]= 9140151162587254617500423300143062904017596949725792932332964683506175340090511002269247291284964·
432
```

■ Euklidischer Algorithmus

```
In[25]:= ggt[a_, b_] := ggt[|a|, |b|] /; a < 0 || b < 0
         ggt[a_, b_] := ggt[b, a] /; a < b
         ggt[a_, 0] := a
         ggt[a_, b_] := ggt[b, Mod[a, b]]

In[29]:= ggt[50!, 2^100]

Out[29]= 140737488355328
```

■ Rechnen mit Polynomen

In[30]:= $(x+y)^{10} - (x-y)^{10}$

Out[30]= $(x+y)^{10} - (x-y)^{10}$

In[31]:= $p = \text{Expand}[(x+y)^{10} - (x-y)^{10}]$

Out[31]= $20\, y\, x^9 + 240\, y^3\, x^7 + 504\, y^5\, x^5 + 240\, y^7\, x^3 + 20\, y^9\, x$

In[32]:= **Factor[p]**

Out[32]= $4\,x\,y\,(5\,x^4 + 10\,y^2\,x^2 + y^4)\,(x^4 + 10\,y^2\,x^2 + 5\,y^4)$

In[33]:= **Together[$\frac{p}{4\,x\,y}$]**

Out[33]= $5\,x^8 + 60\,y^2\,x^6 + 126\,y^4\,x^4 + 60\,y^6\,x^2 + 5\,y^8$

In[34]:= **Together[$\frac{p}{4\,x\,y}$] /. {x -> $\sqrt{u}$, y -> $\sqrt{v}$}**

Out[34]= $5\,u^4 + 60\,v\,u^3 + 126\,v^2\,u^2 + 60\,v^3\,u + 5\,v^4$

In[35]:= **Factor[1 + x^4]**

Out[35]= $x^4 + 1$

In[36]:= **Integrate[$\frac{1}{1+x^4}$, x]**

Out[36]= $\frac{\tan^{-1}\left(\frac{2x-\sqrt{2}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\log\left(-x^2+\sqrt{2}\,x-1\right)}{4\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}$

In[37]:= **Factor[1 + x^4, Extension → {$\sqrt{2}$}]**

Out[37]= $-\left(-x^2+\sqrt{2}\,x-1\right)\left(x^2+\sqrt{2}\,x+1\right)$

■ Kurvendiskussion

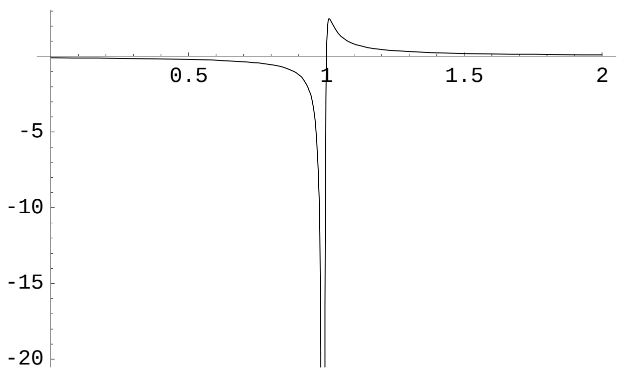
■ Die Funktion ist gegeben durch

In[38]:= **f = $\frac{1000\,(x-1)}{(101\,x-100)\,(100\,x-99)}$**

Out[38]= $\frac{1000\,(x-1)}{(100\,x-99)\,(101\,x-100)}$

■ Wir stellen f graphisch dar

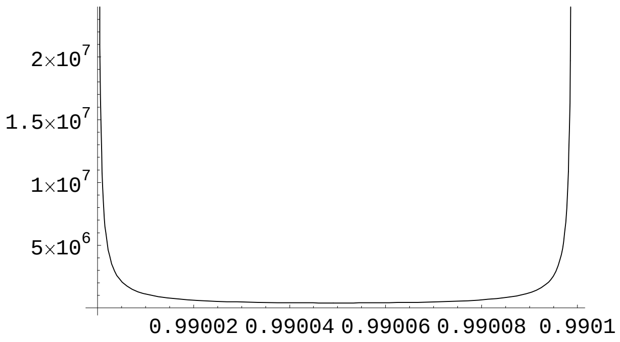
In[39]:= **Plot[f, {x, 0, 2}]**



Out[39]= - Graphics -

■ und zwischen den beiden Polen

In[40]:= Plot[f, {x, $\frac{99}{100}$, $\frac{100}{101}$}]



Out[40]= - Graphics -

■ Wir bestimmen die Nullstellen der Ableitung

In[41]:= sol = Solve[D[f, x] == 0, x]

Out[41]= $\left\{\left\{x \to \frac{1010 - \sqrt{101}}{1010}\right\}, \left\{x \to \frac{1010 + \sqrt{101}}{1010}\right\}\right\}$

In[42]:= N[sol]

Out[42]= $\{\{x \to 0.99005\}, \{x \to 1.00995\}\}$

■ und setzen sie in f ein

In[43]:= wert = f /. {x → $\frac{1010 - \sqrt{101}}{1010}$} // Simplify

Out[43]= $\frac{1000\sqrt{101}}{-2020 + 201\sqrt{101}}$

In[44]:= N[wert]

Out[44]= 401998.

■ Differentiation

In[45]:= f = Sin[2 x^2 - $\frac{1}{1 - x}$] * Cos[$\frac{1}{1 + x}$]

Out[45]= $-\cos\left(\frac{1}{x+1}\right)\sin\left(\frac{1}{1-x} - 2x^2\right)$

In[46]:= D[f, x]

Out[46]= $-\left(\frac{1}{(1-x)^2} - 4x\right)\cos\left(\frac{1}{x+1}\right)\cos\left(\frac{1}{1-x} - 2x^2\right) - \frac{\sin(\frac{1}{x+1})\sin(\frac{1}{1-x} - 2x^2)}{(x+1)^2}$

In[47]:= `D[u[x] * v[x] * w[x], x]`

Out[47]= $v(x)\, w(x)\, u'(x) + u(x)\, w(x)\, v'(x) + u(x)\, v(x)\, w'(x)$

■ Integration

In[48]:= `f =` $\dfrac{x^3 + x^2 + x - 1}{x^4 + x^2 + 1}$

Out[48]= $\dfrac{x^3 + x^2 + x - 1}{x^4 + x^2 + 1}$

In[49]:= `int =` $\int$ `f` dx

Out[49]=

$$\frac{(4+4 i)\left(3+i\sqrt{3}\right)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+\sqrt{3}}\;x\right)}{\left(-i+\sqrt{3}\right)^{5/2}\left(3 i+\sqrt{3}\right)} - \frac{16 i \tan^{-1}\left(\frac{\sqrt{-i+\sqrt{3}}\;x}{\sqrt{i+\sqrt{3}}}\right)}{\left(-i+\sqrt{3}\right)^{5/2}\sqrt{\frac{1}{3}\left(i+\sqrt{3}\right)}\left(3 i+\sqrt{3}\right)} +$$

$$\frac{4 i\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{-x^2-2}{\sqrt{3}\;x^2}\right)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)} + \frac{8 i\tan^{-1}\left(\frac{1-x^2}{\sqrt{3}\;x^2+\sqrt{3}}\right)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)} + \frac{2\left(1-i\sqrt{3}\right)\log(x^4+x^2+1)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)} + \frac{4\log(x^4+x^2+1)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)}$$

In[50]:= `diff = D[int, x]`

Out[50]=

$$\frac{2\left(1-i\sqrt{3}\right)(4x^3+2x)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)(x^4+x^2+1)} + \frac{4(4x^3+2x)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)(x^4+x^2+1)} +$$

$$\frac{8 i\left(-\frac{2x}{\sqrt{3}\;x^2+\sqrt{3}} - \frac{2\sqrt{3}\;(1-x^2)\,x}{\left(\sqrt{3}\;x^2+\sqrt{3}\right)^2}\right)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)\left(\frac{(1-x^2)^2}{\left(\sqrt{3}\;x^2+\sqrt{3}\right)^2}+1\right)} + \frac{4 i\left(3+i\sqrt{3}\right)}{\left(-i+\sqrt{3}\right)^{2}\left(3 i+\sqrt{3}\right)\left(\frac{1}{2}\,i\left(-i+\sqrt{3}\right)x^2+1\right)} -$$

$$\frac{16 i\sqrt{3}}{\left(-i+\sqrt{3}\right)^{2}\left(i+\sqrt{3}\right)\left(3 i+\sqrt{3}\right)\left(\frac{\left(-i+\sqrt{3}\right)x^2}{i+\sqrt{3}}+1\right)} + \frac{4 i\left(1-i\sqrt{3}\right)\left(-\frac{2(-x^2-2)}{\sqrt{3}\;x^3} - \frac{2}{\sqrt{3}\;x}\right)}{\left(-i+\sqrt{3}\right)^{3}\left(3 i+\sqrt{3}\right)\left(\frac{(-x^2-2)^2}{3x^4}+1\right)}$$

In[51]:= `Simplify[diff]`

Out[51]= $\dfrac{x^3 + x^2 + x - 1}{x^4 + x^2 + 1}$

In[52]:= `f = x * Cos[x] * Sin[2 x]`

Out[52]= $x\cos(x)\sin(2x)$

In[53]:= $\int$ `f` dx

Out[53]= $\frac{1}{2}(\sin(x) - x\cos(x)) + \frac{1}{2}\left(\frac{1}{9}\sin(3x) - \frac{1}{3}x\cos(3x)\right)$

In[54]:= $\int \dfrac{\texttt{Sin[x]}}{\texttt{x}}\, dx$

Out[54]= $\mathrm{Si}(x)$

■ Vereinfachung

■ Vereinfachungsfunktionen: Expand, Factor, Together, Simplify, FullSimplify, TrigExpand, TrigReduce, ...

■ Das Hofstadter-Problem

In[55]:= **hofstadter** = $\left\{\left\{\frac{\text{Sin}[r\,\alpha]}{\text{Sin}[(1-r)\,\alpha]}, \frac{\text{Sin}[2\,\alpha]}{\text{Sin}[(1-2)\,\alpha]}, \frac{\text{Sin}[(2-r)\,\alpha]}{\text{Sin}[(r-1)\,\alpha]}\right\}, \left\{\frac{\text{Sin}[r\,\beta]}{\text{Sin}[(1-r)\,\beta]}, \frac{\text{Sin}[2\,\beta]}{\text{Sin}[(1-2)\,\beta]}, \frac{\text{Sin}[(2-r)\,\beta]}{\text{Sin}[(r-1)\,\beta]}\right\}, \left\{\frac{\text{Sin}[r\,\gamma]}{\text{Sin}[(1-r)\,\gamma]}, \frac{\text{Sin}[2\,\gamma]}{\text{Sin}[(1-2)\,\gamma]}, \frac{\text{Sin}[(2-r)\,\gamma]}{\text{Sin}[(r-1)\,\gamma]}\right\}\right\}$

Out[55]= $$\begin{pmatrix} \csc((1-r)\,\alpha)\sin(r\,\alpha) & -\csc(\alpha)\sin(2\,\alpha) & \csc((r-1)\,\alpha)\sin((2-r)\,\alpha) \\ \csc((1-r)\,\beta)\sin(r\,\beta) & -\csc(\beta)\sin(2\,\beta) & \csc((r-1)\,\beta)\sin((2-r)\,\beta) \\ \csc((1-r)\,\gamma)\sin(r\,\gamma) & -\csc(\gamma)\sin(2\,\gamma) & \csc((r-1)\,\gamma)\sin((2-r)\,\gamma) \end{pmatrix}$$

In[56]:= **det = Det[hofstadter]**

General::spell1 : Possible spelling error: new symbol name "det" is similar to existing symbol "Det".

Out[56]= $$\csc((1-r)\,\alpha)\csc((r-1)\,\beta)\csc(\gamma)\sin(r\,\alpha)\sin((2-r)\,\beta)\sin(2\,\gamma) - \csc((r-1)\,\alpha)\csc((1-r)\,\beta)\csc(\gamma)\sin((2-r)\,\alpha)\sin(r\,\beta)\sin(2\,\gamma) - \csc((1-r)\,\alpha)\csc(\beta)\csc((r-1)\,\gamma)\sin(r\,\alpha)\sin(2\,\beta)\sin((2-r)\,\gamma) + \csc(\alpha)\csc((1-r)\,\beta)\csc((r-1)\,\gamma)\sin(2\,\alpha)\sin(r\,\beta)\sin((2-r)\,\gamma) + \csc((r-1)\,\alpha)\csc(\beta)\csc((1-r)\,\gamma)\sin((2-r)\,\alpha)\sin(2\,\beta)\sin(r\,\gamma) - \csc(\alpha)\csc((r-1)\,\beta)\csc((1-r)\,\gamma)\sin(2\,\alpha)\sin((2-r)\,\beta)\sin(r\,\gamma)$$

In[57]:= **Simplify[det]**

Out[57]= 0

■ Relativistische Energie

In[58]:= **Energie** = $\frac{m\,c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

Out[58]= $$\frac{c^2\,m}{\sqrt{1-\frac{v^2}{c^2}}}$$

In[59]:= **Series[Energie, {v, 0, 3}]**

Out[59]= $$c^2\,m + \frac{m\,v^2}{2} + O(v^4)$$