```
> restart;
```

What is a Computer Algebra System about?

```
> 40!;
```

$$815915283247897734345611269596115894272000000000$$

```
> binomial(123,45);
```

$$8966473191018617158916954970192684$$

```
> 40!/binomial(123,45);
```

$$\frac{2595835018726623874037043324518400000000}{285268404472916876134028573}$$

```
> evalf(Pi,100);
```

3.141592653589793238462643383279502884197169399375105820974944592307816406\
286208998628034825342117068

```
> p:=(x+y)^10-(x-y)^10;
```

$$p := (x+y)^{10} - (x-y)^{10}$$

```
> expand(p);
```

$$20\,x^9\,y + 240\,x^7\,y^3 + 504\,x^5\,y^5 + 240\,x^3\,y^7 + 20\,x\,y^9$$

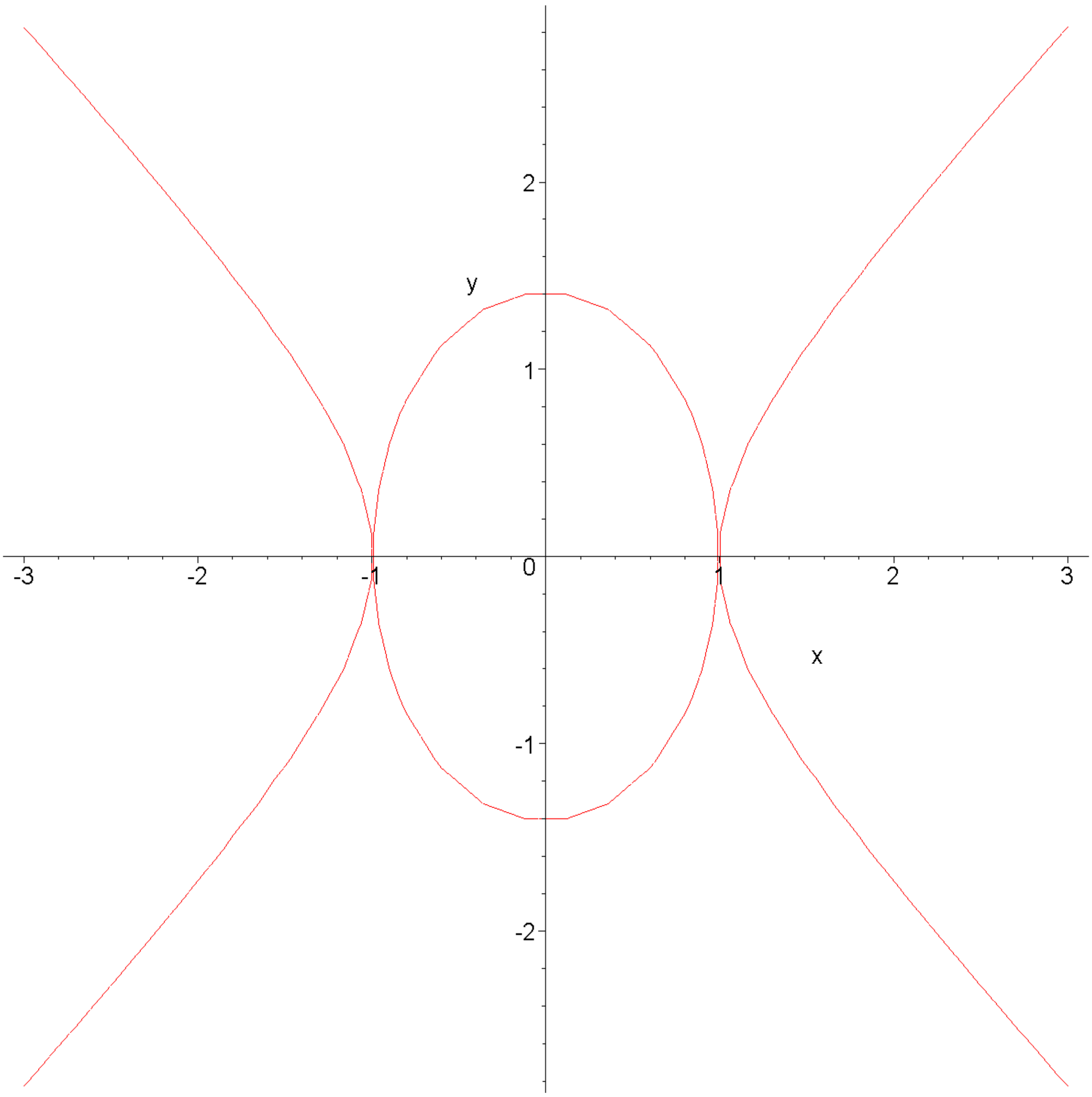
```
> factor(p);
```

$$4\,x\,y\,(5\,y^4 + 10\,x^2\,y^2 + x^4)\,(y^4 + 10\,x^2\,y^2 + 5\,x^4)$$

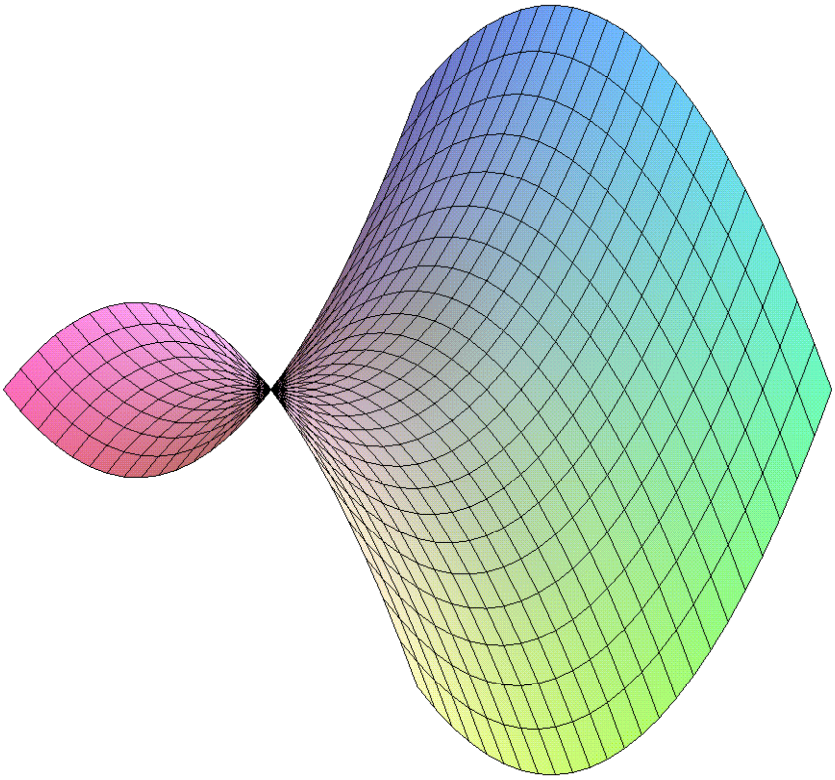
```
> solve({x^2+y^2/2=1,-x^2+y^2+1=0},{x,y});
```

$$\{y=0, x=1\}, \{y=0, x=-1\}$$

```
> plots[implicitplot]({x^2+y^2/2=1,-x^2+y^2+1=0},x=-3..3,y=-3..
3);
```




```
> plot3d(x^2-y^2,x=-1..1,y=-1..1);
```



```
>
```

Computing the Recurrence Coefficients

We define the forward and backward difference operators:

```
> Delta:=(f,x)->subs(x=x+1,f)-f;
```

$$\Delta := (f, x) \rightarrow \mathrm{subs}(x = x + 1, f) - f$$

```
> nabla:=(f,x)->f-subs(x=x-1,f);
```

$$nabla := (f, x) \rightarrow f - \mathrm{subs}(x = x - 1, f)$$

We consider the three highest coefficients of the orthogonal polynomial:

```
> p:=k[n]*x^n+kprime[n]*x^(n-1)+kprimeprime[n]*x^(n-2);
```

$$p := k_n x^n + kprime_n x^{(n-1)} + kprimeprime_n x^{(n-2)}$$

We define the polynomials σ and τ with arbitrary coefficients a,b,c,d,e:

```
> sigma:=a*x^2+b*x+c;
  tau:=d*x+e;
```

$$\sigma := a\,x^2 + b\,x + c$$

$$\tau := d\,x + e$$

The polynomial satisfies the difference equation DE=0 with:

```
> DE:=sigma*Delta(nabla(p,x),x)+tau*Delta(p,x)+lambda[n]*p;
```

$$DE := (a\,x^2 + b\,x + c)\,(k_n\,(x+1)^n + kprime_n\,(x+1)^{(n-1)} + kprimeprime_n\,(x+1)^{(n-2)} - 2\,k_n\,x^n - 2\,kprime_n\,x^{(n-1)} - 2\,kprimeprime_n\,x^{(n-2)} + k_n\,(x-1)^n + kprime_n\,(x-1)^{(n-1)} + kprimeprime_n\,(x-1)^{(n-2)}) + (d\,x+e)\,(k_n\,(x+1)^n + kprime_n\,(x+1)^{(n-1)} + kprimeprime_n\,(x+1)^{(n-2)} - k_n\,x^n - kprime_n\,x^{(n-1)} - kprimeprime_n\,x^{(n-2)}) + \lambda_n\,(k_n\,x^n + kprime_n\,x^{(n-1)} + kprimeprime_n\,x^{(n-2)})$$

We replace the powers (x+1)^n and (x-1)^n by the binomial theorem:

```
> DE:=subs({(x+1)^n=x^n+n*x^(n-1)+n*(n-1)/2*x^(n-2),(x+1)^(n-1)
  =subs(n=n-1,x^n+n*x^(n-1)+n*(n-1)/2*x^(n-2)),(x+1)^(n-2)=subs
  (n=n-2,x^n+n*x^(n-1)+n*(n-1)/2*x^(n-2)),(x-1)^n=x^n-n*x^(n-1)
  +n*(n-1)/2*x^(n-2),(x-1)^(n-1)=subs(n=n-1,x^n-n*x^(n-1)+n*(n-
  1)/2*x^(n-2)),(x-1)^(n-2)=subs(n=n-2,x^n-n*x^(n-1)+n*(n-1)/2*
  x^(n-2))},DE);
```

$$DE := (a\,x^2 + b\,x + c)\left(k_n\left(x^n + n\,x^{(n-1)} + \frac{n\,(n-1)\,x^{(n-2)}}{2}\right) + kprime_n\left(x^{(n-1)} + (n-1)\,x^{(n-2)} + \frac{(n-1)\,(n-2)\,x^{(n-3)}}{2}\right) + kprimeprime_n\left(x^{(n-2)} + (n-2)\,x^{(n-3)} + \frac{(n-2)\,(n-3)\,x^{(n-4)}}{2}\right) - 2\,k_n\,x^n - 2\,kprime_n\,x^{(n-1)} - 2\,kprimeprime_n\,x^{(n-2)} + k_n\left(x^n - n\,x^{(n-1)} + \frac{n\,(n-1)\,x^{(n-2)}}{2}\right) + kprime_n\left(x^{(n-1)} - (n-1)\,x^{(n-2)} + \frac{(n-1)\,(n-2)\,x^{(n-3)}}{2}\right) + kprimeprime_n\left(x^{(n-2)} - (n-2)\,x^{(n-3)} + \frac{(n-2)\,(n-3)\,x^{(n-4)}}{2}\right)\right) + (d\,x+e)\left(k_n\left(x^n + n\,x^{(n-1)} + \frac{n\,(n-1)\,x^{(n-2)}}{2}\right) + kprime_n\left(x^{(n-1)} + (n-1)\,x^{(n-2)} + \frac{(n-1)\,(n-2)\,x^{(n-3)}}{2}\right) + kprimeprime_n\left(x^{(n-2)} + (n-2)\,x^{(n-3)} + \frac{(n-2)\,(n-3)\,x^{(n-4)}}{2}\right) - k_n\,x^n\right.$$

$$- kprime_n\, x^{(n-1)} - kprimeprime_n\, x^{(n-2)} \Big)$$
$$+ \lambda_n \left(k_n x^n + kprime_n\, x^{(n-1)} + kprimeprime_n\, x^{(n-2)} \right)$$

and collect coefficients:

```
> de:=collect(simplify(DE/x^(n-4)),x);
```

$$de := (\lambda_n k_n + a k_n n^2 - a k_n n + d k_n n) x^4 + \Big(-d\, kprime_n - b k_n n + a\, kprime_n n^2 - \frac{1}{2} d k_n n + d\, kprime_n n - 3 a\, kprime_n n + b k_n n^2 + 2 a\, kprime_n + \lambda_n kprime_n + \frac{1}{2} d k_n n^2 + e k_n n \Big) x^3 + \Big(-5 a\, kprimeprime_n n - \frac{3}{2} d\, kprime_n n + d\, kprime_n - 2 d\, kprimeprime_n + 2 b\, kprime_n - e\, kprime_n + b\, kprime_n n^2 + a\, kprimeprime_n n^2 + c k_n n^2 - c k_n n + \frac{1}{2} e k_n n^2 - 3 b\, kprime_n n + e\, kprime_n n + 6 a\, kprimeprime_n + \lambda_n kprimeprime_n + d\, kprimeprime_n n + \frac{1}{2} d\, kprime_n n^2 - \frac{1}{2} e k_n n \Big) x^2 + \Big(-\frac{5}{2} d\, kprimeprime_n n + 2 c\, kprime_n + \frac{1}{2} e\, kprime_n n^2 + \frac{1}{2} d\, kprimeprime_n n^2 + e\, kprime_n + 3 d\, kprimeprime_n - 5 b\, kprimeprime_n n + 6 b\, kprimeprime_n + c\, kprime_n n^2 - 2 e\, kprimeprime_n + b\, kprimeprime_n n^2 - \frac{3}{2} e\, kprime_n n - 3 c\, kprime_n n + e\, kprimeprime_n n \Big) x - \frac{5}{2} e\, kprimeprime_n n + 6 c\, kprimeprime_n + 3 e\, kprimeprime_n + c\, kprimeprime_n n^2 - 5 c\, kprimeprime_n n + \frac{1}{2} e\, kprimeprime_n n^2$$

Equating the highest coefficient gives the already mentioned identity for λ:

```
> rule1:=lambda[n]=solve(coeff(de,x,4),lambda[n]);
```

$$rule1 := \lambda_n = -n\,(a n - a + d)$$

This can be substituted:

```
> de:=expand(subs(rule1,de));
```

$$de := \frac{1}{2} x\, e\, kprime_n n^2 + 6 c\, kprimeprime_n - 4 x^2 a\, kprimeprime_n n + x^2 c k_n n^2 - \frac{3}{2} x^2 d\, kprime_n n + 2 x^3 a\, kprime_n + 6 x^2 a\, kprimeprime_n + 2 x^2 b\, kprime_n + 6 x b\, kprimeprime_n + 2 x c\, kprime_n + x^3 e k_n n - \frac{3}{2} x\, e\, kprime_n n - x^3 d\, kprime_n + x^2 d\, kprime_n - 2 x^2 d\, kprimeprime_n + 3 x d\, kprimeprime_n - x^2 e\, kprime_n + x\, e\, kprime_n$$

$$-2\,x\,e\,kprimeprime_n+3\,e\,kprimeprime_n-2\,x^3\,a\,kprime_n\,n+x^3\,b\,k_n\,n^2-x^3\,b\,k_n\,n$$
$$+x^2\,b\,kprime_n\,n^2-3\,x^2\,b\,kprime_n\,n+x\,b\,kprimeprime_n\,n^2-5\,x\,b\,kprimeprime_n\,n$$
$$-x^2\,c\,k_n\,n+x\,c\,kprime_n\,n^2-3\,x\,c\,kprime_n\,n+c\,kprimeprime_n\,n^2-5\,c\,kprimeprime_n\,n$$
$$+\frac{1}{2}x^3\,d\,k_n\,n^2-\frac{1}{2}x^3\,d\,k_n\,n+\frac{1}{2}x^2\,d\,kprime_n\,n^2+\frac{1}{2}x\,d\,kprimeprime_n\,n^2$$
$$-\frac{5}{2}x\,d\,kprimeprime_n\,n+\frac{1}{2}x^2\,e\,k_n\,n^2-\frac{1}{2}x^2\,e\,k_n\,n+x^2\,e\,kprime_n\,n+x\,e\,kprimeprime_n\,n$$
$$+\frac{1}{2}e\,kprimeprime_n\,n^2-\frac{5}{2}e\,kprimeprime_n\,n$$

Equating the second highest coefficient gives k'[n] as rational multiple of k[n]:

```
> rule2:=kprime[n]=solve(coeff(de,x,3),kprime[n]);
```

$$rule2 := kprime_n=\frac{1}{2}\,\frac{k_n\,n\,(2\,b\,n-2\,b-d+2\,e+d\,n)}{d-2\,a+2\,a\,n}$$

Equating the third highest coefficient gives k''[n] as rational multiple of k[n]:

```
> rule3:=kprimeprime[n]=solve(coeff(subs(rule2,de),x,2),kprimep
  rime[n]);
```

$$rule3 := kprimeprime_n=\frac{1}{8}\,k_n\,n\,(-20\,e\,b\,n+20\,b^2\,n+4\,b^2\,n^3+20\,b\,d\,n-16\,d\,n^2\,b$$
$$-8\,d\,n\,e-16\,c\,a\,n+8\,c\,n^2\,a+4\,c\,n\,d-8\,e\,a\,n+4\,b\,n^3\,d+8\,b\,n^2\,e+4\,e\,n^2\,a-8\,b^2$$
$$-8\,b\,d+12\,b\,e+5\,d^2\,n-4\,d^2\,n^2-2\,d^2+4\,d\,e-4\,c\,d+8\,c\,a+4\,e\,a-4\,e^2-16\,b^2\,n^2$$
$$+d^2\,n^3+4\,e^2\,n+4\,d\,n^2\,e)\,/\,((d-2\,a+2\,a\,n)\,(2\,a\,n-3\,a+d))$$

We consider the monic case, hence

```
> k[n]:=1;
```

$$k_n := 1$$

and therefore

```
> rule2;
```

$$kprime_n=\frac{n\,(2\,b\,n-2\,b-d+2\,e+d\,n)}{2\,(d-2\,a+2\,a\,n)}$$

```
> rule3;
```

$$kprimeprime_n=n\,(-20\,e\,b\,n+20\,b^2\,n+4\,b^2\,n^3+20\,b\,d\,n-16\,d\,n^2\,b-8\,d\,n\,e-16\,c\,a\,n$$
$$+8\,c\,n^2\,a+4\,c\,n\,d-8\,e\,a\,n+4\,b\,n^3\,d+8\,b\,n^2\,e+4\,e\,n^2\,a-8\,b^2-8\,b\,d+12\,b\,e$$
$$+5\,d^2\,n-4\,d^2\,n^2-2\,d^2+4\,d\,e-4\,c\,d+8\,c\,a+4\,e\,a-4\,e^2-16\,b^2\,n^2+d^2\,n^3+4\,e^2\,n$$
$$+4\,d\,n^2\,e)\,/\,(8\,(d-2\,a+2\,a\,n)\,(2\,a\,n-3\,a+d))$$

We would like to find the coefficients β(n) and γ(n) in the recurrence equation RE=0:

```
> RE:=P(n+1)-(x-beta[n])*P(n)+gamma[n]*P(n-1);
```

$$RE := \mathrm{P}(n+1)-(x-\beta_n)\,\mathrm{P}(n)+\gamma_n\,\mathrm{P}(n-1)$$

```
> RE:=subs({P(n)=p,P(n+1)=subs(n=n+1,p),P(n-1)=subs(n=n-1,p)},R
```

```
E);
```

$$RE := x^{(n+1)} + kprime_{n+1}\, x^n + kprimeprime_{n+1}\, x^{(n-1)} - (x - \beta_n)\,(x^n + kprime_n\, x^{(n-1)} + kprimeprime_n\, x^{(n-2)}) + \gamma_n\,(x^{(n-1)} + kprime_{n-1}\, x^{(n-2)} + kprimeprime_{n-1}\, x^{(n-3)})$$

We substitute the known formulas:

```
> RE:=subs({rule2,subs(n=n+1,rule2),subs(n=n-1,rule2),rule3,subs(n=n+1,rule3),subs(n=n-1,rule3)},RE);
```

$$RE := x^{(n+1)} + \frac{(n+1)\,(2\,b\,(n+1) - 2\,b - d + 2\,e + d\,(n+1))\,x^n}{2\,(d - 2\,a + 2\,a\,(n+1))} + (n+1)\,(20\,b^2\,(n+1) - 8\,b^2 - 8\,b\,d + 12\,b\,e - 2\,d^2 + 4\,d\,e - 4\,c\,d + 8\,c\,a + 4\,e\,a - 4\,e^2 - 20\,e\,b\,(n+1) + 20\,b\,d\,(n+1) - 16\,d\,(n+1)^2\,b - 8\,d\,(n+1)\,e - 16\,c\,a\,(n+1) + 8\,c\,(n+1)^2\,a + 4\,c\,(n+1)\,d - 8\,e\,a\,(n+1) + 4\,b\,(n+1)^3\,d + 8\,b\,(n+1)^2\,e + 4\,e\,(n+1)^2\,a + 4\,d\,(n+1)^2\,e + 4\,b^2\,(n+1)^3 + 5\,d^2\,(n+1) - 4\,d^2\,(n+1)^2 - 16\,b^2\,(n+1)^2 + d^2\,(n+1)^3 + 4\,e^2\,(n+1))\,x^{(n-1)} / (8\,(d - 2\,a + 2\,a\,(n+1))\,(2\,a\,(n+1) - 3\,a + d)) - (x - \beta_n)\left(x^n + \frac{n\,(2\,b\,n - 2\,b - d + 2\,e + d\,n)\,x^{(n-1)}}{2\,(d - 2\,a + 2\,a\,n)} + n\,(-20\,e\,b\,n + 20\,b^2\,n + 4\,b^2\,n^3 + 20\,b\,d\,n - 16\,d\,n^2\,b - 8\,d\,n\,e - 16\,c\,a\,n + 8\,c\,n^2\,a + 4\,c\,n\,d - 8\,e\,a\,n + 4\,b\,n^3\,d + 8\,b\,n^2\,e + 4\,e\,n^2\,a - 8\,b^2 - 8\,b\,d + 12\,b\,e + 5\,d^2\,n - 4\,d^2\,n^2 - 2\,d^2 + 4\,d\,e - 4\,c\,d + 8\,c\,a + 4\,e\,a - 4\,e^2 - 16\,b^2\,n^2 + d^2\,n^3 + 4\,e^2\,n + 4\,d\,n^2\,e)\,x^{(n-2)} / (8\,(d - 2\,a + 2\,a\,n)\,(2\,a\,n - 3\,a + d))\right) + \gamma_n\left(x^{(n-1)} + \frac{(n-1)\,(2\,b\,(n-1) - 2\,b - d + 2\,e + d\,(n-1))\,x^{(n-2)}}{2\,(d - 2\,a + 2\,a\,(n-1))} + (n-1)\,(-20\,e\,b\,(n-1) + 20\,b\,d\,(n-1) - 16\,d\,(n-1)^2\,b + 20\,b^2\,(n-1) - 8\,b^2 - 8\,b\,d + 12\,b\,e - 2\,d^2 + 4\,d\,e - 4\,c\,d + 8\,c\,a + 4\,e\,a - 4\,e^2 - 8\,d\,(n-1)\,e - 16\,c\,a\,(n-1) + 8\,c\,(n-1)^2\,a + 4\,c\,(n-1)\,d - 8\,e\,a\,(n-1) + 4\,b\,(n-1)^3\,d + 8\,b\,(n-1)^2\,e + 4\,e\,(n-1)^2\,a + 4\,d\,(n-1)^2\,e + 4\,b^2\,(n-1)^3 + 5\,d^2\,(n-1) - 4\,d^2\,(n-1)^2 - 16\,b^2\,(n-1)^2 + d^2\,(n-1)^3 + 4\,e^2\,(n-1))\,x^{(n-3)} / (8\,(d - 2\,a + 2\,a\,(n-1))\,(2\,a\,(n-1) - 3\,a + d))\right)$$

```
> re:=simplify(numer(normal(RE))/x^(n-3));
```

$$re := -40\,x^2\,n\,d^3\,a^3 + 472\,x^2\,\beta_n\,n\,d^3\,e\,a^2 - 196\,x\,\beta_n\,n\,c\,d^3\,a^2 - 20\,x\,\beta_n\,n^2\,e\,b\,d^4 - 16\,x\,\beta_n\,n^3\,b^2\,d^4 - 4\,x\,\beta_n\,n\,e^2\,d^4 + 3496\,\gamma_n\,a^2\,n^3\,b\,e\,d^2 - 1800\,x^2\,\gamma_n\,d^3\,a^3 - 240\,x^2\,\beta_n\,n\,d^2\,a^4 - 160\,x\,\beta_n\,n\,c\,a^4\,d - 2816\,x\,a^5\,n^4\,\beta_n\,c + 740\,x^2\,n^5\,d^2\,a^4 + 1162\,x^2\,n^4\,d^3\,a^3 - 992\,x\,a^3\,n^6\,\beta_n\,b^2\,d - 120\,x\,\beta_n\,n\,b\,e\,d^3\,a + 12\,\gamma_n\,d^6 + 160\,x^2\,a^2\,n^4\,\beta_n\,d^4$$

$$- 280\,\gamma_n\,a\,n^4\,b^2\,d^3 - 384\,x\,a^4\,n^5\,\beta_n\,e^2 + 900\,x\,a^4\,n^4\,\beta_n\,d^2 + 640\,x^2\,a^4\,n^5\,\beta_n\,e\,d$$
$$+ 11584\,x\,\gamma_n\,a^5\,n^3\,b + 9584\,x\,\gamma_n\,a^3\,n^3\,b\,d^2 + 1536\,x^2\,\gamma_n\,a^5\,n^5\,d + 640\,x\,\gamma_n\,a^3\,n^4\,e\,d^2$$
$$- 1088\,\gamma_n\,a^5\,n^2\,e + 528\,\gamma_n\,b\,d^3\,a^2 + 76\,\gamma_n\,e\,b\,n\,d^4 + 240\,\gamma_n\,b\,e\,d\,a^3 + 48\,\gamma_n\,b\,d^5$$
$$+ 23\,\gamma_n\,d^6\,n^2 + 256\,x\,\gamma_n\,a^5\,n^7\,b + 32\,\gamma_n\,a\,n^3\,e^2\,d^3 - 1800\,x^3\,\beta_n\,d^3\,a^3 - 1472\,x\,a^3\,n^4\,\beta_n\,c\,d^2$$
$$+ 4496\,x^2\,a^4\,n^4\,\beta_n\,d^2 + 6984\,x\,\gamma_n\,a^4\,n^2\,d^2 - 976\,x\,\gamma_n\,b\,a^3\,d^2 + 128\,x\,a^3\,n^5\,\beta_n\,e^2\,d$$
$$+ 80\,x\,\beta_n\,n\,b\,d^4\,a - 232\,x\,\beta_n\,n\,b\,d^3\,a^2 + 48\,\gamma_n\,b^2\,d^4 + 384\,x^3\,d^2\,n^5\,a^4 + 128\,x^3\,d\,n^6\,a^5$$
$$- 8\,x^2\,\beta_n\,n\,b\,d^5 - 796\,x\,\beta_n\,n^3\,e\,a^3\,d^2 - 480\,x^3\,a^5\,n\,d + 3424\,x^3\,e\,a^5\,n - 3776\,x^3\,e\,a^5\,n^2$$
$$- 1344\,x^3\,e\,n^3\,a^4\,d + 128\,x^3\,e\,n^4\,a^4\,d - 120\,x^3\,\beta_n\,d^5\,a - 10800\,x^2\,\gamma_n\,a^4\,n\,d^2$$
$$- 800\,\gamma_n\,a^2\,n^3\,c\,d^3 + 1556\,x\,a^2\,n^4\,\beta_n\,b^2\,d^2 + 6160\,\gamma_n\,a^4\,n^4\,c\,d - 3072\,x^2\,n^3\,c\,a^5$$
$$- 4\,x\,\beta_n\,n^3\,d^6 + 5\,x\,\beta_n\,n^2\,d^6 + 4\,x^2\,\beta_n\,n^2\,d^6 - 4736\,x^2\,n^3\,b^2\,a^3\,d + 320\,x^2\,n^6\,b^2\,a^3\,d$$
$$+ 32\,x\,a\,n^3\,\beta_n\,e^2\,d^3 + 2576\,x^2\,n^2\,c\,d^2\,a^3 + 256\,x^2\,n^3\,c\,d^3\,a^2 + 16\,x^2\,n^3\,b^2\,d^4 - 2\,x\,\beta_n\,n\,d^6$$
$$+ x\,\beta_n\,n^4\,d^6 + 8768\,x^2\,\gamma_n\,a^6\,n^2 - 9088\,x^2\,\beta_n\,n^3\,b\,a^4\,d + 232\,x^2\,n\,b^2\,d^2\,a^2 + 24\,x^2\,n^2\,e\,b\,d^4$$
$$- 632\,x^2\,n\,b\,e\,d^2\,a^2 - 2188\,\gamma_n\,a^3\,n^3\,d^2\,e - 11640\,\gamma_n\,a^3\,n^4\,b\,d^2 - 9600\,x^3\,a^5\,n^4\,d\,\beta_n$$
$$+ 512\,x^3\,a^6\,n^6\,\beta_n + 40\,x\,\beta_n\,n\,d^3\,a^3 - 2496\,x^2\,a^3\,n^3\,\beta_n\,e\,d^2 + 36\,x^2\,n\,d^4\,e\,a$$
$$+ 768\,x^3\,b\,n^5\,a^4\,d + 4384\,x^3\,e\,a^4\,n^2\,d - 14400\,x^3\,a^6\,n^3\,\beta_n - 7200\,x^2\,a^5\,n^4\,\beta_n\,b$$
$$- 4640\,\gamma_n\,a^3\,n^2\,c\,d^2 + 488\,x\,\gamma_n\,e\,a^3\,d^2 + 588\,x\,a^2\,n^3\,\beta_n\,d^3\,e - 1380\,\gamma_n\,d^3\,n^2\,e\,a^2$$
$$- 19392\,x\,\gamma_n\,a^4\,n^3\,b\,d + 640\,x^2\,a^3\,n^5\,\beta_n\,b\,d^2 + 20\,x\,\beta_n\,n^2\,b^2\,d^4 + 680\,x^3\,\beta_n\,d^4\,a^2$$
$$+ 2192\,x^3\,\beta_n\,a^4\,d^2 - 4032\,x\,\gamma_n\,a^3\,n^4\,b\,d^2 + 13840\,\gamma_n\,a^3\,n^3\,b\,d^2 + 428\,x^2\,\beta_n\,n\,d^3\,a^3$$
$$- 360\,x\,\beta_n\,n^2\,d^3\,e\,a^2 + 480\,x^2\,n\,e^2\,a^4 - 912\,x^2\,n^2\,e^2\,a^4 - 2392\,x\,\gamma_n\,b\,d^3\,a^2\,n - 80\,\gamma_n\,e\,a^3\,d^2$$
$$+ 192\,x^2\,n^4\,e\,a^2\,d^3 + 4496\,x^2\,n^3\,e\,b\,a^3\,d - 2960\,\gamma_n\,a^2\,n\,b^2\,d^2 + 64\,\gamma_n\,a^4\,n^7\,d\,e$$
$$+ 320\,x\,\gamma_n\,a^2\,n^4\,b\,d^3 - 2368\,x\,\gamma_n\,a^4\,n^5\,d^2 - 3328\,x^2\,a^4\,n^4\,\beta_n\,e\,d - 1328\,x\,a^3\,n^2\,\beta_n\,b\,d^2$$
$$+ 8\,x^2\,n\,b^2\,d^4 - 92\,x^2\,a\,n^2\,\beta_n\,d^5 + 4\,x\,\beta_n\,n^4\,b^2\,d^4 - 480\,x^2\,n^5\,e\,a^5 + 88\,x\,\gamma_n\,d^4\,e\,a$$
$$- 96\,x^2\,n\,e^2\,d^3\,a + 80\,x^2\,n^6\,d^3\,a^3 + 8\,x^3\,d^5\,e + 8\,x\,\gamma_n\,d^6 + 8\,x^2\,\gamma_n\,d^6 - 960\,x^3\,e\,a^5 + 8\,x^3\,d^6\,n$$
$$+ 8\,x^3\,\beta_n\,d^6 + 576\,x^3\,d^4\,a^2\,n - 792\,x^3\,d^4\,a^2\,n^2 - 112\,x^3\,d^5\,a\,n + 2\,x^2\,n\,d^6 + 4\,x^2\,n^3\,d^6$$
$$- 6\,x^2\,n^2\,d^6 - 184\,x^2\,a\,n^2\,\beta_n\,b\,d^4 - 1984\,x^2\,a^4\,n^5\,\beta_n\,d^2 + 8\,x\,a\,n^5\,\beta_n\,d^5 - 672\,\gamma_n\,e^2\,n^4\,a^3\,d$$
$$+ 2432\,x\,\gamma_n\,a^5\,n^2\,e + 232\,x^2\,n\,b\,d^3\,a^2 - 12640\,x\,\gamma_n\,a^5\,n^4\,b + 580\,x\,\beta_n\,n^2\,c\,a^2\,d^3$$
$$- 228\,\gamma_n\,a^2\,n^5\,d^4 + 192\,x\,a^2\,n^5\,\beta_n\,b\,e\,d^2 - 576\,x^2\,a^2\,n^3\,\beta_n\,d^4 - 20\,x^2\,n\,d^5\,a$$
$$+ 1592\,\gamma_n\,a^4\,n^2\,e\,d + 3080\,x\,\gamma_n\,a^2\,n^2\,b\,d^3 + 680\,x^2\,\gamma_n\,d^4\,a^2 - 1920\,x^2\,a^5\,n^6\,\beta_n\,b$$
$$+ 1312\,x^2\,n^4\,e\,a^5 - 3392\,x^2\,n^2\,c\,a^4\,d + 856\,x^2\,n\,b\,e\,a^3\,d - 472\,x^2\,\beta_n\,n\,b\,a^2\,d^3$$
$$+ 32\,\gamma_n\,a^3\,n\,e\,d^2 - 8\,x\,\gamma_n\,d^5\,e - 856\,x^2\,\beta_n\,n\,e\,a^3\,d^2 + 36\,x\,\beta_n\,n\,e\,a^3\,d^2 - 88\,x^2\,n^3\,d^5\,a$$
$$+ 480\,x^3\,a^2\,n^2\,\beta_n\,d^4 - 9600\,x^3\,a^4\,n^3\,\beta_n\,d^2 + 10880\,x^3\,a^6\,n^4\,\beta_n + 400\,x^2\,n\,e^2\,d^2\,a^2$$

$$
\begin{aligned}
&- 16 x^2 n b e d^4 + 160 x^2 n^3 e b a d^3 + 1556 x \beta_n n^4 b d^3 a^2 + 1568 x \beta_n n^3 c a^5 \\
&+ 2664 x^2 a^3 n^3 \beta_n d^3 - 1844 x^2 a^3 n^2 \beta_n d^3 - 1152 x^2 a^2 n^3 \beta_n b d^3 - 104 x^2 \beta_n n d^4 e a \\
&- 432 x a^4 n^3 \beta_n d^2 - 1916 \gamma_n a^4 n^5 d^2 + 668 \gamma_n e^2 n^2 d^2 a^2 - 96 \gamma_n e^2 a^4 n + 40 x \gamma_n n^3 d^5 a \\
&+ \gamma_n d^6 n^4 - 88 x \gamma_n d^5 a + 16 x \gamma_n b d^5 - 4544 x^2 a^4 n^3 \beta_n d^2 - 264 x a n^2 \beta_n b^2 d^3 \\
&- 120 x^3 d^4 e a - 4 x^2 \beta_n n d^6 + 160 x a^2 n^4 \beta_n c d^3 - 2672 x \gamma_n a^5 n^2 d + 3456 \gamma_n a^4 n^2 b d \\
&+ 4 \gamma_n d^5 n^3 e + 640 x^2 n c d a^4 + 176 x^2 n b e d^3 a + 796 \gamma_n a^4 n^6 d^2 + 924 \gamma_n a^3 n^2 e d^2 \\
&+ 656 x \gamma_n b a^2 d^3 - 112 \gamma_n b^2 n d^4 - 3968 x^2 \beta_n n^5 b a^4 d - 2336 x^3 d^2 a^4 n^4 \\
&- 2064 x^3 d^3 a^3 n^3 + 192 \gamma_n a^2 n^5 b e d^2 - 1920 x^2 \gamma_n a^6 n - 288 \gamma_n b d^2 a^3 - 288 \gamma_n b^2 d a^3 \\
&+ 8 \gamma_n b n^3 e d^4 + 32 x a n^5 \beta_n b^2 d^3 + 80 x^2 n e a^4 d + 1408 x a^4 n^2 \beta_n c d \\
&- 3424 x^2 a^4 n^2 \beta_n e d - 4 x^2 n d^5 e - 1024 \gamma_n a^5 n^6 c + 128 \gamma_n a^5 n^7 c - 1856 x \beta_n n^5 c a^4 d \\
&+ 464 x a^4 n^2 \beta_n e d - 1328 x a^3 n^2 \beta_n b^2 d + 16 x^2 n^3 d^5 b + 5792 \gamma_n a^4 n^3 b e \\
&- 6320 \gamma_n a^4 n^4 b e + 21760 x^2 \gamma_n a^5 n^3 d - 9600 x^2 \gamma_n a^5 n^4 d - 8936 \gamma_n a^3 n^2 b d^2 \\
&+ 132 \gamma_n d^4 a^2 - 3840 x^2 \gamma_n a^6 n^5 - 896 x a^5 n^6 \beta_n c - 1856 x a^2 n^3 \beta_n b d^3 + 8 x^2 n b d^5 \\
&- 3600 x \beta_n n^3 c a^4 d + 4 x \gamma_n n^2 d^6 - 232 x \beta_n n b^2 d^2 a^2 - 336 x a^2 n^3 \beta_n e^2 d^2 \\
&- 200 x^2 n^2 d^2 a^4 - 832 x^2 \beta_n n^2 e a^2 d^3 + 2832 x^2 \beta_n n^2 e a^3 d^2 - 16 \gamma_n d^5 e \\
&- 240 x \beta_n n b e d a^3 + 256 x^2 a^5 n^7 \beta_n b - 1064 x^2 n^4 a^4 d^2 - 992 x^2 n^6 b a^4 d \\
&+ 128 x^2 n^7 b a^4 d - 2240 x^2 n^4 e b a^3 d + 480 x \gamma_n b a^4 d - 576 \gamma_n b d a^4 n \\
&+ 1536 x^3 a^5 n^5 d \beta_n - 3840 x^3 a^6 n^5 \beta_n + 2960 x^2 n^3 b^2 a^4 + 128 x^2 n^7 b^2 a^4 \\
&+ 486 x^2 n^2 d^3 a^3 - 1184 x^2 n^3 d^3 a^3 + 480 \gamma_n b e a^4 n - 20 \gamma_n d^5 n^2 e - 164 x \gamma_n n^2 d^5 a \\
&+ 16320 x^2 \gamma_n n^2 d^2 a^4 + 480 x^2 \gamma_n n^2 d^4 a^2 - 44 \gamma_n b n^2 e d^4 + 448 x^2 n^4 c d^2 a^3 \\
&- 1968 x^2 n^3 c d^2 a^3 + 384 x^2 n^4 e b a^2 d^2 + 4 \gamma_n b n^4 d^5 + 640 x \gamma_n n^5 e a^4 d \\
&- 11640 \gamma_n b^2 n^4 a^3 d + 5424 \gamma_n b^2 n^5 a^3 d - 28 \gamma_n d^6 n - 1368 x^2 n^4 d^3 b a^2 \\
&+ 320 x^2 n^6 b a^3 d^2 + 384 x^2 n^5 e b a^3 d - 1088 \gamma_n a^4 n^6 b e - 704 \gamma_n a^4 n^7 b d \\
&+ 104 x^2 \beta_n n b d^4 a + 20 x \beta_n n d^5 a - 320 x \beta_n n^2 c a^5 - 480 x \beta_n n^2 b e a^4 \\
&+ 384 x^2 n^5 c a^4 d - 48 \gamma_n e^2 d^3 a + 320 x^2 a^4 n^6 \beta_n d^2 + 280 \gamma_n c a^2 d^3 + 1952 x^2 \beta_n n^2 d^2 a^4 \\
&+ 128 \gamma_n a^4 n^7 b e - 696 x^2 n e^2 a^3 d - 7664 \gamma_n a^4 n^5 b d + 128 \gamma_n a^3 n^6 d^2 e - 80 x^2 n b d^4 a \\
&- 21600 x^3 a^5 n^2 d \beta_n + 21760 x^3 a^5 n^3 d \beta_n + 8768 x^3 a^6 n^2 \beta_n + 2276 \gamma_n e b n d^2 a^2 \\
&+ 3136 x^3 e a^3 d^2 n + 5968 x^3 b n^2 a^3 d^2 - 5488 x^3 e a^4 n d - 4672 x^3 b n^4 a^4 d \\
&+ 2672 x^3 b n a^4 d + 16320 x^3 a^4 n^2 \beta_n d^2 + 1280 x^3 a^3 n^3 \beta_n d^3 - 4800 x^3 a^3 n^2 \beta_n d^3 \\
&+ 96 x^3 a n \beta_n d^5 + 1920 x^3 a^4 n^4 \beta_n d^2 - 1584 x^3 b n^2 a^2 d^3 - 752 x^3 d^3 e a^2 n \\
&+ 1152 x^3 b n a^2 d^3 - 4128 x^3 b n^3 a^3 d^2 + 512 x^3 d^3 n^3 b a^2 + 896 x^3 b n^4 a^3 d^2
\end{aligned}
$$

$$+ 256 x^3 e n^3 a^3 d^2 - 1632 x^3 e n^2 a^3 d^2 + 192 x^3 e n^2 a^2 d^3 + 144 x^3 d^4 n^2 b a$$
$$+ 10048 x^3 b n^3 a^4 d - 8912 x^3 b n^2 a^4 d + 64 x^3 d^4 e a n + 5440 x^3 a^3 n \beta_n d^3$$
$$- 10800 x^3 a^4 n \beta_n d^2 - 1200 x^3 a^2 n \beta_n d^4 + 8768 x^3 d \beta_n a^5 n - 2656 x^3 b n a^3 d^2$$
$$- 224 x^3 b d^4 n a - 960 x^3 \beta_n a^5 d - 1920 x^3 \beta_n a^6 n + 2192 x^3 a^5 n^2 d + 256 x^3 d^4 n^3 a^2$$
$$+ 72 x^3 d^5 n^2 a + 2720 x^3 a^5 n^4 d - 3600 x^3 a^5 n^3 d + 5024 x^3 a^4 n^3 d^2 + 1336 x^3 d^2 a^4 n$$
$$- 1328 x^3 d^3 a^3 n - 4456 x^3 d^2 a^4 n^2 + 2984 x^3 d^3 a^3 n^2 - 960 x^3 a^5 n^5 d + 5440 x^3 b n^4 a^5$$
$$- 7200 x^3 b n^3 a^5 - 1920 x^3 b n^5 a^5 + 4384 x^3 b n^2 a^5 + 256 x^3 b n^6 a^5 + 448 x^3 d^3 n^4 a^3$$
$$+ 1664 x^3 e a^5 n^3 - 256 x^3 e n^4 a^5 - 960 x^3 b n a^5 + 16 x^3 b d^5 n - 1800 x^3 e a^3 d^2$$
$$+ 680 x^3 e a^2 d^3 + 2192 x^3 e a^4 d + 48 x \beta_n n c d^4 a + 4 x \beta_n n^2 c d^5 + 2720 x a^4 n^5 \beta_n e b$$
$$+ 2960 x^2 n^3 b a^4 d + 136 \gamma_n e^2 n d^3 a - 88 x a n^2 \beta_n c d^4 - 80 x^2 n b^2 d^3 a$$
$$+ 5440 x^2 a^5 n^5 \beta_n b - 576 \gamma_n b^2 a^4 n + 32 \gamma_n a n^5 b d^4 - 5344 x \gamma_n a^5 n^2 b + 740 x^2 n^3 d^2 a^4$$
$$+ 192 x^2 n^5 e a^3 d^2 + 24 x a^2 n^6 \beta_n d^4 + 3296 \gamma_n a^5 n^5 c - 1664 x^2 a^5 n^5 \beta_n e$$
$$- 776 \gamma_n a n b e d^3 - 1920 \gamma_n a^3 n^4 c d^2 + 4360 \gamma_n a^3 n^3 c d^2 + 2960 x^2 n^5 b^2 a^4$$
$$- 960 x^2 \beta_n n^2 b a^5 + 2416 \gamma_n a^5 n^3 e - 342 x^2 n^4 d^4 a^2 + 3680 \gamma_n a^4 n^5 b e - 72 \gamma_n d^5 a$$
$$- 72 \gamma_n d^3 a^3 + 320 x^2 n^2 b^2 d^3 a + 944 x a^3 n^3 \beta_n e^2 d + 756 x a^3 n^5 \beta_n d^3 + 128 x^2 a^5 n^7 \beta_n d$$
$$- 3424 x^2 a^5 n^3 \beta_n e + 152 x a^3 n^2 \beta_n e d^2 - 4180 \gamma_n a^2 n^2 b e d^2 - 36 x^2 n e a^3 d^2$$
$$- 76 x^2 n e a^2 d^3 - 1600 x a^3 n^5 \beta_n e b d + 3680 x \gamma_n d n^5 a^5 - 960 x^2 n^5 c a^5$$
$$+ 1420 \gamma_n a^2 n^2 c d^3 - 4592 x a^3 n^4 \beta_n b d^2 - 3744 x a^4 n^5 \beta_n b^2 + 851 \gamma_n a^2 n^4 d^4$$
$$+ 8768 x^2 \gamma_n a^5 n d + 1920 x^2 n^2 b e d^2 a^2 - 104 x^2 n c d^4 a + 192 \gamma_n e a^5 n$$
$$- 2234 \gamma_n a^3 n^2 d^3 + 160 x \beta_n n b d^2 a^3 + 256 \gamma_n a^3 n^6 b e d + 296 x \beta_n n^2 e b d^3 a$$
$$+ 64 x a^5 n^7 \beta_n e + 960 x^2 \beta_n n^2 e a^5 + 3712 x a^3 n^4 \beta_n e b d + 288 x^2 n^5 d^3 b a^2$$
$$- 1588 \gamma_n d^4 n^3 a^2 - 4576 x \gamma_n n^3 e a^5 + 96 x^2 n^3 e^2 a^2 d^2 - 960 x^2 \gamma_n a^5 d + 1056 \gamma_n a n b^2 d^3$$
$$- 160 x^2 n b^2 d a^3 + 64 x^2 n^6 e a^5 + 12 x^2 n^2 d^5 e - 248 x^2 n^6 d^2 a^4 - 420 \gamma_n a^2 n e^2 d^2$$
$$- 236 x^2 \beta_n n d^4 a^2 - 120 x^2 \gamma_n d^5 a + 512 x^2 \gamma_n a^6 n^6 + 10880 x^2 \gamma_n a^6 n^4 - 12 x \gamma_n n d^6$$
$$+ 8 \gamma_n c d^5 - 14400 x^2 \gamma_n a^6 n^3 + 328 x \gamma_n d^4 a^2 + 80 x \gamma_n a n^3 b d^4 - 488 x \gamma_n d^3 a^3$$
$$+ 8 \gamma_n e^2 d^4 + 240 x \gamma_n d^2 a^4 - 8 \gamma_n d^6 n^3 + 2192 x^2 \gamma_n a^4 d^2 - 160 x^2 n b d^2 a^3$$
$$- 480 x^2 \beta_n n b a^4 d + 389 x a^2 n^4 \beta_n d^4 + 80 x^2 a n^3 \beta_n b d^4 + 32 x a n^5 \beta_n b d^4$$
$$+ 80 x \beta_n n^2 d^2 a^4 + 160 x \beta_n n^2 e^2 a^4 - 248 x a^3 n^6 \beta_n d^3 - 1148 x a^3 n^4 \beta_n d^3$$
$$+ 32 x a^3 n^7 \beta_n d^3 + 320 x \beta_n n^2 b d a^4 - 160 x \beta_n n^2 e a^5 + 2336 x a^5 n^5 \beta_n c$$
$$+ 480 x^2 \beta_n n e a^4 d + 160 x \beta_n n b^2 d a^3 - 80 x \beta_n n e a^4 d - 480 x^2 \beta_n n^2 d a^5$$
$$+ 652 x^2 a^2 n^2 \beta_n d^4 + 256 x a^3 n^6 \beta_n b e d + 76 x \beta_n n e a^2 d^3 + 312 x \beta_n n c a^3 d^2$$
$$+ 12 x \beta_n n b e d^4 - 1568 x^2 a^3 n^4 \beta_n d^3 + 356 x a^2 n^2 \beta_n e^2 d^2 + 640 x^2 a^3 n^4 \beta_n e d^2$$

$$- 624\,x\,a^2\,n^5\,\beta_n\,b\,d^3 + 1060\,x\,a^2\,n^2\,\beta_n\,b\,d^3 + 320\,x^2\,a^3\,n^5\,\beta_n\,d^3 + 1120\,x\,a^3\,n^4\,\beta_n\,e\,d^2$$
$$- 640\,x\,a^3\,n^5\,\beta_n\,e\,d^2 + 40\,x^2\,a\,n^3\,\beta_n\,d^5 - 400\,x\,a^2\,n^4\,\beta_n\,e\,d^3 + 265\,x\,a^2\,n^2\,\beta_n\,d^4$$
$$+ 1720\,x\,a^2\,n^3\,\beta_n\,e\,b\,d^2 - 464\,x\,a^2\,n^3\,\beta_n\,d^4 + 80\,x\,a\,n^3\,\beta_n\,d^5 - 42\,x\,a\,n^4\,\beta_n\,d^5$$
$$+ 900\,x\,a^3\,n^3\,\beta_n\,d^3 - 66\,x\,a\,n^2\,\beta_n\,d^5 - 448\,x\,a^5\,n^6\,\beta_n\,e - 328\,x\,\gamma_n\,e\,a^2\,d^3 + 264\,\gamma_n\,a\,n\,d^5$$
$$- 336\,x^2\,n^2\,d^4\,a^2 + 548\,x^2\,n^3\,d^4\,a^2 + 80\,x^2\,n^2\,d^5\,a + 28\,x^2\,n^4\,d^5\,a + 640\,x^2\,n^2\,e\,a^5$$
$$+ 72\,x^2\,n^5\,d^4\,a^2 - 1536\,x^2\,n^3\,e\,a^5 - 504\,x^2\,n^5\,d^3\,a^3 + 32\,x^2\,n^7\,d^2\,a^4 - 24\,x^2\,n^2\,b\,d^5$$
$$+ 8\,x^2\,n\,c\,d^5 + 8\,x^2\,n\,e^2\,d^4 - 96\,x^2\,n^4\,e^2\,a^4 - 4256\,x^2\,n^4\,b^2\,a^4 - 992\,x^2\,n^6\,b^2\,a^4$$
$$+ 2624\,x^2\,n^4\,c\,a^5 + 1280\,x^2\,n^2\,c\,a^5 + 128\,x^2\,n^6\,c\,a^5 + 528\,x^2\,n^3\,e^2\,a^4 - 24\,x^2\,n^2\,b^2\,d^4$$
$$- 800\,x^2\,n^2\,b^2\,a^4 - 744\,x^2\,n^2\,c\,d^3\,a^2 + 480\,x^2\,n\,c\,d^3\,a^2 - 1344\,x^2\,n^2\,b\,a^2\,d^3 + 80\,x^2\,n^3\,d^4\,e\,a$$
$$+ 64\,x^2\,n^6\,e\,a^4\,d + 112\,x^2\,n^4\,d^4\,b\,a - 4736\,x^2\,n^3\,b\,a^3\,d^2 + 1944\,x^2\,n^2\,b\,a^3\,d^2$$
$$+ 4648\,x^2\,n^4\,b\,a^3\,d^2 - 2016\,x^2\,n^5\,b\,a^3\,d^2 - 896\,x^2\,n^4\,e\,a^3\,d^2 + 1264\,x^2\,n^3\,e\,a^3\,d^2$$
$$- 664\,x^2\,n^3\,e\,a^2\,d^3 - 352\,x^2\,n^3\,d^4\,b\,a - 600\,x^2\,n^2\,e\,a^4\,d - 4256\,x^2\,n^4\,b\,a^4\,d$$
$$+ 2960\,x^2\,n^5\,b\,a^4\,d - 80\,\gamma_n\,c\,a\,d^4 + 472\,x^2\,n^3\,e\,a^4\,d + 224\,x^2\,n^4\,e\,a^4\,d - 288\,x^2\,n^5\,e\,a^4\,d$$
$$- 156\,x^2\,n^2\,d^4\,e\,a + 320\,x^2\,n^2\,b\,d^4\,a + 72\,x^2\,n^2\,c\,d^4\,a + 48\,x^2\,n^2\,e^2\,d^3\,a - 408\,x^2\,n^2\,e^2\,d^2\,a^2$$
$$+ 1064\,x^2\,n^2\,e^2\,d\,a^3 - 352\,x^2\,n^3\,b^2\,d^3\,a + 2192\,x^2\,n^2\,b\,e\,a^4 - 3600\,x^2\,n^3\,b\,e\,a^4$$
$$+ 2720\,x^2\,n^4\,b\,e\,a^4 - 960\,x^2\,n^5\,e\,b\,a^4 + 128\,x^2\,n^6\,e\,b\,a^4 - 1368\,x^2\,n^4\,b^2\,a^2\,d^2$$
$$+ 2192\,x^2\,n^3\,b^2\,a^2\,d^2 + 288\,x^2\,n^5\,b^2\,a^2\,d^2 - 3544\,x^2\,n^2\,b\,e\,d\,a^3 - 480\,x^2\,n\,b\,e\,a^4$$
$$- 928\,x^2\,n\,c\,d^2\,a^3 - 1344\,x^2\,n^2\,b^2\,d^2\,a^2 + 64\,x^2\,n^4\,e^2\,a^3\,d + 4648\,x^2\,n^4\,b^2\,a^3\,d$$
$$- 2016\,x^2\,n^5\,b^2\,a^3\,d + 4544\,x^2\,n^3\,c\,a^4\,d - 2272\,x^2\,n^4\,c\,a^4\,d - 480\,x^2\,n^3\,e^2\,a^3\,d$$
$$+ 1944\,x^2\,n^2\,b^2\,a^3\,d + 112\,x^2\,n^4\,b^2\,d^3\,a + 160\,x\,\gamma_n\,a^2\,n^4\,d^4 + 58\,x^2\,n\,d^4\,a^2$$
$$- 2016\,x\,\gamma_n\,a^3\,n^4\,d^3 - 7664\,\gamma_n\,a^4\,n^5\,b^2 + 1356\,\gamma_n\,a^3\,n^5\,d^3 - 328\,\gamma_n\,a^3\,n^6\,d^3$$
$$+ 3184\,\gamma_n\,a^4\,n^6\,b^2 + 6832\,x\,\gamma_n\,d^2\,n^4\,a^4 + 864\,\gamma_n\,a^4\,n^2\,d^2 - 2084\,\gamma_n\,a^4\,n^3\,d^2 + 496\,\gamma_n\,a^4\,n^2\,e^2$$
$$- 2176\,x\,\gamma_n\,a^5\,n^6\,b + 8\,x\,\beta_n\,n^3\,b\,e\,d^4 - 1344\,\gamma_n\,b\,n^4\,e\,d^2\,a^2 + 64\,\gamma_n\,b\,n^4\,e\,d^3\,a$$
$$+ 40\,\gamma_n\,a\,n^3\,c\,d^4 + 13664\,x\,\gamma_n\,a^4\,n^4\,b\,d - 432\,\gamma_n\,a^2\,n^3\,e^2\,d^2 - 8400\,\gamma_n\,a^4\,n^3\,c\,d$$
$$+ 8\,x\,\gamma_n\,n^2\,b\,d^5 - 112\,\gamma_n\,b\,d^5\,n - 1024\,x\,\gamma_n\,n^2\,d^3\,e\,a^2 + 6196\,\gamma_n\,b\,d^3\,n^2\,a^2 - 24\,x\,\gamma_n\,n\,b\,d^5$$
$$+ 8\,x\,\gamma_n\,n\,d^5\,e + 4\,\gamma_n\,e^2\,n^2\,d^4 + 92\,\gamma_n\,b^2\,n^2\,d^4 - 21600\,x^2\,\gamma_n\,a^5\,n^2\,d + 92\,\gamma_n\,d^5\,n^2\,b$$
$$- 9696\,x\,\gamma_n\,a^4\,n^3\,d^2 + 32\,\gamma_n\,d^5\,n\,e + 80\,x\,\gamma_n\,a\,n^2\,d^4\,e - 1664\,x\,\gamma_n\,a^2\,n^3\,b\,d^3 - 288\,\gamma_n\,b\,d^4\,a$$
$$+ 88\,\gamma_n\,e^2\,d^2\,a^2 - 288\,\gamma_n\,b^2\,d^3\,a + 384\,\gamma_n\,c\,a^5\,n - 48\,\gamma_n\,e^2\,d\,a^3 + 480\,x\,\gamma_n\,d\,a^5\,n$$
$$+ 640\,x^2\,\beta_n\,n^6\,b\,a^4\,d - 176\,x\,\gamma_n\,b\,d^4\,a - 440\,\gamma_n\,b\,e\,d^2\,a^2 + 16\,\gamma_n\,a^4\,n^8\,d^2 + 24\,\gamma_n\,a^2\,n^6\,d^4$$
$$+ 3888\,x\,\beta_n\,n^4\,c\,a^4\,d + 2784\,\gamma_n\,a^3\,n\,b^2\,d - 2960\,\gamma_n\,a^2\,n\,b\,d^3 + 2784\,\gamma_n\,a^3\,n\,b\,d^2$$
$$- 180\,\gamma_n\,a\,n^3\,d^4\,e + 3456\,\gamma_n\,b^2\,n^2\,a^4 + 3184\,\gamma_n\,a^4\,n^6\,b\,d + 128\,x\,\gamma_n\,a^5\,n^7\,d$$
$$+ 128\,\gamma_n\,a^3\,n^7\,b\,d^2 - 2672\,\gamma_n\,a^4\,n^2\,b\,e + 2016\,\gamma_n\,a^3\,n^4\,d^2\,e + 80\,x\,\beta_n\,n\,b^2\,d^3\,a$$

$$+96\,\gamma_n\,a^2\,n^6\,b\,d^3-8336\,\gamma_n\,a^4\,n^3\,b\,d-2176\,\gamma_n\,a^5\,n^2\,c-176\,\gamma_n\,a^4\,n^7\,d^2+64\,\gamma_n\,a^4\,n^8\,b^2$$
$$+32\,\gamma_n\,a^3\,n^7\,d^3+4832\,\gamma_n\,a^5\,n^3\,c+5792\,x\,\gamma_n\,a^5\,n^3\,d+5424\,\gamma_n\,a^3\,n^5\,d^2\,b$$
$$+10576\,\gamma_n\,a^4\,n^4\,b\,d-832\,\gamma_n\,a^3\,n^5\,d^2\,e-2752\,x\,\gamma_n\,a^3\,n^3\,e\,d^2+640\,x\,\gamma_n\,a^3\,n^5\,b\,d^2$$
$$+432\,x\,\beta_n\,n^5\,e\,a^4\,d+144\,x\,\beta_n\,n^4\,e\,a^4\,d-320\,x\,\beta_n\,n^6\,e\,a^4\,d+8992\,x^2\,\beta_n\,n^4\,b\,a^4\,d$$
$$-4800\,x^2\,\gamma_n\,n^2\,d^3\,a^3-9600\,x^2\,\gamma_n\,n^3\,d^2\,a^4+1280\,x^2\,\gamma_n\,n^3\,d^3\,a^3+528\,\gamma_n\,b^2\,d^2\,a^2$$
$$+80\,\gamma_n\,d^4\,e\,a+1920\,x^2\,\gamma_n\,n^4\,a^4\,d^2-36\,x\,\beta_n\,n\,d^4\,e\,a-10616\,x\,\gamma_n\,a^3\,n^2\,b\,d^2$$
$$+13968\,x\,\gamma_n\,a^4\,n^2\,b\,d+3024\,x\,a^3\,n^5\,\beta_n\,b^2\,d-144\,x\,a^4\,n^7\,\beta_n\,d^2+2064\,x\,a^4\,n^6\,\beta_n\,b^2$$
$$-608\,x\,a^3\,n^4\,\beta_n\,e^2\,d+3600\,x\,a^3\,n^3\,\beta_n\,b\,d^2+64\,x\,a^4\,n^8\,\beta_n\,b^2+3600\,x\,a^4\,n^4\,\beta_n\,b^2$$
$$-1728\,x\,a^4\,n^3\,\beta_n\,b^2-576\,x\,a^4\,n^7\,\beta_n\,b^2-544\,x\,a^3\,n^2\,\beta_n\,e^2\,d-936\,x\,a^4\,n^5\,\beta_n\,d^2$$
$$-384\,x^2\,n^2\,b\,e\,d^3\,a+8\,\gamma_n\,a\,n^5\,d^5-4592\,x\,a^3\,n^4\,\beta_n\,b^2\,d+5664\,x^2\,a^4\,n^3\,\beta_n\,e\,d$$
$$+784\,x\,a^4\,n^4\,\beta_n\,e^2+16\,x\,a^4\,n^8\,\beta_n\,d^2-624\,x\,a^4\,n^3\,\beta_n\,e^2+516\,x\,a^4\,n^6\,\beta_n\,d^2$$
$$+64\,x\,a^4\,n^6\,\beta_n\,e^2+3904\,x^2\,a^4\,n^2\,\beta_n\,b\,d+320\,x\,a^4\,n^6\,\beta_n\,c\,d-1584\,x^2\,n^3\,b\,e\,d^2\,a^2$$
$$+2720\,x^2\,a^5\,n^5\,\beta_n\,d+128\,x\,a^3\,n^7\,\beta_n\,b^2\,d+128\,x\,a^5\,n^7\,\beta_n\,c+3600\,x\,a^3\,n^3\,d\,\beta_n\,b^2$$
$$-576\,x\,a^4\,n^7\,\beta_n\,b\,d+64\,x\,a^4\,n^8\,\beta_n\,b\,d+2064\,x\,a^4\,n^6\,\beta_n\,b\,d+784\,x\,a^5\,n^3\,\beta_n\,e$$
$$-960\,x^2\,a^5\,n^6\,\beta_n\,d+1060\,x\,a^2\,n^2\,\beta_n\,b^2\,d^2-3600\,x^2\,a^5\,n^4\,\beta_n\,d+96\,x\,a^2\,n^5\,\beta_n\,d^3\,e$$
$$+128\,x\,a^3\,n^6\,\beta_n\,d^2\,e-704\,x\,a^4\,n^3\,d\,\beta_n\,e+3776\,x^2\,a^5\,n^4\,\beta_n\,e+256\,x^2\,a^5\,n^6\,\beta_n\,e$$
$$+2192\,x^2\,a^5\,n^3\,\beta_n\,d-3744\,x\,a^4\,n^5\,\beta_n\,b\,d+3600\,x\,a^4\,n^4\,\beta_n\,b\,d+128\,x\,a^4\,n^7\,\beta_n\,b\,e$$
$$-1728\,x\,a^4\,n^3\,\beta_n\,b\,d+2192\,x\,a^4\,n^3\,\beta_n\,b\,e+64\,x\,a^4\,n^7\,\beta_n\,d\,e-960\,x\,a^4\,n^6\,\beta_n\,e\,b$$
$$-3600\,x\,a^4\,n^4\,\beta_n\,e\,b+320\,x\,a\,n^3\,\beta_n\,b^2\,d^3+320\,x\,\beta_n\,n^2\,b^2\,a^4+1792\,x\,a^3\,n^2\,\beta_n\,b\,e\,d$$
$$-624\,x\,a^2\,n^5\,\beta_n\,b^2\,d^2-960\,x\,a^2\,n^4\,\beta_n\,e\,b\,d^2+80\,x\,\beta_n\,n\,e^2\,d\,a^3-156\,x\,a^2\,n^5\,\beta_n\,d^4$$
$$-800\,x^2\,n^2\,b\,a^4\,d-484\,x^2\,n^2\,e\,a^3\,d^2+2192\,x^2\,n^3\,b\,a^2\,d^3+588\,x^2\,n^2\,d^3\,e\,a^2$$
$$+96\,x^2\,\gamma_n\,a\,n\,d^5-120\,\gamma_n\,a\,n^2\,e^2\,d^3+3460\,\gamma_n\,a^3\,n^3\,d^3-70\,\gamma_n\,a\,n^4\,d^5-704\,\gamma_n\,a^4\,n^7\,b^2$$
$$-912\,\gamma_n\,a^2\,n^5\,b^2\,d^2+880\,\gamma_n\,a^4\,n^4\,e^2+232\,\gamma_n\,a\,n^3\,d^5+320\,x\,\gamma_n\,a^3\,n^5\,d^3-384\,\gamma_n\,a^4\,n^5\,e^2$$
$$+1549\,\gamma_n\,a^2\,n^2\,d^4+5400\,x\,\gamma_n\,b\,d^2\,a^3\,n-4624\,x\,\gamma_n\,b\,a^4\,d\,n+872\,\gamma_n\,b\,n^2\,e\,d^3\,a$$
$$-2216\,\gamma_n\,e\,b\,n\,d\,a^3+32\,\gamma_n\,a\,n^4\,d^4\,e+4\,\gamma_n\,b^2\,n^4\,d^4-240\,x\,\gamma_n\,e\,a^4\,d+608\,\gamma_n\,e\,a^2\,d^3\,n$$
$$+6196\,\gamma_n\,b^2\,n^2\,a^2\,d^2-1196\,x\,\gamma_n\,d^4\,a^2\,n-2312\,x\,\gamma_n\,d^2\,a^4\,n+2700\,x\,\gamma_n\,d^3\,a^3\,n$$
$$-80\,\gamma_n\,e\,a^2\,d^3+96\,\gamma_n\,e\,a^4\,d+2644\,\gamma_n\,d^2\,n^4\,a^4-960\,\gamma_n\,a^4\,n^3\,e^2-832\,x\,\gamma_n\,a^2\,n^3\,d^4$$
$$+1168\,x\,a^5\,n^5\,\beta_n\,e-72\,x\,a\,n^2\,\beta_n\,e^2\,d^3-168\,x\,a\,n^4\,\beta_n\,b^2\,d^3+80\,x^2\,a\,n^2\,\beta_n\,d^4\,e$$
$$+320\,x\,a\,n^3\,\beta_n\,b\,d^4-100\,x\,a\,n^3\,\beta_n\,d^4\,e-168\,x\,a\,n^4\,\beta_n\,d^4\,b+40\,x\,a\,n^3\,\beta_n\,c\,d^4$$
$$-264\,x\,a\,n^2\,\beta_n\,b\,d^4+32\,x\,a\,n^4\,\beta_n\,d^4\,e+104\,x\,a\,n^2\,\beta_n\,d^4\,e-240\,x\,a\,n^3\,\beta_n\,e\,b\,d^3$$
$$-992\,x\,a^3\,n^6\,\beta_n\,b\,d^2+320\,x\,a^3\,n^5\,\beta_n\,c\,d^2+96\,x\,a^2\,n^6\,\beta_n\,b^2\,d^2-544\,x\,a^2\,n^3\,\beta_n\,c\,d^3$$

$$+96\,x\,a^2\,n^4\,\beta_n\,e^2\,d^2+128\,x\,a^3\,n^7\,\beta_n\,b\,d^2+96\,x\,a^2\,n^6\,\beta_n\,b\,d^3+320\,x^2\,a^2\,n^4\,\beta_n\,b\,d^3$$
$$-3688\,x^2\,a^3\,n^2\,\beta_n\,b\,d^2+5328\,x^2\,a^3\,n^3\,\beta_n\,b\,d^2+1304\,x^2\,a^2\,n^2\,\beta_n\,b\,d^3$$
$$+2328\,x\,a^3\,n^3\,\beta_n\,c\,d^2-3136\,x^2\,a^3\,n^4\,\beta_n\,b\,d^2+64\,x\,a\,n^4\,\beta_n\,b\,e\,d^3-1300\,x\,a^2\,n^2\,\beta_n\,b\,e\,d^2$$
$$+8\,x^2\,\beta_n\,n^2\,b\,d^5+4\,x\,\beta_n\,n^4\,b\,d^5+4\,x\,\beta_n\,n^2\,e^2\,d^4-8\,x\,\beta_n\,n\,b^2\,d^4+52\,x^2\,\beta_n\,n\,d^5\,a$$
$$-1856\,x\,\beta_n\,n^3\,b^2\,d^2\,a^2-116\,x\,\beta_n\,n\,e^2\,d^2\,a^2+40\,x\,\beta_n\,n\,e^2\,d^3\,a+320\,x^2\,\beta_n\,n^3\,e\,a^2\,d^3$$
$$-3920\,x\,\beta_n\,n^3\,e\,b\,d\,a^3-58\,x\,\beta_n\,n\,d^4\,a^2-332\,x\,\beta_n\,n^2\,d^3\,a^3-1408\,x\,\beta_n\,n^4\,e\,a^5$$
$$+4384\,x^2\,\beta_n\,n^3\,b\,a^5-8\,x\,\beta_n\,n\,b\,d^5+4\,x\,\beta_n\,n\,d^5\,e+4\,x\,\beta_n\,n^3\,d^5\,e+2192\,x\,\gamma_n\,a^4\,n\,e\,d$$
$$+1540\,x\,\gamma_n\,a^2\,n^2\,d^4-8736\,\gamma_n\,a^3\,n^3\,b\,e\,d+96\,\gamma_n\,a^2\,n^4\,e^2\,d^2-5888\,x\,\gamma_n\,a^4\,n^2\,e\,d$$
$$+6752\,x\,\gamma_n\,a^4\,n^3\,e\,d-6352\,\gamma_n\,a^2\,n^3\,b^2\,d^2-328\,x\,\gamma_n\,a\,n^2\,b\,d^4+928\,\gamma_n\,a\,n^3\,b^2\,d^3$$
$$+7360\,x\,\gamma_n\,a^5\,n^5\,b-512\,\gamma_n\,a^5\,n^6\,e+1648\,\gamma_n\,a^5\,n^5\,e-12\,\gamma_n\,c\,n\,d^5+348\,x\,\beta_n\,n\,b\,e\,d^2\,a^2$$
$$+96\,\gamma_n\,a^2\,n^5\,d^3\,e-2720\,\gamma_n\,a^5\,n^4\,e-592\,\gamma_n\,a^2\,n^4\,d^3\,e-912\,\gamma_n\,a^2\,n^5\,b\,d^3$$
$$-6320\,x\,\gamma_n\,a^5\,n^4\,d-480\,x\,\gamma_n\,e\,a^5\,n+64\,\gamma_n\,a^5\,n^7\,e-5440\,\gamma_n\,a^5\,n^4\,c+6488\,\gamma_n\,a^3\,n^2\,e\,b\,d$$
$$+3024\,x\,\beta_n\,n^5\,b\,d^2\,a^3-4\,x\,\beta_n\,n\,c\,d^5+8\,x^2\,\beta_n\,n\,d^5\,e-16\,x\,\beta_n\,n^3\,d^5\,b+20\,x\,\beta_n\,n^2\,b\,d^5$$
$$-8\,x\,\beta_n\,n^2\,d^5\,e-1304\,\gamma_n\,a^4\,n^3\,e\,d+1032\,x\,\gamma_n\,a^2\,n\,e\,d^3-704\,\gamma_n\,a^4\,n\,e\,d$$
$$+4080\,x\,\gamma_n\,a^3\,n^2\,e\,d^2+5856\,\gamma_n\,a^4\,n^2\,c\,d-1060\,\gamma_n\,a^2\,n\,c\,d^3+856\,x^2\,\beta_n\,n\,b\,a^3\,d^2$$
$$+212\,x\,\gamma_n\,a\,n\,d^5-5308\,x\,\gamma_n\,a^3\,n^2\,d^3+64\,\gamma_n\,a^4\,n^6\,e^2+4792\,x\,\gamma_n\,a^3\,n^3\,d^3$$
$$+10576\,\gamma_n\,a^4\,n^4\,b^2-8336\,\gamma_n\,a^4\,n^3\,b^2-2456\,x\,\gamma_n\,a^3\,n\,e\,d^2+2280\,\gamma_n\,a^3\,n\,c\,d^2$$
$$-1888\,\gamma_n\,a^4\,n\,c\,d-1488\,x\,\beta_n\,n^2\,c\,a^3\,d^2+320\,\gamma_n\,a^3\,n^5\,c\,d^2-2240\,\gamma_n\,a^4\,n^5\,c\,d$$
$$-168\,x\,\gamma_n\,a\,n\,d^4\,e+320\,\gamma_n\,a^4\,n^6\,c\,d+256\,x\,\gamma_n\,a^5\,n^6\,e+160\,\gamma_n\,a^2\,n^4\,c\,d^3$$
$$+320\,x\,\gamma_n\,a^2\,n^3\,d^3\,e-1448\,\gamma_n\,a\,n^2\,b\,d^4-6352\,\gamma_n\,a^2\,n^3\,b\,d^3+3404\,\gamma_n\,a^2\,n^4\,d^3\,b$$
$$+96\,\gamma_n\,a^2\,n^6\,b^2\,d^2+1348\,\gamma_n\,a^2\,n^3\,d^3\,e+928\,\gamma_n\,a\,n^3\,d^4\,b+320\,x\,\gamma_n\,a^4\,n^6\,d^2$$
$$-1088\,x\,\gamma_n\,a^5\,n^6\,d+696\,\gamma_n\,a^3\,n\,d^3+5952\,\gamma_n\,a^3\,n^4\,b\,e\,d+356\,\gamma_n\,a\,n^2\,d^4\,e$$
$$+4032\,x\,\gamma_n\,a^5\,n^4\,e+32\,\gamma_n\,a\,n^5\,b^2\,d^3-1664\,x\,\gamma_n\,a^5\,n^5\,e+5440\,x^2\,\gamma_n\,a^3\,n\,d^3$$
$$-2910\,\gamma_n\,a^3\,n^4\,d^3-1984\,\gamma_n\,a^3\,n^5\,b\,e\,d-400\,\gamma_n\,a\,n^3\,b\,e\,d^3+64\,\gamma_n\,a^4\,n^8\,b\,d$$
$$-1312\,\gamma_n\,a^3\,n^6\,b\,d^2+720\,\gamma_n\,a^4\,n^5\,e\,d-80\,\gamma_n\,a^4\,n^4\,e\,d+128\,\gamma_n\,a^3\,n^7\,b^2\,d$$
$$-384\,\gamma_n\,a^4\,n^6\,e\,d-160\,\gamma_n\,c\,n^2\,d^4\,a-280\,\gamma_n\,b\,n^4\,d^4\,a-32\,\gamma_n\,b\,n^3\,d^5+192\,\gamma_n\,c\,a^4\,d$$
$$-400\,\gamma_n\,c\,a^3\,d^2+3404\,\gamma_n\,b^2\,n^4\,d^2\,a^2-40\,\gamma_n\,b\,e\,d^4-12\,\gamma_n\,e^2\,n\,d^4-32\,\gamma_n\,b^2\,n^3\,d^4$$
$$+200\,\gamma_n\,c\,a\,n\,d^4-1200\,x^2\,\gamma_n\,d^4\,a^2\,n-740\,\gamma_n\,d^4\,a^2\,n-144\,\gamma_n\,d^2\,a^4\,n+4\,\gamma_n\,c\,n^2\,d^5$$
$$+960\,x\,\gamma_n\,b\,a^5\,n+1056\,\gamma_n\,b\,d^4\,n\,a-288\,\gamma_n\,d^4\,n\,e\,a-8936\,\gamma_n\,b^2\,n^2\,a^3\,d$$
$$+424\,x\,\gamma_n\,n\,b\,d^4\,a-3456\,x\,\gamma_n\,n^4\,e\,a^4\,d-362\,\gamma_n\,d^5\,n^2\,a+240\,\gamma_n\,b\,e\,d^3\,a$$
$$-1312\,\gamma_n\,b^2\,n^6\,a^3\,d+128\,\gamma_n\,e^2\,n^5\,a^3\,d+424\,\gamma_n\,e^2\,a^3\,n\,d-4736\,x\,\gamma_n\,b\,n^5\,a^4\,d$$

$$+640\,x\,\gamma_n\,b\,n^6\,a^4\,d-1128\,\gamma_n\,e^2\,n^2\,a^3\,d+1296\,\gamma_n\,e^2\,n^3\,a^3\,d+13840\,\gamma_n\,b^2\,n^3\,a^3\,d$$
$$-1448\,\gamma_n\,b^2\,n^2\,d^3\,a$$

Equating the highest coefficient gives

```
> rule4:=beta[n]=factor(solve(coeff(re,x,3),beta[n]));
```

$$rule4 := \beta_n = -\frac{d\,n^2\,a+2\,b\,n^2\,a-d\,a\,n-2\,b\,n\,a-2\,e\,a+2\,b\,d\,n+d^2\,n+d\,e}{(d-2\,a+2\,a\,n)\,(d+2\,a\,n)}$$

and equating the second highest coefficient yields

```
> rule5:=gamma[n]=factor(subs(rule4,solve(coeff(re,x,2),gamma[n
])));
```

$$rule5 := \gamma_n = -(-2\,a+a\,n+d)\,(-d^3\,n-4\,b\,n\,d^2-4\,b\,e\,d-4\,b^2\,n\,d+2\,a\,d^2\,n-a\,d^2$$
$$-n^2\,a\,d^2+16\,a\,c\,n\,d-4\,a\,b^2+4\,a\,e^2-4\,a\,b^2\,n^2+16\,a^2\,c-32\,a^2\,n\,c+8\,a\,b^2\,n-4\,a\,b\,d$$
$$-16\,a\,c\,d-8\,a\,d\,e-4\,a\,d\,n^2\,b+16\,a^2\,c\,n^2-16\,a^2\,n\,e+8\,a\,d\,n\,e+8\,a\,b\,d\,n+d^3$$
$$+8\,e\,a^2+8\,e\,n^2\,a^2+4\,c\,d^2+4\,b\,d^2+4\,b^2\,d)\,n\Big/(4\,(2\,a\,n-3\,a+d)\,(d-a+2\,a\,n)$$
$$(d-2\,a+2\,a\,n)^2)$$

```
>
```

Orthogonal Polynomial Solutions of Recurrence Equations

```
> read "hsum6.mpl";
```

Package "Hypergeometric Summation", Maple V - Maple 8

Copyright 2002, Wolfram Koepf, University of Kassel

```
> read "retode.mpl";
```

Package "REtoDE", Maple V - Maple 8

Copyright 2002, Wolfram Koepf, University of Kassel

First example

```
> RE:=P(n+2)-(x-n-1)*P(n+1)+alpha*(n+1)^2*P(n)=0;
```

$$RE := \mathrm{P}(n+2)-(x-n-1)\,\mathrm{P}(n+1)+\alpha\,(n+1)^2\,\mathrm{P}(n)=0$$

```
> REtoDE(RE,P(n),x);
```

Warning: parameters have the values, $\{a=0,\ e=0,\ b=2\,c,\ \alpha=\frac{1}{4},\ c=c,\ d=-4\,c\}$

$$\left[\frac{1}{2}(2\,x+1)\left(\frac{\partial^2}{\partial x^2}\mathrm{P}(n,x)\right)-2\,x\left(\frac{\partial}{\partial x}\mathrm{P}(n,x)\right)+2\,n\,\mathrm{P}(n,x)=0,\right.$$
$$\left.\left[I=\left[\frac{-1}{2},\infty\right],\rho(x)=2\,\mathbf{e}^{(-2\,x)}\right],\frac{k_{n+1}}{k_n}=1\right]$$

```
> REtodiscreteDE(RE,P(n),x);
```

Warning: parameters have the values, $\{g=g,\ b=-\frac{1}{2}f\,d-\frac{1}{2}d,$

$$c=\frac{1}{4}d-\frac{1}{4}f^2\,d+\frac{1}{2}g\,d\,f+\frac{1}{2}g\,d,\ a=0,\ e=-g\,d,\ d=d,\ f=f,\ \alpha=\frac{-1+f^2}{4\,f^2}\}$$

$$\left[\frac{1}{2}\,\frac{(f+2\,f\,x-1)\,(\mathrm{Nabla}(\mathrm{P}(n,f\,x+f+g),x+1)-\mathrm{Nabla}(\mathrm{P}(n,f\,x+g),x))}{f}\right.$$
$$+\frac{2\,x\,(-\mathrm{P}(n,f\,x+f+g)+\mathrm{P}(n,f\,x+g))}{1+f}+\frac{2\,n\,\mathrm{P}(n,f\,x+g)}{(1+f)\,f}=0,$$
$$\left[\sigma(x)=\frac{f}{2}+x-\frac{1}{2}-g,\ \sigma(x)+\tau(x)=-\frac{(f-1)\,(-1+2\,g-f-2\,x)}{2\,(1+f)}\right],\ \rho(x)=\left(\frac{f-1}{1+f}\right)^{x},$$
$$\left.\frac{k_{n+1}}{k_n}=\frac{1}{f}\right]$$

```
> strict:=true;
```

$$strict := true$$

```
> REtodiscreteDE(RE,P(n),x);
Error, (in REtodiscreteDE) this recurrence equation has no classical
discrete orthogonal polynomial solutions
```

Second example

```
> RE:=P(n+2)-x*P(n+1)+alpha*q^n*(q^(n+1)-1)*P(n)=0;
```

$$RE := \mathrm{P}(n+2)-\mathrm{P}(n+1)\,x+\alpha\,q^{n}\,(q^{(n+1)}-1)\,\mathrm{P}(n)=0$$

```
> REtoqDE(RE,P(n),q,x);
```

Warning: parameters have the values, $\{a=-d\,q+d,\ c=-\alpha\,d\,q+\alpha\,d,\ b=0,\ e=0,\ d=d\}$

$$\left[(x^2+\alpha)\,\mathrm{Dq}\left(\mathrm{Dq}\left(\mathrm{P}(n,x),\frac{1}{q},x\right),q,x\right)-\frac{x\,\mathrm{Dq}(\mathrm{P}(n,x),q,x)}{q-1}+\frac{q\,(-1+q^{n})\,\mathrm{P}(n,x)}{(q-1)^2\,q^{n}}=0,\right.$$
$$\left.\frac{\rho(q\,x)}{\rho(x)}=\frac{\alpha}{q^2\,x^2+\alpha},\ \frac{k_{n+1}}{k_n}=1\right]$$

```
>
>
```