■ Taylorpolynome

In[1]:= `f = Cos[x]`

Out[1]= $\cos(x)$

In[2]:= $\texttt{taylor} = \sum_{k=0}^{5} \frac{(\texttt{D[f, \{x, k\}] /. x} \to 0)}{k!}\, x^k$

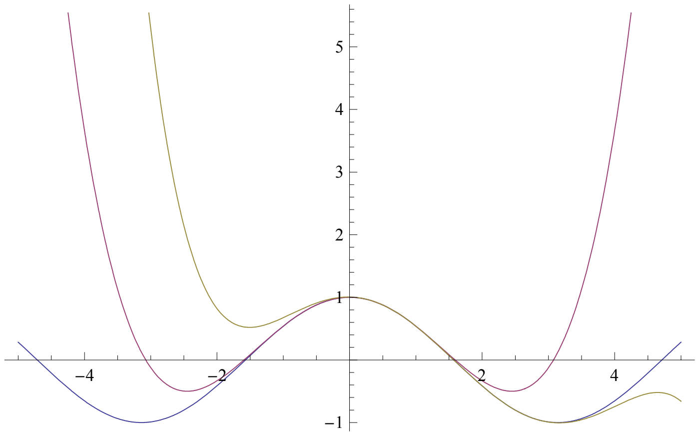
Out[2]= $\frac{x^4}{24} - \frac{x^2}{2} + 1$

In[3]:= $\texttt{taylor2} = \sum_{k=0}^{5} \frac{\left(\texttt{D[f, \{x, k\}] /. x} \to \frac{\pi}{2}\right)}{k!} \left(x - \frac{\pi}{2}\right)^k$

Out[3]= $-\frac{1}{120}\left(x - \frac{\pi}{2}\right)^5 + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3 - x + \frac{\pi}{2}$

In[4]:= `Plot[{f, taylor, taylor2}, {x, -5, 5}]`

Out[4]=



In[5]:= `Sin[85 Degree] // N`

Out[5]= 0.996195

In[6]:= $\texttt{taylor = Series}\left[\texttt{Sin[x]}, \left\{x, \frac{\pi}{2}, 5\right\}\right]$

Out[6]= $1 - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{24}\left(x - \frac{\pi}{2}\right)^4 + O\left(\left(x - \frac{\pi}{2}\right)^6\right)$

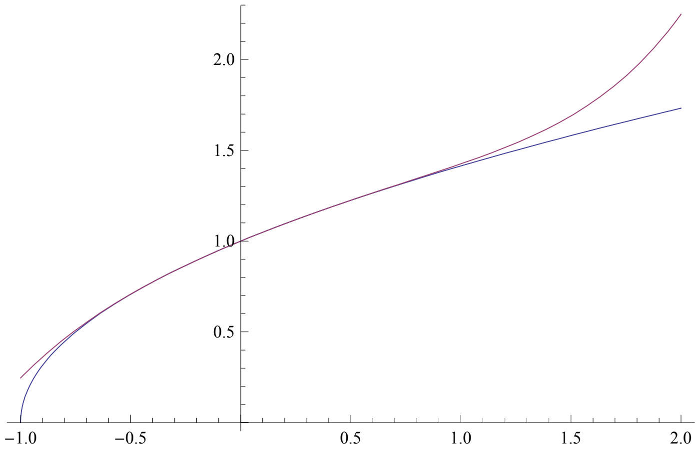
In[7]:= `Normal[taylor] /. x → 85 Degree // N`

Out[7]= 0.996195

In[8]:= `Plot[Evaluate[{Sqrt[1 + x], Sum[(D[Sqrt[1 + x], {x, k}] /. x → 0)/k! x^k, {k, 0, 5}]}], {x, -1, 2}]`

$$\text{Plot}\left[\text{Evaluate}\left[\left\{\sqrt{1+x}\,,\ \sum_{k=0}^{5}\frac{\left(D\left[\sqrt{1+x}\,,\ \{x,k\}\right]\ /.\ x\to 0\right)}{k\,!}\ x^k\right\}\right],\ \{x,-1,2\}\right]$$

Out[8]=



In[9]:=

$$\sum_{k=0}^{5}\frac{\left(D\left[\sqrt{1+x}\,,\ \{x,k\}\right]\ /.\ x\to 0\right)}{k\,!}\ x^k\ /.\ x\to 0.01$$

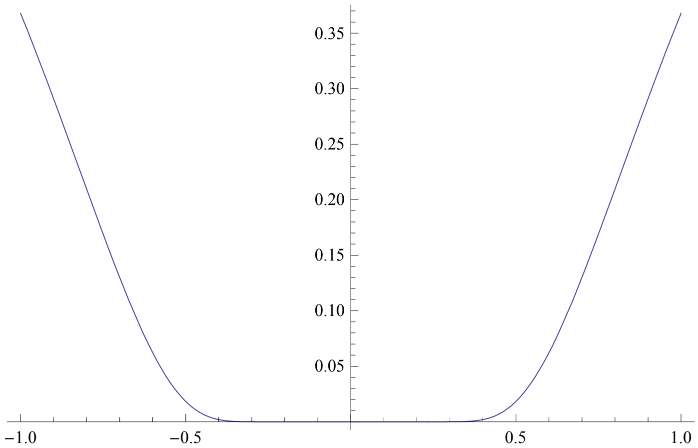
Out[9]= 1.00499

- **Gegenbeispiel: Taylorpolynom konvergiert nicht gegen die Funktion**

In[10]:=

$$\text{Plot}\left[\text{Exp}\left[-\frac{1}{x^2}\right],\ \{x,-1,1\}\right]$$

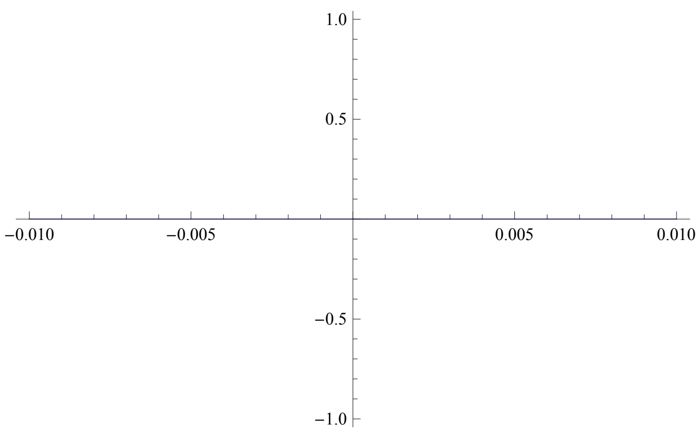
Out[10]=



In[11]:=

$$\text{Plot}\left[\text{Exp}\left[-\frac{1}{x^2}\right],\ \{x,-0.01,0.01\}\right]$$

Out[11]=



Exponentialreihe

In[12]:=
```
Series[E^x, {x, 0, 10}]
```

Out[12]= $1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\frac{x^6}{720}+\frac{x^7}{5040}+\frac{x^8}{40320}+\frac{x^9}{362880}+\frac{x^{10}}{3628800}+O(x^{11})$

Logarithmusreihe

In[13]:=
```
Series[-Log[1 - x], {x, 0, 10}]
```

Out[13]= $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}+\frac{x^6}{6}+\frac{x^7}{7}+\frac{x^8}{8}+\frac{x^9}{9}+\frac{x^{10}}{10}+O(x^{11})$

Arkustangensreihe

In[14]:=
```
Series[ArcTan[x], {x, 0, 10}]
```

Out[14]= $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}+O(x^{11})$

Weitere Reihen:

In[15]:=
```
Series[√(1 + x), {x, 0, 10}]
```

Out[15]= $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}+\frac{7x^5}{256}-\frac{21x^6}{1024}+\frac{33x^7}{2048}-\frac{429x^8}{32768}+\frac{715x^9}{65536}-\frac{2431x^{10}}{262144}+O(x^{11})$

In[16]:=
```
Series[ArcSin[x], {x, 0, 10}]
```

Out[16]= $x+\frac{x^3}{6}+\frac{3x^5}{40}+\frac{5x^7}{112}+\frac{35x^9}{1152}+O(x^{11})$

In[17]:=
```
Series[1/(1 - x), {x, 0, 10}]
```

Out[17]= $1+x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10}+O(x^{11})$

In[18]:=
```
Series[Cos[x], {x, 0, 10}]
```

Out[18]= $1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800}+O(x^{11})$

In[19]:=
```
Series[Sin[x], {x, 0, 10}]
```

Out[19]= $x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}+O(x^{11})$

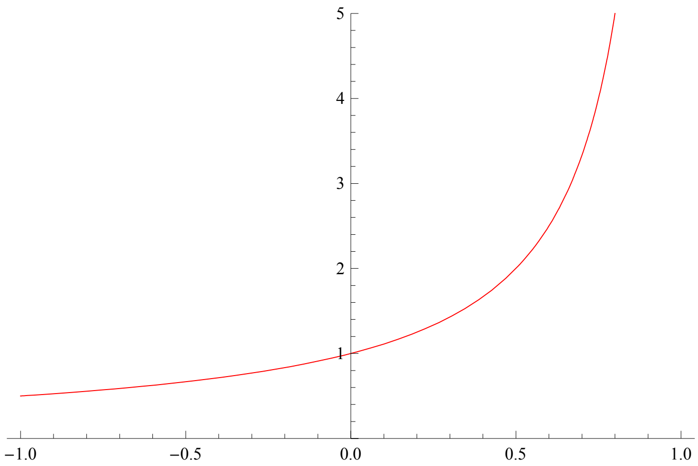
Graphische Darstellung

In[20]:=
```
f = 1/(1 - x)
```

Out[20]= $\frac{1}{1-x}$

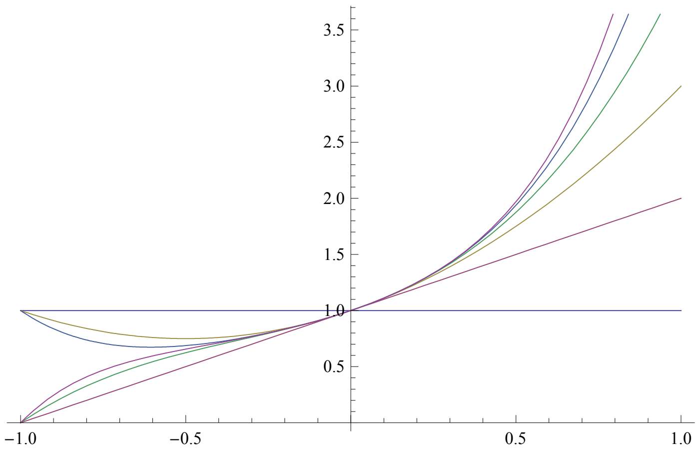
In[21]:=
```
plot1 = Plot[f, {x, -1, 1}, PlotRange → {0, 5}, PlotStyle → RGBColor[1, 0, 0]]
```

Out[21]=



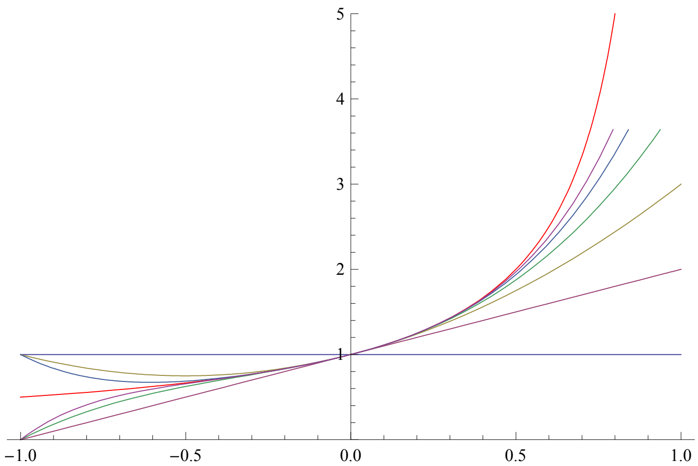
In[22]:=
```
plot2 = Plot[Evaluate[Table[Normal[Series[f, {x, 0, k}]], {k, 0, 5}]], {x, -1, 1}]
```

Out[22]=



In[23]:=
```
Show[plot1, plot2]
```

Out[23]=



- **Integration durch Reihenentwicklung**

- **1. Beispiel**

In[27]:= NIntegrate$\left[e^{x^2}, \{x, 0, 1\}\right]$

Out[27]= 1.46265

In[30]:= $\int_0^1 e^{x^2}\, dx$

Out[30]= $e\, F(1)$

In[31]:= `N[%, 20]`

Out[31]= 1.4626517459071816088

In[32]:= $res = \int_0^1 \left(\sum_{k=0}^{10} \frac{x^{2k}}{k!}\right) dx$

Out[32]= $\frac{5\,148\,275\,993\,941}{3\,519\,823\,507\,200}$

In[33]:= `N[res, 20]`

Out[33]= 1.4626517447280829388

In[34]:= $fehler = \frac{e}{(2n+3)\,(n+1)!}$ `/. {n → 10} // N`

Out[34]= 2.96081×10^{-9}

- **2. Beispiel**

In[35]:= $NIntegrate\left[\frac{Sin[x]}{x}, \{x, 0, 1\}\right]$

Out[35]= 0.946083

In[36]:= $\int_0^1 \frac{Sin[x]}{x}\, dx$

Out[36]= Si(1)

In[37]:= `N[%, 20]`

Out[37]= 0.94608307036718301494

In[38]:= $res = \int_0^1 \left(\sum_{k=0}^{10} (-1)^k \frac{x^{2k}}{(2k+1)!}\right) dx$

Out[38]= $\frac{46\,884\,688\,750\,091\,597\,861\,333\,881}{49\,556\,630\,087\,350\,833\,807\,360\,000}$

In[39]:= `N[res, 20]`

Out[39]= 0.94608307036718301494

In[40]:= $fehler = \frac{1}{(2n+3)\,(2n+3)!}$ `/. {n → 10} // N`

Out[40]= 1.68181×10^{-24}