Übungsaufgabe

$$\text{Apart}\left[\frac{1}{z^2\,(z-1)\,(z+1)}\right]$$

$$-\frac{1}{z^2}-\frac{1}{2\,(z+1)}+\frac{1}{2\,(z-1)}$$

$$\int_{\left|z-\frac{1}{2}\right|=1}^{\square}\frac{f[z]}{z^2\,(z+1)\,(z-1)}\,dz == 2\,\pi\,i\left(\frac{1}{2}\,f[1]-f'[0]\right)$$

Beispiel 3.10

$$\text{Apart}\left[\frac{1}{(z-1)\,(z+1)\,(z+2\,i)}\right]$$

$$-\frac{1}{5\,(z+2\,i)}+\frac{\frac{1}{10}+\frac{i}{5}}{z+1}+\frac{\frac{1}{10}-\frac{i}{5}}{z-1}$$

Potenzreihen

$$\text{Konvergenzradius}[a\_,\ k\_] := \text{Limit}\left[\text{FullSimplify}\left[\text{Abs}\left[\frac{a}{(a\ /.\ k\to k+1)}\right]\right],\ k\to\infty\right]$$

■ Beispiel (1)

$$\text{Series}\left[f=\frac{1}{1+z^2},\ \{z,\ 0,\ 20\}\right]$$

$$1-z^2+z^4-z^6+z^8-z^{10}+z^{12}-z^{14}+z^{16}-z^{18}+z^{20}+O\left(z^{21}\right)$$

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Konvergenzradius[1, k]
```

1

$$\text{Solve}\left[1+z^2 == 0,\ z\right]$$

$$\{\{z\to -i\},\ \{z\to i\}\}$$

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Series[f, {z, √2 , 10}]
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$$\frac{1}{3}-\frac{2}{9}\sqrt{2}\left(z-\sqrt{2}\right)+\frac{5}{27}\left(z-\sqrt{2}\right)^2-\frac{4}{81}\sqrt{2}\left(z-\sqrt{2}\right)^3+\frac{1}{243}\left(z-\sqrt{2}\right)^4+\frac{10}{729}\sqrt{2}\left(z-\sqrt{2}\right)^5-\frac{43\left(z-\sqrt{2}\right)^6}{2187}+\frac{56\sqrt{2}\left(z-\sqrt{2}\right)^7}{6561}-\frac{95\left(z-\sqrt{2}\right)^8}{19\,683}+\frac{22\sqrt{2}\left(z-\sqrt{2}\right)^9}{59\,049}+\frac{197\left(z-\sqrt{2}\right)^{10}}{177\,147}+O\left(\left(z-\sqrt{2}\right)^{11}\right)$$

■ Beispiel (2)

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Series[f = z/(e^z - 1), {z, 0, 10}]
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$$1-\frac{z}{2}+\frac{z^2}{12}-\frac{z^4}{720}+\frac{z^6}{30\,240}-\frac{z^8}{1\,209\,600}+\frac{z^{10}}{47\,900\,160}+O(z^{11})$$

$$\sum_{k=0}^{10}\frac{\text{Limit[D[f, \{z, k\}], z} \to 0]}{k\,!}\,z^k$$

$$\frac{z^{10}}{47\,900\,160}-\frac{z^8}{1\,209\,600}+\frac{z^6}{30\,240}-\frac{z^4}{720}+\frac{z^2}{12}-\frac{z}{2}+1$$

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Denominator[f]
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$e^z - 1$

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Solve[Denominator[f] == 0, z]
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— *Solve::ifun :*
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$\{\{z \to 0\}\}$

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Reduce[Denominator[f] == 0, z]
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$c_1 \in \mathbb{Z} \wedge z = 2\, i\, \pi\, c_1$

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Reduce[Cos[z] == 0, z]
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$$c_1 \in \mathbb{Z} \bigwedge \left(z = 2\pi c_1 - \frac{\pi}{2} \bigvee z = 2\pi c_1 + \frac{\pi}{2}\right)$$

■ Beispiel (3)

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Series[f = 1/(z^2 - 3 z), {z, 0, 10}]
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$$-\frac{1}{3z}-\frac{1}{9}-\frac{z}{27}-\frac{z^2}{81}-\frac{z^3}{243}-\frac{z^4}{729}-\frac{z^5}{2187}-\frac{z^6}{6561}-\frac{z^7}{19\,683}-\frac{z^8}{59\,049}-\frac{z^9}{177\,147}-\frac{z^{10}}{531\,441}+O(z^{11})$$

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Apart[f]
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$$\frac{1}{3(z-3)}-\frac{1}{3z}$$

$$-\frac{1}{3z}-\frac{1}{9}\sum_{k=0}^{10}\left(\frac{z}{3}\right)^k \text{ // Expand}$$

$$-\frac{z^{10}}{531\,441}-\frac{z^9}{177\,147}-\frac{z^8}{59\,049}-\frac{z^7}{19\,683}-\frac{z^6}{6561}-\frac{z^5}{2187}-\frac{z^4}{729}-\frac{z^3}{243}-\frac{z^2}{81}-\frac{z}{27}-\frac{1}{3z}-\frac{1}{9}$$

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Series[f, {z, -1, 10}]
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$$\frac{1}{4}+\frac{5(z+1)}{16}+\frac{21}{64}(z+1)^2+\frac{85}{256}(z+1)^3+\frac{341(z+1)^4}{1024}+\frac{1365(z+1)^5}{4096}+\frac{5461(z+1)^6}{16\,384}+\frac{21\,845(z+1)^7}{65\,536}+\frac{87\,381(z+1)^8}{262\,144}+\frac{349\,525(z+1)^9}{1\,048\,576}+\frac{1\,398\,101(z+1)^{10}}{4\,194\,304}+O((z+1)^{11})$$

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Konvergenzradius[-1/12 1/4^k + 1/3, k]
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1

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Konvergenzradius[-1/12 1/4^k, k]
```

4

```
Konvergenzradius[1/3, k]
```

1

```
Series[f, {z, -1, 5}]
```

$$\frac{1}{4}+\frac{5(z+1)}{16}+\frac{21}{64}(z+1)^2+\frac{85}{256}(z+1)^3+\frac{341(z+1)^4}{1024}+\frac{1365(z+1)^5}{4096}+O((z+1)^6)$$

$$\sum_{k=0}^{5} \left(-\frac{1}{12}\ \frac{1}{4^k} + \frac{1}{3}\right)\ (z+1)^k$$

$$\frac{1365\,(z+1)^5}{4096} + \frac{341\,(z+1)^4}{1024} + \frac{85}{256}\,(z+1)^3 + \frac{21}{64}\,(z+1)^2 + \frac{5\,(z+1)}{16} + \frac{1}{4}$$