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> restart:
> read "ODE3solve.mpl":
    Package "Solving third-order holonomic differential equations", Maple 16
    Copyright 2017, Mouafo Wouodjie Merlin, University of Kassel
    Package "Hypergeometric Summation", Maple V - Maple 17
    Copyright 1998-2013, Wolfram Koepf, University of Kassel

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(1)

[Here is the Maple example in Introduction.

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> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));

$$F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) = 0$$


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(2)

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> L12:=de2diffop(F,J(x));

$$L12 := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1$$


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(3)

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> L12:=subs({a1=1/4,b1=1/2,b2=3/4},L12);

$$L12 := x^2 Dx^3 + \frac{9 x Dx^2}{4} + \left( \frac{3}{8} - x \right) Dx - \frac{1}{4}$$


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(4)

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> f:=(x-1)/(x^2);

$$f := \frac{x-1}{x^2}$$


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(5)

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> L2:=ChangeOfVariables(L12,f);

$$L2 := 8 Dx^3 (x-1)^2 x^5 (x-2)^2 + 6 (5 x^2 - 20 x + 12) Dx^2 (x-1) x^4 (x-2) + (15 x^6 - 128 x^5 + 420 x^4 - 592 x^3 + 544 x^2 - 384 x + 128) Dx x + 2 (x-2)^5$$


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(6)

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> r:=-1/x;

$$r := -\frac{1}{x}$$


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(7)

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> L3:=ExpProduct(L2,r);

$$L3 := 8 Dx^3 (x-1)^2 x^5 (x-2)^2 + 6 (9 x^2 - 32 x + 20) Dx^2 (x-1) x^4 (x-2) + (75 x^6 - 548 x^5 + 1404 x^4 - 1504 x^3 + 832 x^2 - 384 x + 128) Dx x + 15 x^6 - 126 x^5 + 400 x^4 - 512 x^3 + 384 x^2 - 224 x + 64$$


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(8)

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> r0:=2*x;

$$r0 := 2 x$$


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(9)

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> r1:=x^2;

$$r1 := x^2$$


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(10)

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> r2:=0;

$$r2 := 0$$


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(11)

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> L:=GaugeTransf(L3,r0,r1,r2);

$$L := 8 x^5 (x-2)^2 (3 x^6 + 2 x^5 + 80 x^4 - 336 x^3 + 608 x^2 - 544 x + 192)^2 (x-1)^5 Dx^3$$


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(12)

$$\begin{aligned}
& + 2x^4(x-2)(3x^6+2x^5+80x^4-336x^3+608x^2-544x+192)(45x^8-70x^7 \\
& + 1756x^6-13848x^5+44640x^4-79520x^3+81600x^2-44672x+9984)(x-1)^4 Dx^2 \\
& + x(x-2)(3x^6+2x^5+80x^4-336x^3+608x^2-544x+192)(45x^{11}-54x^{10} \\
& + 1976x^9-22440x^8+100336x^7-257888x^6+415872x^5-437632x^4+318464x^3 \\
& - 172032x^2+65536x-12288)(x-1)^3 Dx + 32(2x^6+11x^5-33x^4+68x^3-94x^2 \\
& + 72x-24)(3x^6+2x^5+80x^4-336x^3+608x^2-544x+192)(x-1)^3(x-2)^4
\end{aligned}$$

> dsolve(diffop2de(L,y(x)),y(x));

$$y(x) = DESol \left\{ \left(64x^9 - 32x^8 - 2400x^7 + 12224x^6 - 31552x^5 + 54912x^4 - 68096x^3 \right. \right. \quad (13)$$

$$+ 56320x^2 - 27648x + 6144) _Y(x) + (45x^{12} - 54x^{11} + 1976x^{10} - 22440x^9$$

$$+ 100336x^8 - 257888x^7 + 415872x^6 - 437632x^5 + 318464x^4 - 172032x^3 + 65536x^2$$

$$- 12288x) \left(\frac{d}{dx} _Y(x) \right) + (90x^{13} - 230x^{12} + 3652x^{11} - 31208x^{10} + 116976x^9$$

$$- 248320x^8 + 322240x^7 - 252544x^6 + 109312x^5 - 19968x^4) \left(\frac{d^2}{dx^2} _Y(x) \right) + (24x^{14}$$

$$- 80x^{13} + 696x^{12} - 5216x^{11} + 18784x^{10} - 38528x^9 + 48640x^8 - 37632x^7 + 16384x^6$$

$$- 3072x^5) \left(\frac{d^3}{dx^3} _Y(x) \right) \Big\}, \{ _Y(x) \}$$

> TIME := time();

HyplF2Solutions(L);

time()-TIME;

$$TIME := 5.734$$

$$\left\{ \left[\left[\left[\left[\frac{1}{4} \right], \left[\frac{1}{2}, \frac{3}{4} \right], \left[-\frac{1}{x} \right], [x^2 Dx + 2x] \right] \right], \frac{x-1}{x^2} \right] \right\}$$

$$0.297$$

(14)

> y1:=expand(exp(int(-1/x,x))*(2*x*hypergeom([1/4],[1/2,3/4],(x-1)/x^2)+x^2*normal(diff(hypergeom([1/4],[1/2,3/4],(x-1)/x^2),x))+0*normal(diff(hypergeom([1/4],[1/2,3/4],(x-1)/x^2),x\$2))));

$$\begin{aligned}
y1 := & 2 \operatorname{hypergeom} \left(\left[\frac{1}{4} \right], \left[\frac{1}{2}, \frac{3}{4} \right], \frac{x-1}{x^2} \right) - \frac{2 \operatorname{hypergeom} \left(\left[\frac{5}{4} \right], \left[\frac{3}{2}, \frac{7}{4} \right], \frac{x-1}{x^2} \right)}{3x} \\
& + \frac{4 \operatorname{hypergeom} \left(\left[\frac{5}{4} \right], \left[\frac{3}{2}, \frac{7}{4} \right], \frac{x-1}{x^2} \right)}{3x^2}
\end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{eq} := \text{HolonomicDE}(\text{BessellI}(\text{nu}, \sqrt{x})^2, Y(x)); \\ &eq := -Y(x) + (-2v^2 - 2x + 2) \left(\frac{d}{dx} Y(x) \right) + 2 \left(\frac{d^3}{dx^3} Y(x) \right) x^2 + 6 \left(\frac{d^2}{dx^2} Y(x) \right) x \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{LBB} := \text{de2diffop}(eq, Y(x)); \\ &LBB := 2x^2 Dx^3 + 6x Dx^2 + (-2v^2 - 2x + 2) Dx - 1 \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{LBB} := \text{subs}(\text{nu}=1/7, \text{LBB}); \\ &LBB := 2x^2 Dx^3 + 6x Dx^2 + \left(\frac{96}{49} - 2x \right) Dx - 1 \end{aligned} \quad (18)$$

$$\begin{aligned} &> f := 2*(x-7)/x; \\ &f := \frac{2(x-7)}{x} \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{L1} := \text{ChangeOfVariables}(\text{LBB}, f); \\ &L1 := Dx^3 (x-7)^2 x^4 + 3(-7+2x) Dx^2 (x-7) x^3 + 2(3x^3 - 21x^2 - 25x + 343) Dx x - 343 \end{aligned} \quad (20)$$

$$\begin{aligned} &> r := x^2; \\ &r := x^2 \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{L2} := \text{ExpProduct}(\text{L1}, r); \\ &L2 := Dx^3 (x-7)^2 x^4 - 3(x^4 - 7x^3 - 2x + 7) Dx^2 (x-7) x^3 + (3x^9 - 42x^8 + 147x^7 - 18x^6 + 210x^5 - 588x^4 + 6x^3 - 42x^2 - 50x + 686) Dx x - x^{12} + 14x^{11} - 49x^{10} + 12x^9 - 147x^8 + 441x^7 - 20x^6 + 196x^5 - 342x^4 - 686x^3 - 343 \end{aligned} \quad (22)$$

$$\begin{aligned} &> r0 := 1; \\ &r0 := 1 \end{aligned} \quad (23)$$

$$\begin{aligned} &> r1 := x; \\ &r1 := x \end{aligned} \quad (24)$$

$$\begin{aligned} &> r2 := 0; \\ &r2 := 0 \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{L} := \text{GaugeTransf}(\text{L2}, r0, r1, r2); \\ &L := x^4 (x^{12} - 14x^{11} + 49x^{10} - 9x^9 + 105x^8 - 294x^7 + 8x^6 - 70x^5 + 48x^4 + 686x^3 - 50x + 1029)^2 (x-7)^3 Dx^3 - x^3 (3x^{16} - 63x^{15} + 441x^{14} - 1053x^{13} + 420x^{12} - 2352x^{11} + 4140x^{10} - 168x^9 - 738x^8 + 7200x^7 - 14238x^6 + 100x^5 + 21x^4 - 16807x^3 + 400x^2 - 10311x + 28812) (x^{12} - 14x^{11} + 49x^{10} - 9x^9 + 105x^8 - 294x^7 + 8x^6 - 70x^5 + 48x^4 + 686x^3 - 50x + 1029) (x-7)^2 Dx^2 + x(x-7) (x^{12} - 14x^{11} + 49x^{10} - 9x^9 + 105x^8 - 294x^7 + 8x^6 - 70x^5 + 48x^4 + 686x^3 - 50x + 1029) (3x^{21} - 84x^{20} + 882x^{19} - 4143x^{18} + 7854x^{17} - 5733x^{16} + 21693x^{15} - 30576x^{14} + 13324x^{13} - 39768x^{12} + 10332x^{11} + 127136x^{10} - 339x^9 + 209958x^8 - 554707x^7 - 42840x^6 + 418804x^5 - 1212076x^4 + 489181x^3 - 83936x^2 - 85750x + 705894) Dx - (x - 7) (x^{24} - 28x^{23} + 294x^{22} - 1384x^{21} + 2695x^{20} - 2646x^{19} + 10354x^{18} - 15862x^{17} + 12248x^{16} - 43900x^{15} + 43456x^{14} + 68428x^{13} - 46754x^{12} + 281204x^{11} - 484946x^{10} - 132496x^9 + 924680x^8 - 2562036x^7 + 649936x^6 - 1182464x^5 + 1607298x^4 + 2352980x^3 + 352947) (x^{12} - 14x^{11} + 49x^{10} - 9x^9 + 105x^8 - 294x^7 + 8x^6 - 70x^5 \end{aligned} \quad (26)$$

$$+ 48 x^4 + 686 x^3 - 50 x + 1029)$$

> dsolve(diffop2de(L,y(x)),y(x));

$$y(x) = DESol \left(\left\{ \begin{aligned} &(-x^{24} + 28 x^{23} - 294 x^{22} + 1384 x^{21} - 2695 x^{20} + 2646 x^{19} - 10354 x^{18} \\ &+ 15862 x^{17} - 12248 x^{16} + 43900 x^{15} - 43456 x^{14} - 68428 x^{13} + 46754 x^{12} - 281204 x^{11} \\ &+ 484946 x^{10} + 132496 x^9 - 924680 x^8 + 2562036 x^7 - 649936 x^6 + 1182464 x^5 \\ &- 1607298 x^4 - 2352980 x^3 - 352947) \end{aligned} \right. \right) _Y(x) + (3 x^{22} - 84 x^{21} + 882 x^{20} - 4143 x^{19} \\ + 7854 x^{18} - 5733 x^{17} + 21693 x^{16} - 30576 x^{15} + 13324 x^{14} - 39768 x^{13} + 10332 x^{12} \\ + 127136 x^{11} - 339 x^{10} + 209958 x^9 - 554707 x^8 - 42840 x^7 + 418804 x^6 - 1212076 x^5 \\ + 489181 x^4 - 83936 x^3 - 85750 x^2 + 705894 x) \left(\frac{d}{dx} _Y(x) \right) + (-3 x^{20} + 84 x^{19} \\ - 882 x^{18} + 4140 x^{17} - 7791 x^{16} + 5292 x^{15} - 20604 x^{14} + 29148 x^{13} - 438 x^{12} \\ - 12366 x^{11} + 64638 x^{10} - 99766 x^9 + 679 x^8 + 16954 x^7 - 118049 x^6 + 13111 x^5 \\ - 100989 x^4 + 201684 x^3) \left(\frac{d^2}{dx^2} _Y(x) \right) + (x^{18} - 28 x^{17} + 294 x^{16} - 1381 x^{15} \\ + 2632 x^{14} - 2205 x^{13} + 9269 x^{12} - 14588 x^{11} + 1420 x^{10} - 3416 x^9 - 7252 x^8 + 33564 x^7 \\ + 1729 x^6 - 16856 x^5 + 50421 x^4) \left(\frac{d^3}{dx^3} _Y(x) \right) \Bigg\}, \{ _Y(x) \} \right) \quad (27)$$

**> TIME := time();
BesSqRootSolutions(L);
time()-TIME;**

$$TIME := 7.046$$

$$\left\{ \left[\frac{1}{7}, [x^2], [x Dx + 1], \frac{2(x-7)}{x} \right] \right\}$$

$$0.860$$

(28)

**> y1:=collect(expand(exp(int(r,x))*(r0*BessellI(1/7,f)^2 + r1*normal
(diff(BessellI(1/7,f)^2,x)) +r2*normal(diff(BessellI(1/7,f)^2,x\$2))
)),BessellI,factor);**

$$y1 := \frac{e^{\frac{x^3}{3}} (x-5) \text{BessellI}\left(\frac{1}{7}, 2 - \frac{14}{x}\right)^2}{x-7} \quad (29)$$

$$\left[+ \frac{28 \, e^{\frac{x^3}{3}} \operatorname{BesselI}\left(\frac{8}{7}, 2 - \frac{14}{x}\right) \operatorname{BesselI}\left(\frac{1}{7}, 2 - \frac{14}{x}\right)}{x} \right]$$