

```
[> restart;
```

```
[> read "ODE3solve.mpl":
```

*Package "Solving third-order holonomic differential equations", Maple 16*  
*Copyright 2017, Mouafo Wouodjie Merlin, University of Kassel*  
*Package "Hypergeometric Summation", Maple V - Maple 17*  
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(1)

[Here are the Maple implementations in chapter 5 related just to the 1F2 functions.

```
[> ##### THE EXPONENT DIFFERENCES #####
```

[In chapter 5, section 5.2 which is called "Exponent differences", we have the following Maple implementations:

```
[> L12 := x^2*Dx^3+(x*b2+x*x*b1)*Dx^2+(b2*b1-x)*Dx-a1;
```

$$L12 := x^2 Dx^3 + (x b1 + x b2 + x) Dx^2 + (b2 b1 - x) Dx - a1$$

(2)

```
[> gen_exp(L12,t,x=0);
```

$$[[0, t=x], [-b1+1, t=x], [-b2+1, t=x]]$$

(3)

```
[> gen_exp(L12,t,x=infinity);
```

$$\left[ \left[ a1, t = \frac{1}{x} \right], \left[ \frac{1}{t} - \frac{a1}{2} + \frac{b1}{2} + \frac{b2}{2} - \frac{1}{4}, t^2 = \frac{1}{x} \right] \right]$$

(4)

```
[> ##### EXAMPLE IN THE THESIS #####
```

[In chapter 5, section 5.5.7 which is called "Examples", those are the Maple implementations for the example that we have used in the 1F2 type solutions:

```
[> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));
```

$$F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x)$$

(5)

```
[> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1$$

(6)

```
[> L12:=subs({a1=1/2,b1=1/3,b2=RootOf(x^2+1)},LA);
```

$$L12 := x^2 Dx^3 + \left( \frac{4x}{3} + \text{RootOf}(\_Z^2 + 1) x \right) Dx^2 + \left( \frac{\text{RootOf}(\_Z^2 + 1)}{3} - x \right) Dx - \frac{1}{2}$$

(7)

```
[> f:=(2*(x-1)^3)/((x-9));
```

$$f := \frac{2(x-1)^3}{x-9}$$

(8)

```
[> L:=ChangeOfVariables(L12,f);
```

$$L := 3 D x^3 (x-1)^2 (x-13)^2 (x-9)^4 + \frac{1}{37} ((6 \text{RootOf}(\_Z^2 + 1) - 1) (37 x^2 \quad (9)$$

$$+ 2592 \text{RootOf}(\_Z^2 + 1) - 962 x + 6685) (x-1) (x-13) (x-9)^3 D x^2) - (x - 9) (24 x^7 + 2 \text{RootOf}(\_Z^2 + 1) x^5 - 1320 x^6 - 122 \text{RootOf}(\_Z^2 + 1) x^4 + 28151 x^5 + 3252 \text{RootOf}(\_Z^2 + 1) x^3 - 287627 x^4 - 45908 \text{RootOf}(\_Z^2 + 1) x^2 + 1390494 x^3 + 331370 \text{RootOf}(\_Z^2 + 1) x - 2683358 x^2 - 952146 \text{RootOf}(\_Z^2 + 1) + 2046851 x - 50847) D x - 24 (x-13)^5 (x-1)^2$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$\text{ext} := \{\text{RootOf}(\_Z^2 + 1)\} \quad (10)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ } ),ext),{RootOf,name});

$$\text{ext} := \{\text{RootOf}(\_Z^2 + 1)\} \quad (11)$$

> extppp:={};

$$\text{extppp} := \emptyset \quad (12)$$

> E:= Singular(L,extppp);

$$E := [[x-13, 13], [\infty, \infty], [x-1, 1], [x-9, 9]] \quad (13)$$

> F:=NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-1, 1], [x-9, 9]] \quad (14)$$

> Sirr:=irrsinglF2(L,t,F,ext);

$$\text{Sirr} := \left[ [[\infty, \infty], [x-9, 9]], \left[ \left[ 1, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 \quad (15)$$

$$+ 1), \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1) \right], \left[ \frac{1}{2}, \frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\text{RootOf}(\_Z^2 + 1)}{2} \right],$$

$$\left[ \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{5}{3} + \text{RootOf}(\_Z^2 + 1), \right.$$

$$\left. \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{5}{3} + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \right], \left[ \frac{32}{\sqrt{t}} - \frac{5}{6} + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{32}{\sqrt{t}} - \frac{5}{6} \right.$$

$$+ \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{64}{\sqrt{t}} \left. \right], \left[ 1, \frac{1}{2} \right], [1, 2], \left[ \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1), 1 \right], \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1), 1 \right], \right.$$

$$\left. \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1) \right], \left[ \frac{32}{\sqrt{t}} - \frac{1}{3} \right.$$

$$\left. \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1) \right], \left[ \frac{32}{\sqrt{t}} - \frac{1}{3} \right.$$

$$\left. \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1) \right], \left[ \frac{32}{\sqrt{t}} - \frac{1}{3} \right.$$

$$\begin{aligned}
& + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, \frac{1}{2} \Big], \Big[ -\frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, \frac{1}{2} \Big], \Big[ -\frac{32}{\sqrt{t}} - \frac{1}{3} \\
& + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, \frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\text{RootOf}(\_Z^2 + 1)}{2} \Big] \Big], \Big[ [2 \text{RootOf}(\_Z^2 - 2, \text{index} \\
& = 1) t, 2 \text{RootOf}(\_Z^2 - 2, \text{index} = 2) t, -4 \text{RootOf}(\_Z^2 - 2, \text{index} = 1) t], [32 t, -32 t, \\
& -64 t] \Big], \Big[ \Big[ -\frac{5}{3} + \text{RootOf}(\_Z^2 + 1), -\frac{5}{3} + \text{RootOf}(\_Z^2 + 1), 0 \Big], \Big[ -\frac{5}{6} \\
& + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{5}{6} + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, 0 \Big] \Big], \Big[ [x - 1, 1], [[0, 2, \\
& -3 \text{RootOf}(\_Z^2 + 1) + 3], [2, -3 \text{RootOf}(\_Z^2 + 1) + 3, -3 \text{RootOf}(\_Z^2 + 1) + 1], [1, \\
& 1, 1], [[2, 0], [-3 \text{RootOf}(\_Z^2 + 1) + 3, 0], [-3 \text{RootOf}(\_Z^2 + 1) + 3, 2], 3] \Big] \Big]
\end{aligned}$$

**> Sreg:=regsingtrue1F2(L,t,Sirr[-1],ext);**

$$\text{Sreg} := \Big[ [[x - 1, 1], [[0, 2, -3 \text{RootOf}(\_Z^2 + 1) + 3], [2, -3 \text{RootOf}(\_Z^2 + 1) + 3, -3 \text{RootOf}(\_Z^2 + 1) + 1]], [[2, 0], [-3 \text{RootOf}(\_Z^2 + 1) + 3, 0], [-3 \text{RootOf}(\_Z^2 + 1) + 3, 2]] \Big] \Big] \quad (16)$$

**> RSreg:=Sregseptrue1F2(L,Sreg,ext);**

$$\text{RSreg} := \Big[ [[x - 1, 1], [[0, 2, -3 \text{RootOf}(\_Z^2 + 1) + 3], [[[-3 \text{RootOf}(\_Z^2 + 1) + 3, -3 \text{RootOf}(\_Z^2 + 1) + 1], [2]]], [ ], [ ] \Big] \Big] \quad (17)$$

**> R1:=IrrRegAppsing1F2(L,t,E,ext);**

$$\begin{aligned}
\text{R1} := & \Bigg[ \Big[ [\infty, \infty], [x - 9, 9], \Big[ \Big[ 1, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 1)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 \\
& + 1), \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 2)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1) \Big], \Big[ \frac{1}{2}, \frac{32}{\sqrt{t}} - \frac{1}{3} \\
& + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\text{RootOf}(\_Z^2 + 1)}{2} \Big] \Big], \\
& \Big[ \Big[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 1)}{t} - \frac{5}{3} + \text{RootOf}(\_Z^2 + 1), \\
& \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 2)}{t} - \frac{5}{3} + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 2)}{t} \\
& - \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 1)}{t} \Big], \Big[ \frac{32}{\sqrt{t}} - \frac{5}{6} + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{32}{\sqrt{t}} - \frac{5}{6} \\
& + \frac{\text{RootOf}(\_Z^2 + 1)}{2}, -\frac{64}{\sqrt{t}} \Big], \Big[ 1, \frac{1}{2} \Big], [1, 2], \Big[ \Big[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 1)}{t} - \frac{2}{3} \\
& + \text{RootOf}(\_Z^2 + 1), 1 \Big], \Big[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index} = 2)}{t} - \frac{2}{3} + \text{RootOf}(\_Z^2 + 1), 1 \Big],
\end{aligned} \quad (18)$$

$$\begin{aligned}
& \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1), \right. \\
& \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1) \right], \left[ \left[ \frac{32}{\sqrt{t}} - \frac{1}{3} \right. \right. \\
& \left. \left. + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, \frac{1}{2} \right], \left[ -\frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, \frac{1}{2} \right], \left[ -\frac{32}{\sqrt{t}} - \frac{1}{3} \right. \right. \\
& \left. \left. + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, \frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2} \right] \right], \left[ [2 \operatorname{RootOf}(\_Z^2 - 2, \text{index} \right. \\
& = 1) t, 2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2) t, -4 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1) t], [32 t, -32 t, \\
& -64 t], \left[ \left[ -\frac{5}{3} + \operatorname{RootOf}(\_Z^2 + 1), -\frac{5}{3} + \operatorname{RootOf}(\_Z^2 + 1), 0 \right], \left[ -\frac{5}{6} \right. \right. \\
& \left. \left. + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, -\frac{5}{6} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, 0 \right] \right], \left[ [x - 1, 1], [0, 2, \right. \\
& -3 \operatorname{RootOf}(\_Z^2 + 1) + 3], [[2, -3 \operatorname{RootOf}(\_Z^2 + 1) + 3, -3 \operatorname{RootOf}(\_Z^2 + 1) + 1]], \\
& [[ [2, 0], [-3 \operatorname{RootOf}(\_Z^2 + 1) + 3, 0], [-3 \operatorname{RootOf}(\_Z^2 + 1) + 3, 2]]], \left[ [ [x - 1, \right. \\
& 1], [0, 2, -3 \operatorname{RootOf}(\_Z^2 + 1) + 3], [[ -3 \operatorname{RootOf}(\_Z^2 + 1) + 3, -3 \operatorname{RootOf}(\_Z^2 \\
& + 1) + 1], [2]]], [ ], [ ]], \left[ [ [x - 13, 13], [0, 2, 4], [2, 4, 2], [[ [2, 0], [4, 0], [4, \right. \\
& 2]]], \left[ [[ \infty, \infty], [x - 1, 1], [x - 9, 9]], \left[ \left[ 1, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} \right. \right. \right. \\
& \left. \left. + \operatorname{RootOf}(\_Z^2 + 1), \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1) \right], [0, 2, \right. \\
& -3 \operatorname{RootOf}(\_Z^2 + 1) + 3], \left[ \frac{1}{2}, \frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, -\frac{32}{\sqrt{t}} - \frac{1}{3} \right. \\
& \left. \left. + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2} \right] \right], \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{5}{3} + \operatorname{RootOf}(\_Z^2 + 1), \right. \right. \\
& \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{5}{3} + \operatorname{RootOf}(\_Z^2 + 1), \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} \right. \right. \\
& \left. \left. - \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \right] \right], [2, -3 \operatorname{RootOf}(\_Z^2 + 1) + 3, -3 \operatorname{RootOf}(\_Z^2 + 1) \\
& + 1], \left[ \frac{32}{\sqrt{t}} - \frac{5}{6} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, -\frac{32}{\sqrt{t}} - \frac{5}{6} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, -\frac{64}{\sqrt{t}} \right],
\end{aligned}$$

$$\left[ \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1), 1 \right], \right. \right. \\ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1), 1 \right], \\ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1), \right. \\ \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{2}{3} + \operatorname{RootOf}(\_Z^2 + 1) \right] \right], \left[ [2, 0], [-3 \operatorname{RootOf}(\_Z^2 + 1) \right. \\ \left. + 1) + 3, 0], [-3 \operatorname{RootOf}(\_Z^2 + 1) + 3, 2] \right], \left[ \left[ \frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, \frac{1}{2} \right], \left[ \right. \right. \\ \left. \left. - \frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, \frac{1}{2} \right], \left[ -\frac{32}{\sqrt{t}} - \frac{1}{3} + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2}, \frac{32}{\sqrt{t}} - \frac{1}{3} \right. \right. \\ \left. \left. + \frac{\operatorname{RootOf}(\_Z^2 + 1)}{2} \right] \right] \right], \left[ [1, 1, 1], [1, 1, 1], [1, 2, 2] \right] \right]$$

**> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);**

$$\text{info1} := \left[ \left[ \left[ \infty, \frac{1}{x}, [2 x^2], 2, \{ \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1) \}, \{ \operatorname{RootOf}(\_Z^2 + 1), \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1) \} \right], \left[ 9, x - 9, \left[ \frac{1024}{x - 9} \right], 1, \emptyset, \{ \operatorname{RootOf}(\_Z^2 + 1) \} \right] \right], 3, 3, x - 9, 1 \right] \quad (19)$$

**> find1F2Irr(L,R1,info1,x,t,ext);**

$$\left[ \left[ \left[ \left\{ \frac{1}{18}, \frac{7}{18}, \frac{13}{18} \right\}, \left[ \frac{1}{3}, \operatorname{RootOf}(\_Z^2 + 1) + \frac{2}{3} \right], \left[ \left\{ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right\}, \left[ \frac{2}{3}, \operatorname{RootOf}(\_Z^2 + 1) \right. \right. \right. \right. \\ \left. \left. \left. + \frac{2}{3} \right] \right], \left[ \left\{ \frac{5}{18}, \frac{11}{18}, \frac{17}{18} \right\}, \left[ \frac{2}{3}, \operatorname{RootOf}(\_Z^2 + 1) \right], \left[ \left\{ \frac{5}{18}, \frac{11}{18}, \frac{17}{18} \right\}, \left[ \frac{1}{3}, \frac{1}{3} \right. \right. \right. \\ \left. \left. \left. + \operatorname{RootOf}(\_Z^2 + 1) \right] \right], \left[ \left\{ \frac{1}{18}, \frac{7}{18}, \frac{13}{18} \right\}, \left[ \frac{2}{3}, \frac{1}{3} + \operatorname{RootOf}(\_Z^2 + 1) \right], \left[ \left\{ \frac{1}{2}, \frac{1}{6}, \right. \right. \right. \\ \left. \left. \left. \frac{5}{6} \right\}, \left[ \frac{1}{3}, \operatorname{RootOf}(\_Z^2 + 1) \right], \frac{2 (x - 1)^3}{x - 9} \right], \left[ \left[ \left\{ \frac{1}{18}, \frac{7}{18}, \frac{13}{18} \right\}, \left[ \frac{1}{3}, \operatorname{RootOf}(\_Z^2 + 1) \right. \right. \right. \right. \\ \left. \left. \left. + \frac{2}{3} \right] \right], \left[ \left\{ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right\}, \left[ \frac{2}{3}, \operatorname{RootOf}(\_Z^2 + 1) + \frac{2}{3} \right], \left[ \left\{ \frac{5}{18}, \frac{11}{18}, \frac{17}{18} \right\}, \left[ \frac{2}{3}, \right. \right. \right. \\ \left. \left. \left. \operatorname{RootOf}(\_Z^2 + 1) \right] \right], \left[ \left\{ \frac{5}{18}, \frac{11}{18}, \frac{17}{18} \right\}, \left[ \frac{1}{3}, \frac{1}{3} + \operatorname{RootOf}(\_Z^2 + 1) \right], \left[ \left\{ \frac{1}{18}, \frac{7}{18}, \right. \right. \right. \\ \left. \left. \left. \frac{13}{18} \right\}, \left[ \frac{2}{3}, \frac{1}{3} + \operatorname{RootOf}(\_Z^2 + 1) \right], \left[ \left\{ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right\}, \left[ \frac{1}{3}, \operatorname{RootOf}(\_Z^2 + 1) \right], \right. \right. \\ \left. \left. \left. - \frac{2 (x - 1)^3}{x - 9} \right] \right] \right] \right] \right] \quad (20)$$

**> TIME :=time();**  
**Hyp1F2Solutions(L);**  
**time() - TIME;**

TIME := 39.281

$$\left[ \left\{ \left\{ \left[ \left[ \frac{1}{6} \right], \left[ \frac{2}{3}, \text{RootOf}(\_Z^2 + 1) + \frac{2}{3} \right], \left[ -\frac{2}{3(x-9)} \right], \left[ \frac{(x-1)(x-9)^3 Dx^2}{(x-13)^2} \right. \right. \right. \right. \\
+ \frac{1}{183(x-13)^3} \left( (6 \text{RootOf}(\_Z^2 + 1) + 5) (61x^4 - 2444x^3 + 35638x^2 \right. \\
- 288 \text{RootOf}(\_Z^2 + 1)x^3 - 225468x + 9792 \text{RootOf}(\_Z^2 + 1)x^2 + 523989 \\
+ 373248 \text{RootOf}(\_Z^2 + 1) - 106272 \text{RootOf}(\_Z^2 + 1)x) Dx) \\
+ \frac{1}{3(x-13)^3} \left( 4 \text{RootOf}(\_Z^2 + 1) (2 \text{RootOf}(\_Z^2 + 1)x^2 + x^3 - 20 \text{RootOf}(\_Z^2 \\
+ 1)x - 35x^2 + 18 \text{RootOf}(\_Z^2 + 1) + 403x - 1521) \right) \left. \left. \left. \right. \right\}, \frac{2(x-1)^3}{x-9} \right] \Bigg\} \\
10.672 \tag{21}$$

[Here are another examples related to the 1F2 type solutions. Those examples are not in my PhD thesis.

```
[> ##### THE EASY CASE #####
```

$$\begin{aligned}
& > \mathbf{F} := \text{sumdiffeq}(\text{hyperterm}([\mathbf{a1}], [\mathbf{b1}, \mathbf{b2}], \mathbf{x}, \mathbf{k}), \mathbf{k}, \mathbf{J}(\mathbf{x})); \\
& F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \tag{22} \\
& = 0
\end{aligned}$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \tag{23}$$

```
> L12:=subs({a1=1/5,b1=1/3,b2=1/2},LA);
```

$$L12 := x^2 Dx^3 + \frac{11 Dx^2 x}{6} + \left( \frac{1}{6} - x \right) Dx - \frac{1}{5} \tag{24}$$

```
> f:=(2*(x-1)^2*(x-7))/((x-12));
```

$$f := \frac{2(x-1)^2(x-7)}{x-12} \tag{25}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$\begin{aligned}
L := & 30 Dx^3 (x-1) (x-7)^2 (x-12)^4 (2x^2 - 43x + 173)^2 + 5 (8x^4 - 344x^3 + 3327x^2 \\
& - 14366x + 46223) Dx^2 (x-7) (x-12)^3 (2x^2 - 43x + 173) - 15 (64x^{10} - 6016x^9 \\
& + 244128x^8 - 5608224x^7 + 80433302x^6 - 749439912x^5 + 4572786787x^4 \\
& - 17910590021x^3 + 42503518443x^2 - 53550275639x + 25143269248) Dx (x-12) \\
& - 12 (2x^2 - 43x + 173)^5 \tag{26}
\end{aligned}$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\mathbf{L}, \{\text{RootOf}, \text{name}\}) \text{ minus } \{\mathbf{x}, \text{Dx}\}; \\ &\text{ext} := \emptyset \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\text{map}(\mathbf{s} \rightarrow \text{ReplirrRoot}(\mathbf{s}, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\ &\text{ext} := \emptyset \end{aligned} \quad (28)$$

$$\begin{aligned} &> \text{extppp} := \{\}; \\ &\text{extppp} := \emptyset \end{aligned} \quad (29)$$

$$\begin{aligned} &> \mathbf{E} := \text{Singular}(\mathbf{L}, \text{extppp}); \\ \mathbf{E} := &\left[ \left[ x^2 - \frac{43}{2}x + \frac{173}{2}, \text{RootOf}(2\_Z^2 - 43\_Z + 173) \right], [\infty, \infty], [x - 7, 7], [x - 12, \right. \\ &\left. 12], [x - 1, 1] \right] \end{aligned} \quad (30)$$

$$\begin{aligned} &> \mathbf{F} := \text{NotAppSing}(\mathbf{L}, \mathbf{E}, \text{ext}); \\ \mathbf{F} := &[[x - 12, 12], [\infty, \infty], [x - 1, 1], [x - 7, 7]] \end{aligned} \quad (31)$$

$$> \text{Sirr} := \text{irringsinglF2}(\mathbf{L}, \mathbf{t}, \mathbf{F}, \text{ext});$$

$$\begin{aligned} \text{Sirr} := &\left[ [[x - 12, 12], [\infty, \infty]], \left[ \left[ \frac{1}{5}, \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \frac{2}{5}, \right. \right. \\ &\left. \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{2}{15}, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{2}{15} \right], \left[ \left[ \frac{11\sqrt{10}}{\sqrt{t}} - \frac{2}{15}, -\frac{11\sqrt{10}}{\sqrt{t}} - \frac{2}{15}, -\frac{22\sqrt{10}}{\sqrt{t}} \right], \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \right. \right. \\ &- \frac{4}{15}, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{4}{15}, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} \\ &- \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \left. \right], \left[ \frac{1}{2}, 1 \right], [2, 1], \left[ \left[ \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ \right. \right. \\ &- \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \left. \right], \left[ -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{2}{15}, \frac{2}{5} \right], \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{2}{15}, \right. \right. \\ &\frac{2}{5} \left. \right], \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{2}{15}, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{2}{15} \right] \left. \right], \\ &[[11\sqrt{10}t, -11\sqrt{10}t, -22\sqrt{10}t], [2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)t, 2 \text{RootOf}(\_Z^2 \\ &- 2, \text{index}=2)t, -4 \text{RootOf}(\_Z^2 - 2, \text{index}=1)t], \left[ \left[ -\frac{2}{15}, -\frac{2}{15}, 0 \right], \left[ -\frac{4}{15}, -\frac{4}{15}, \right. \right. \\ &0 \left. \right], \left[ [[x - 1, 1], [x - 7, 7]], \left[ \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 1, \frac{4}{3}, \frac{1}{3} \right], [1, 1, 1], \left[ [1, 0], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, \right. \right. \right. \right. \right. \\ &1 \left. \right], 3 \left. \right], \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], [1, 1, 1], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right], 2 \right] \left. \right] \right] \end{aligned} \quad (32)$$

$$\begin{aligned} &> \text{Sreg} := \text{regsingtruelF2}(\mathbf{L}, \mathbf{t}, \text{Sirr}[-1], \text{ext}); \\ \text{Sreg} := &\left[ [[x - 1, 1], [x - 7, 7]], \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ 1, \frac{4}{3}, \frac{1}{3} \right], \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right] \right], \end{aligned} \quad (33)$$

$$\left[ \left[ [1, 0], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, 1 \right] \right], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right] \right]$$

**> RSreg:=Sregseptrue1F2(L,Sreg,ext);**

$$RSreg := \left[ \left[ [x-1, 1], [x-7, 7] \right], \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ \left[ \frac{4}{3}, \frac{1}{3} \right], [1] \right], \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], [ ] \right] \right] \right], [ ], [ ] \quad (34)$$

**> R1:=IrrRegAppsing1F2(L,t,E,ext);**

$$R1 := \left[ \left[ [x-12, 12], [\infty, \infty] \right], \left[ \left[ \frac{1}{5}, \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \frac{2}{5}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=1)}{t} + \frac{2}{15}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=2)}{t} + \frac{2}{15} \right] \right], \right. \\ \left[ \left[ \frac{11\sqrt{10}}{\sqrt{t}} - \frac{2}{15}, -\frac{11\sqrt{10}}{\sqrt{t}} - \frac{2}{15}, -\frac{22\sqrt{10}}{\sqrt{t}} \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=1)}{t} - \frac{4}{15}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=2)}{t} - \frac{4}{15}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=2)}{t} - \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=1)}{t} \right] \right], \\ \left[ \frac{1}{2}, 1 \right], [2, 1], \left[ \left[ \left[ \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right] \right], \left[ -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15} \right] \right], \\ \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=1)}{t} + \frac{2}{15}, \frac{2}{5} \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=2)}{t} + \frac{2}{15}, \frac{2}{5} \right] \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=2)}{t} + \frac{2}{15}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, index=1)}{t} + \frac{2}{15} \right] \right], \\ [ [11\sqrt{10} t, -11\sqrt{10} t, -22\sqrt{10} t], [2 \operatorname{RootOf}(\_Z^2 - 2, index=1) t, 2 \operatorname{RootOf}(\_Z^2 - 2, index=2) t, -4 \operatorname{RootOf}(\_Z^2 - 2, index=1) t] ], \\ \left[ \left[ -\frac{2}{15}, -\frac{2}{15}, 0 \right], \left[ -\frac{4}{15}, -\frac{4}{15}, 0 \right] \right], \left[ [x-1, 1], [x-7, 7] \right], \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ 1, \frac{4}{3}, \frac{1}{3} \right], \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right] \right], \\ \left[ \left[ [1, 0], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, 1 \right] \right], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right] \right], \left[ [x-1, 1], [x-7, 7] \right], \\ \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ \left[ \frac{4}{3}, \frac{1}{3} \right], [1] \right], \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], [ ] \right] \right], [ ], [ ] \right], \left[ \left[ x^2 - \frac{43}{2} x + \frac{173}{2}, \operatorname{RootOf}(2\_Z^2 - 43\_Z + 173) \right] \right], [ [0, 2, 4], [2, 4, 2], [[2, 0], [4, 2, 0], [4, 2, 0], [4, 2, 0]] ], [ ] \quad (35)$$



$$\begin{aligned}
& 0], [4, 2]]], \left[ [[x-12, 12], [\infty, \infty], [x-1, 1], [x-7, 7]], \left[ \left[ \frac{1}{5}, \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \right. \right. \right. \\
& \left. \left. \left. - \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \frac{2}{5}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{2}{15}, \right. \right. \right. \\
& \left. \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{2}{15} \right], \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ \frac{11\sqrt{10}}{\sqrt{t}} - \frac{2}{15}, \right. \right. \right. \\
& \left. \left. \left. - \frac{11\sqrt{10}}{\sqrt{t}} - \frac{2}{15}, -\frac{22\sqrt{10}}{\sqrt{t}} \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} - \frac{4}{15}, \right. \right. \right. \\
& \left. \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{4}{15}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} \right. \right. \right. \\
& \left. \left. \left. - \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \right], \left[ 1, \frac{4}{3}, \frac{1}{3} \right], \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right] \right], \left[ \left[ \left[ \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1}{5} \right], \left[ -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ -\frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15}, \frac{11\sqrt{10}}{\sqrt{t}} + \frac{1}{15} \right] \right], \right. \\
& \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{2}{15}, \frac{2}{5} \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{2}{15}, \right. \right. \\
& \left. \left. \frac{2}{5} \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{2}{15}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{2}{15} \right] \right], \\
& \left[ [1, 0], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, 1 \right] \right], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right], [[1, 2, 2], [1, 1, 1], [1, 1, 1], \\
& [1, 1, 1]]]
\end{aligned}$$

```
> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);
```

$$\text{info1} := \left[ \left[ \left[ 12, x-12, \left[ \frac{1210}{x-12} \right], 1, \emptyset, \emptyset \right], \left[ \infty, \frac{1}{x}, [2x^2], 2, \{ \operatorname{RootOf}(\_Z^2 - 2, \text{index} \right. \right. \right. \quad (36)$$

$$= 1) \}, \{ \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1) \} \right], 3, 4, x-12, 1 ]$$

```
> easy1F2(L,R1,info1,x,t,ext);
```

$$\left[ \left[ \left[ \left\{ \frac{1}{5}, \frac{8}{15}, \frac{13}{15} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], -\frac{2(x-1)^2(x-7)}{x-12} \right], \left[ \left[ \left\{ \frac{1}{5}, \frac{8}{15}, \frac{13}{15} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], \right. \quad (37)$$

$$\left. \frac{2(x-1)^2(x-7)}{x-12} \right]$$

```
> find1F2Rat(L,R1,info1,x,t,T,ext);
```

$$\left[ \left[ \left[ \left\{ \frac{1}{5}, \frac{8}{15}, \frac{13}{15} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], \frac{2(x-1)^2(x-7)}{x-12} \right] \right] \quad (38)$$

```
> TIME :=time();
Hyp1F2Solutions(L);
time() - TIME;
```

TIME := 52.031

$$\left[ \left[ \left[ \left[ \frac{1}{5} \right], \left[ \frac{1}{2}, \frac{1}{3} \right], [0], [1] \right] \right], \frac{2(x-1)^2(x-7)}{x-12} \right] \right] \\ 0.297 \quad (39)$$

$$\begin{aligned} & \text{> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));} \\ F &:= \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \\ &= 0 \end{aligned} \quad (40)$$

$$\begin{aligned} & \text{> LA:=de2diffop(F,J(x));} \\ LA &:= x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \end{aligned} \quad (41)$$

$$\begin{aligned} & \text{> L12:=subs({a1=-1,b1=1/3,b2=1/2},LA);} \\ L12 &:= x^2 Dx^3 + \frac{11 Dx^2 x}{6} + \left( \frac{1}{6} - x \right) Dx + 1 \end{aligned} \quad (42)$$

$$\begin{aligned} & \text{> f:=(2*(x-1)*(x-3)^2)/((x-7));} \\ f &:= \frac{2(x-1)(x-3)^2}{x-7} \end{aligned} \quad (43)$$

$$\begin{aligned} & \text{> L:=ChangeOfVariables(L12,f);} \\ L &:= 3 Dx^3 (x-1)^2 (x-3) (x-7)^4 (x^2 - 11x + 16)^2 + (2x^4 - 44x^3 + 171x^2 - 326x \\ &+ 593) Dx^2 (x-1) (x-7)^3 (x^2 - 11x + 16) - 3 (8x^{10} - 384x^9 + 7752x^8 - 85824x^7 \\ &+ 572149x^6 - 2390640x^5 + 6365511x^4 - 10752200x^3 + 11139483x^2 - 6475544x \\ &+ 1617097) Dx (x-7) + 48 (x^2 - 11x + 16)^5 \end{aligned} \quad (44)$$

$$\begin{aligned} & \text{> ext:=indets(L,{RootOf,name}) minus {x,Dx};} \\ ext &:= \emptyset \end{aligned} \quad (45)$$

$$\begin{aligned} & \text{> ext:= indets(map(s-> ReplirrRoot(s,{ } ),ext),{RootOf,name});} \\ ext &:= \emptyset \end{aligned} \quad (46)$$

$$\begin{aligned} & \text{> extppp:={};} \\ extppp &:= \emptyset \end{aligned} \quad (47)$$

$$\begin{aligned} & \text{> E:= Singular(L,extppp);} \\ E &:= [[\infty, \infty], [x^2 - 11x + 16, \text{RootOf}(\_Z^2 - 11\_Z + 16)], [x - 3, 3], [x - 7, 7], [x - 1, \\ &1]] \end{aligned} \quad (48)$$

$$\begin{aligned} & \text{> F:=NotAppSing(L,E,ext);} \\ F &:= [[x - 3, 3], [\infty, \infty], [x - 1, 1], [x - 7, 7]] \end{aligned} \quad (49)$$

$$\begin{aligned} & \text{> Sirr:=irrsglF2(L,t,F,ext);} \\ Sirr &:= \left[ [[\infty, \infty], [x - 7, 7]], \left[ \left[ -2, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{4}{3}, \right. \right. \right. \\ & \left. \left. \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{4}{3} \right], \left[ -1, \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3} \right] \right], \\ & \left[ \left[ \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{10}{3}, \frac{2 \text{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{10}{3}, \right. \right. \end{aligned} \quad (50)$$

$$\begin{aligned}
& \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \Bigg], \left[ \frac{8\sqrt{3}}{\sqrt{t}} + \frac{5}{3}, \right. \\
& \left. -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{5}{3}, -\frac{16\sqrt{3}}{\sqrt{t}} \right] \Bigg], \left[ 1, \frac{1}{2} \right], [1, 2], \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{4}{3}, \right. \right. \\
& \left. \left. -2 \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{4}{3}, -2 \right], \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{4}{3}, \right. \right. \\
& \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{4}{3} \right] \right], \left[ \left[ \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -1 \right], \left[ -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -1 \right], \left[ \right. \right. \\
& \left. \left. -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3} \right] \right] \Bigg], \left[ [2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1) t, 2 \operatorname{RootOf}(\_Z^2 - 2, \right. \\
& \text{index}=2) t, -4 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1) t], [8\sqrt{3} t, -8\sqrt{3} t, -16\sqrt{3} t], \left[ \left[ \frac{10}{3}, \right. \right. \\
& \left. \left. \frac{10}{3}, 0 \right], \left[ \frac{5}{3}, \frac{5}{3}, 0 \right] \right], \left[ [x-3, 3], [x-1, 1], \left[ \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 1, \frac{4}{3}, \frac{1}{3} \right], [1, 1, 1], \left[ [1, \right. \right. \right. \right. \\
& \left. \left. \left. 0, \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, 1 \right] \right], 3 \right], \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], [1, 1, 1], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{2}{3} \right] \right], 2 \right] \right] \Bigg]
\end{aligned}$$

**> Sreg:=regsingtrue1F2(L,t,Sirr[-1],ext);**

$$\begin{aligned}
Sreg := & \left[ [x-3, 3], [x-1, 1], \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ 1, \frac{4}{3}, \frac{1}{3} \right], \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right] \right], \right. \\
& \left. \left[ \left[ 1, 0 \right], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, 1 \right] \right], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right] \right] \quad (51)
\end{aligned}$$

**> RSreg:=SregseptruelF2(L,Sreg,ext);**

$$\begin{aligned}
RSreg := & \left[ \left[ [x-3, 3], [x-1, 1], \left[ \left[ 0, 1, \frac{4}{3} \right], \left[ 0, \frac{2}{3}, \frac{1}{2} \right] \right], \left[ \left[ \left[ \frac{4}{3}, \frac{1}{3} \right], [1] \right], \left[ \left[ \frac{2}{3}, \frac{1}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{1}{6} \right], [ ] \right] \right], [ ], [ ] \right] \quad (52)
\end{aligned}$$

**> R1:=IrrRegAppsing1F2(L,t,E,ext);**

$$\begin{aligned}
R1 := & \left[ \left[ [\infty, \infty], [x-7, 7], \left[ -2, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{4}{3}, \right. \right. \right. \\
& \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{4}{3} \right], \left[ -1, \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3} \right] \right], \\
& \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{10}{3}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} + \frac{10}{3}, \right. \right. \\
& \left. \left. \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=2)}{t} - \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} \right], \left[ \frac{8\sqrt{3}}{\sqrt{t}} + \frac{5}{3}, \right. \right. \\
& \left. \left. -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{5}{3}, -\frac{16\sqrt{3}}{\sqrt{t}} \right] \right] \Bigg], \left[ 1, \frac{1}{2} \right], [1, 2], \left[ \left[ \left[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \text{index}=1)}{t} + \frac{4}{3}, \right. \right. \right. \quad (53)
\end{aligned}$$

$$\begin{aligned}
& -2 \Big], \Big[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} + \frac{4}{3}, -2 \Big], \Big[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} + \frac{4}{3}, \\
& \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1)}{t} + \frac{4}{3} \Big], \Big[ \Big[ \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -1 \Big], \Big[ -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -1 \Big], \Big[ \\
& -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3} \Big] \Big], \Big[ [2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1) \, t, 2 \operatorname{RootOf}(\_Z^2 - 2, \\
& \operatorname{index}=2) \, t, -4 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1) \, t], [8\sqrt{3} \, t, -8\sqrt{3} \, t, -16\sqrt{3} \, t], \Big[ \Big[ \frac{10}{3}, \\
& \frac{10}{3}, 0 \Big], \Big[ \frac{5}{3}, \frac{5}{3}, 0 \Big] \Big], \Big[ [[x-3, 3], [x-1, 1]], \Big[ \Big[ 0, 1, \frac{4}{3} \Big], \Big[ 0, \frac{2}{3}, \frac{1}{2} \Big] \Big], \Big[ \Big[ 1, \frac{4}{3}, \frac{1}{3} \Big], \\
& \Big[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \Big] \Big], \Big[ \Big[ [1, 0], \Big[ \frac{4}{3}, 0 \Big], \Big[ \frac{4}{3}, 1 \Big] \Big], \Big[ \Big[ \frac{2}{3}, 0 \Big], \Big[ \frac{1}{2}, 0 \Big], \Big[ \frac{1}{2}, \frac{2}{3} \Big] \Big] \Big], \Big[ \Big[ [x-3, \\
& 3], [x-1, 1] \Big], \Big[ \Big[ 0, 1, \frac{4}{3} \Big], \Big[ 0, \frac{2}{3}, \frac{1}{2} \Big] \Big], \Big[ \Big[ \Big[ \frac{4}{3}, \frac{1}{3} \Big], [1] \Big], \Big[ \Big[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \Big], [\ ] \Big] \Big], [\ ] \\
& [\ ] \Big], \Big[ [[x^2 - 11x + 16, \operatorname{RootOf}(\_Z^2 - 11\_Z + 16)], [[0, 2, 4]], [[2, 4, 2]], [[2, 0], \\
& [4, 0], [4, 2]] \Big], \Big[ [[x-3, 3], [\infty, \infty], [x-1, 1], [x-7, 7]], \Big[ \Big[ 0, 1, \frac{4}{3} \Big], \Big[ -2, \\
& \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1)}{t} + \frac{4}{3}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} + \frac{4}{3} \Big], \Big[ 0, \frac{2}{3}, \frac{1}{2} \Big], \\
& \Big[ -1, \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3} \Big], \Big[ \Big[ 1, \frac{4}{3}, \frac{1}{3} \Big], \Big[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1)}{t} \\
& + \frac{10}{3}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} + \frac{10}{3}, \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} \\
& - \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1)}{t} \Big], \Big[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \Big], \Big[ \frac{8\sqrt{3}}{\sqrt{t}} + \frac{5}{3}, -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{5}{3}, \\
& -\frac{16\sqrt{3}}{\sqrt{t}} \Big], \Big[ \Big[ [1, 0], \Big[ \frac{4}{3}, 0 \Big], \Big[ \frac{4}{3}, 1 \Big] \Big], \Big[ \Big[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1)}{t} + \frac{4}{3}, -2 \Big], \\
& \Big[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} + \frac{4}{3}, -2 \Big], \Big[ \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=2)}{t} + \frac{4}{3}, \\
& \frac{2 \operatorname{RootOf}(\_Z^2 - 2, \operatorname{index}=1)}{t} + \frac{4}{3} \Big], \Big[ \Big[ \frac{2}{3}, 0 \Big], \Big[ \frac{1}{2}, 0 \Big], \Big[ \frac{1}{2}, \frac{2}{3} \Big] \Big], \Big[ \Big[ \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, \\
& -1 \Big], \Big[ -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, -1 \Big], \Big[ -\frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3}, \frac{8\sqrt{3}}{\sqrt{t}} + \frac{2}{3} \Big] \Big], [[1, 1, 1], [1, 1, 1], [1, 1,
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. 1], [1, 2, 2] \right] \right] \right] \\
& \text{info1} := \left[ \left[ \left[ \infty, \frac{1}{x}, [2x^2], 2, \{ \text{RootOf}(\_Z^2 - 2, \text{index}=1) \}, \{ \text{RootOf}(\_Z^2 - 2, \text{index}=1) \} \right], \right. \right. \\
& \quad \left. \left. \left[ 7, x-7, \left[ \frac{192}{x-7} \right], 1, \emptyset, \emptyset \right], 3, 4, x-7, 1 \right] \right] \\
& \text{easy1F2(L,R1,info1,x,t,ext);} \\
& \left[ \left[ \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], \frac{2(x-1)(x-3)^2}{x-7} \right], \left[ \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], \right. \right. \\
& \quad \left. \left. - \frac{2(x-1)(x-3)^2}{x-7} \right] \right] \\
& \text{find1F2Rat(L,R1,info1,x,t,T,ext);} \\
& \left[ \left[ \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], \frac{2(x-1)(x-3)^2}{x-7} \right] \right] \\
& \text{TIME := time();} \\
& \text{Hyp1F2Solutions(L);} \\
& \text{time() - TIME;} \\
& \text{TIME := 54.296} \\
& \left\{ \left[ \left[ \left[ [-1], \left[ \frac{1}{2}, \frac{1}{3} \right], [0], [1] \right] \right], \frac{2(x-1)(x-3)^2}{x-7} \right] \right\} \\
& 0.360
\end{aligned}
\tag{54}$$

$$\begin{aligned}
& \text{F:=sumdiffEq(hyperterm([a1],[b1,b2],x,k),k,J(x));} \\
& F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \\
& = 0
\end{aligned}
\tag{58}$$

$$\begin{aligned}
& \text{LA:=de2diffop(F,J(x));} \\
& LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1
\end{aligned}
\tag{59}$$

$$\begin{aligned}
& \text{L12:=subs({a1=1/2,b1=1/3,b2=1/4},LA);} \\
& L12 := x^2 Dx^3 + \frac{19 Dx^2 x}{12} + \left( \frac{1}{12} - x \right) Dx - \frac{1}{2}
\end{aligned}
\tag{60}$$

$$\begin{aligned}
& \text{f:=(2*(x-1)^2*(x-7)^3*(x-3))/((x-9)^2*(x-12)^3);} \\
& f := \frac{2(x-1)^2(x-7)^3(x-3)}{(x-9)^2(x-12)^3}
\end{aligned}
\tag{61}$$

$$\begin{aligned}
& \text{L:=ChangeOfVariables(L12,f);} \\
& L := 12 Dx^3 (x-1)^2 (x-7)^2 (x-3)^2 (x-9)^5 (x-12)^6 (x^4 - 60 x^3 + 830 x^2 - 3852 x \\
& \quad + 5193)^2 + (19 x^8 - 2280 x^7 + 56020 x^6 - 508920 x^5 + 1130098 x^4 + 9445608 x^3 \\
& \quad - 61260588 x^2 + 120205944 x - 64605357) Dx^2 (x-1) (x-7) (x-3) (x-9)^4 (x \\
& \quad - 12)^5 (x^4 - 60 x^3 + 830 x^2 - 3852 x + 5193) - (24 x^{22} - 6385 x^{21} + 754350 x^{20}
\end{aligned}
\tag{62}$$

$$\begin{aligned}
& -52579365 x^{19} + 2426921300 x^{18} - 79115657228 x^{17} + 1899893943080 x^{16} \\
& - 34621839407788 x^{15} + 489210278977592 x^{14} - 5445522690713510 x^{13} \\
& + 48307532521723644 x^{12} - 344411425921517550 x^{11} + 1985433017334660048 x^{10} \\
& - 9293797910485299948 x^9 + 35419917139640211600 x^8 - 109972429379040566988 x^7 \\
& + 277304017109700242160 x^6 - 562148970650500351761 x^5 \\
& + 895891243076785476198 x^4 - 1076867910973553914485 x^3 \\
& + 909141792620260982940 x^2 - 474652399511244981600 x \\
& + 114341628182233769784) Dx (x-9) (x-12) - 12 (x^4 - 60 x^3 + 830 x^2 \\
& - 3852 x + 5193)^5 (x-1) (x-7)^2
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:=indets(L,{RootOf,name}) minus \{x,Dx\};} \\
& \text{ext := } \emptyset \tag{63}
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:= indets(map(s-> ReplirrRoot(s,\{ \}),ext),\{RootOf,name\});} \\
& \text{ext := } \emptyset \tag{64}
\end{aligned}$$

$$\begin{aligned}
& \text{> extppp:=\{ \};} \\
& \text{extppp := } \emptyset \tag{65}
\end{aligned}$$

$$\begin{aligned}
& \text{> E:= Singular(L,extppp);} \\
& E := \left[ [\infty, \infty], [x-3, 3], [x-7, 7], [x^4 - 60 x^3 + 830 x^2 - 3852 x + 5193, \text{RootOf}(\_Z^4 \right. \\
& \quad \left. - 60 \_Z^3 + 830 \_Z^2 - 3852 \_Z + 5193)], [x-12, 12], [x-1, 1], [x-9, 9] \right] \tag{66}
\end{aligned}$$

$$\begin{aligned}
& \text{> F:=NotAppSing(L,E,ext);} \\
& F := [[x-3, 3], [x-9, 9], [\infty, \infty], [x-1, 1], [x-12, 12], [x-7, 7]] \tag{67}
\end{aligned}$$

$$\begin{aligned}
& \text{> Sirr:=irrsinglF2(L,t,F,ext);} \\
& Sirr := \left[ [[x-9, 9], [\infty, \infty], [x-12, 12]], \left[ \left[ 1, \frac{64 \text{RootOf}(\_Z^2 + 2, index=1)}{3 t} - \frac{5}{12}, \right. \right. \right. \\
& \quad \left. \frac{64 \text{RootOf}(\_Z^2 + 2, index=2)}{3 t} - \frac{5}{12} \right], \left[ \frac{1}{2}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24} \right], \left[ \frac{3}{2}, \right. \\
& \quad \left. \frac{165 \sqrt{10}}{t^{3/2}} + \frac{56 \sqrt{10}}{9 \sqrt{t}} - \frac{5}{8}, -\frac{165 \sqrt{10}}{t^{3/2}} - \frac{56 \sqrt{10}}{9 \sqrt{t}} - \frac{5}{8} \right] \Bigg], \\
& \left[ \left[ \frac{64 \text{RootOf}(\_Z^2 + 2, index=1)}{3 t} - \frac{17}{12}, \frac{64 \text{RootOf}(\_Z^2 + 2, index=2)}{3 t} - \frac{17}{12}, \right. \right. \\
& \quad \left. \frac{64 \text{RootOf}(\_Z^2 + 2, index=2)}{3 t} - \frac{64 \text{RootOf}(\_Z^2 + 2, index=1)}{3 t} \right], \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{17}{24}, \right. \\
& \quad \left. -\frac{\sqrt{2}}{\sqrt{t}} - \frac{17}{24}, -\frac{2 \sqrt{2}}{\sqrt{t}} \right], \left[ \frac{165 \sqrt{10}}{t^{3/2}} + \frac{56 \sqrt{10}}{9 \sqrt{t}} - \frac{17}{8}, -\frac{165 \sqrt{10}}{t^{3/2}} - \frac{56 \sqrt{10}}{9 \sqrt{t}} \right. \\
& \quad \left. - \frac{17}{8}, -\frac{330 \sqrt{10}}{t^{3/2}} - \frac{112 \sqrt{10}}{9 \sqrt{t}} \right] \Bigg], \left[ 1, \frac{1}{2}, \frac{3}{2} \right], [1, 2, 2], \\
& \left[ \left[ \left[ \frac{64 \text{RootOf}(\_Z^2 + 2, index=1)}{3 t} - \frac{5}{12}, 1 \right], \left[ \frac{64 \text{RootOf}(\_Z^2 + 2, index=2)}{3 t} - \frac{5}{12}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 1], \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} - \frac{5}{12}, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} - \frac{5}{12} \right], \\
& \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{1}{2} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{1}{2} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24} \right] \right], \left[ \left[ \frac{165\sqrt{10}}{t^{3/2}} \right. \right. \\
& + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{3}{2} \right], \left[ -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{3}{2} \right], \left[ -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} \right. \\
& - \frac{5}{8}, \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8} \left. \right] \left. \right], \left[ \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)t}{3}, \right. \right. \\
& \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)t}{3}, -\frac{128 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)t}{3} \left. \right], [\sqrt{2}t, -\sqrt{2}t, \\
& -2\sqrt{2}t], \left[ \frac{\sqrt{10}t(1485t^2 + 56)}{9}, -\frac{\sqrt{10}t(1485t^2 + 56)}{9}, \right. \\
& \left. -\frac{2\sqrt{10}(1485t^3 + 56t)}{9} \right], \left[ \left[ -\frac{17}{12}, -\frac{17}{12}, 0 \right], \left[ -\frac{17}{24}, -\frac{17}{24}, 0 \right], \left[ -\frac{17}{8}, -\frac{17}{8}, 0 \right] \right], \\
& \left[ [x-3, 3], [x-1, 1], [x-7, 7] \right], \left[ \left[ \left[ 0, \frac{2}{3}, \frac{3}{4} \right], \left[ \frac{2}{3}, \frac{3}{4}, \frac{1}{12} \right], [1, 1, 1], \left[ \left[ \frac{2}{3}, 0 \right], \right. \right. \right. \\
& \left[ \frac{3}{4}, 0 \right], \left[ \frac{3}{4}, \frac{2}{3} \right] \left. \right], 2 \right], \left[ \left[ 0, \frac{3}{2}, \frac{4}{3} \right], \left[ \frac{3}{2}, \frac{4}{3}, -\frac{1}{6} \right], [1, 1, 1], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, \right. \right. \right. \\
& \left. \left. \frac{3}{2} \right] \right], 2 \right], \left[ \left[ 0, 2, \frac{9}{4} \right], \left[ 2, \frac{9}{4}, \frac{1}{4} \right], [1, 1, 1], \left[ [2, 0], \left[ \frac{9}{4}, 0 \right], \left[ \frac{9}{4}, 2 \right] \right], 3 \right] \left. \right] \left. \right]
\end{aligned}$$

**> Sreg:=regsingtrue1F2(L,t,Sirr[-1],ext);**

$$\begin{aligned}
Sreg := & \left[ [x-3, 3], [x-1, 1], [x-7, 7] \right], \left[ \left[ 0, \frac{2}{3}, \frac{3}{4} \right], \left[ 0, \frac{3}{2}, \frac{4}{3} \right], \left[ 0, 2, \frac{9}{4} \right] \right], \left[ \left[ \frac{2}{3}, \right. \right. \\
& \frac{3}{4}, \frac{1}{12} \right], \left[ \frac{3}{2}, \frac{4}{3}, -\frac{1}{6} \right], \left[ 2, \frac{9}{4}, \frac{1}{4} \right] \left. \right], \left[ \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{3}{4}, 0 \right], \left[ \frac{3}{4}, \frac{2}{3} \right] \right], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{4}{3}, 0 \right], \right. \right. \\
& \left. \left[ \frac{4}{3}, \frac{3}{2} \right] \right], \left[ [2, 0], \left[ \frac{9}{4}, 0 \right], \left[ \frac{9}{4}, 2 \right] \right] \left. \right] \left. \right]
\end{aligned} \tag{69}$$

**> RSreg:=SregseptruelF2(L,Sreg,ext);**

$$\begin{aligned}
RSreg := & \left[ [x-3, 3], [x-1, 1], [x-7, 7] \right], \left[ \left[ 0, \frac{2}{3}, \frac{3}{4} \right], \left[ 0, \frac{3}{2}, \frac{4}{3} \right], \left[ 0, 2, \frac{9}{4} \right] \right], \\
& \left[ \left[ \left[ \frac{2}{3}, \frac{3}{4}, \frac{1}{12} \right], [] \right], \left[ \left[ \frac{3}{2}, \frac{4}{3}, -\frac{1}{6} \right], [] \right], \left[ \left[ \frac{9}{4}, \frac{1}{4} \right], [2] \right] \right], [], []
\end{aligned} \tag{70}$$

**> R1:=IrrRegAppsing1F2(L,t,E,ext);**

$$\begin{aligned}
R1 := & \left[ [x-9, 9], [\infty, \infty], [x-12, 12] \right], \left[ \left[ 1, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} - \frac{5}{12}, \right. \right. \\
& \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} - \frac{5}{12} \left. \right], \left[ \frac{1}{2}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24} \right], \left[ \frac{3}{2}, \right. \\
& \left. \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8} \right] \left. \right],
\end{aligned} \tag{71}$$

$$\begin{aligned}
& \left[ \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3 t} - \frac{17}{12}, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3 t} - \frac{17}{12}, \right. \right. \\
& \left. \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3 t} - \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3 t} \right], \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{17}{24}, \right. \\
& \left. - \frac{\sqrt{2}}{\sqrt{t}} - \frac{17}{24}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{17}{8}, -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} \right. \\
& \left. - \frac{17}{8}, -\frac{330\sqrt{10}}{t^{3/2}} - \frac{112\sqrt{10}}{9\sqrt{t}} \right], \left[ 1, \frac{1}{2}, \frac{3}{2} \right], [1, 2, 2], \\
& \left[ \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3 t} - \frac{5}{12}, 1 \right], \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3 t} - \frac{5}{12}, \right. \right. \\
& \left. 1 \right], \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3 t} - \frac{5}{12}, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3 t} - \frac{5}{12} \right], \\
& \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{1}{2} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{1}{2} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24} \right] \right], \left[ \left[ \frac{165\sqrt{10}}{t^{3/2}} \right. \right. \\
& \left. \left. + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{3}{2} \right], \left[ -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{3}{2} \right], \left[ -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} \right. \right. \\
& \left. \left. - \frac{5}{8}, \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8} \right] \right], \left[ \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1) t}{3}, \right. \right. \\
& \left. \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2) t}{3}, -\frac{128 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1) t}{3} \right], [\sqrt{2} t, -\sqrt{2} t, \\
& -2\sqrt{2} t], \left[ \frac{\sqrt{10} t (1485 t^2 + 56)}{9}, -\frac{\sqrt{10} t (1485 t^2 + 56)}{9}, \right. \\
& \left. -\frac{2\sqrt{10} (1485 t^3 + 56 t)}{9} \right], \left[ \left[ -\frac{17}{12}, -\frac{17}{12}, 0 \right], \left[ -\frac{17}{24}, -\frac{17}{24}, 0 \right], \left[ -\frac{17}{8}, -\frac{17}{8}, 0 \right] \right], \\
& \left[ [x-3, 3], [x-1, 1], [x-7, 7], \left[ 0, \frac{2}{3}, \frac{3}{4} \right], \left[ 0, \frac{3}{2}, \frac{4}{3} \right], \left[ 0, 2, \frac{9}{4} \right], \left[ \frac{2}{3}, \frac{3}{4}, \right. \right. \\
& \left. \frac{1}{12} \right], \left[ \frac{3}{2}, \frac{4}{3}, -\frac{1}{6} \right], \left[ 2, \frac{9}{4}, \frac{1}{4} \right], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{3}{4}, 0 \right], \left[ \frac{3}{4}, \frac{2}{3} \right] \right], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{4}{3}, 0 \right], \right. \\
& \left. \left[ \frac{4}{3}, \frac{3}{2} \right] \right], \left[ [2, 0], \left[ \frac{9}{4}, 0 \right], \left[ \frac{9}{4}, 2 \right] \right], \left[ [x-3, 3], [x-1, 1], [x-7, 7], \left[ 0, \frac{2}{3}, \right. \right. \\
& \left. \frac{3}{4} \right], \left[ 0, \frac{3}{2}, \frac{4}{3} \right], \left[ 0, 2, \frac{9}{4} \right], \left[ \left[ \frac{2}{3}, \frac{3}{4}, \frac{1}{12} \right], [] \right], \left[ \left[ \frac{3}{2}, \frac{4}{3}, -\frac{1}{6} \right], [] \right], \left[ \left[ \frac{9}{4}, \frac{1}{4}, \right. \right. \\
& \left. [2] \right] \right], [ [], []], [[x^4 - 60 x^3 + 830 x^2 - 3852 x + 5193, \operatorname{RootOf}(\_Z^4 - 60 \_Z^3 + 830 \_Z^2
\end{aligned}$$



$$\begin{aligned}
& -3852\_Z + 5193) ]], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]], \left[ [[x-3, 3], [x \right. \\
& -9, 9], [\infty, \infty], [x-1, 1], [x-12, 12], [x-7, 7]], \left[ \left[ 0, \frac{2}{3}, \frac{3}{4} \right], \left[ 1, \right. \right. \\
& \left. \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} - \frac{5}{12}, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} - \frac{5}{12} \right], \left[ \frac{1}{2}, \right. \\
& \left. \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24} \right], \left[ 0, \frac{3}{2}, \frac{4}{3} \right], \left[ \frac{3}{2}, \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \right. \\
& \left. -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8} \right], \left[ 0, 2, \frac{9}{4} \right], \left[ \left[ \frac{2}{3}, \frac{3}{4}, \frac{1}{12} \right], \right. \\
& \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} - \frac{17}{12}, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} - \frac{17}{12}, \right. \\
& \left. \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} - \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} \right], \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{17}{24}, \right. \\
& \left. -\frac{\sqrt{2}}{\sqrt{t}} - \frac{17}{24}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{3}{2}, \frac{4}{3}, -\frac{1}{6} \right], \left[ \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{17}{8}, -\frac{165\sqrt{10}}{t^{3/2}} \right. \\
& \left. -\frac{56\sqrt{10}}{9\sqrt{t}} - \frac{17}{8}, -\frac{330\sqrt{10}}{t^{3/2}} - \frac{112\sqrt{10}}{9\sqrt{t}} \right], \left[ 2, \frac{9}{4}, \frac{1}{4} \right], \left[ \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{3}{4}, 0 \right], \left[ \frac{3}{4}, \right. \right. \right. \\
& \left. \left. \frac{2}{3} \right] \right], \left[ \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} - \frac{5}{12}, 1 \right], \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} \right. \right. \\
& \left. \left. - \frac{5}{12}, 1 \right], \left[ \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=2)}{3t} - \frac{5}{12}, \frac{64 \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1)}{3t} \right. \right. \\
& \left. \left. - \frac{5}{12} \right] \right], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{1}{2} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{1}{2} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{5}{24} \right] \right], \\
& \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{4}{3}, 0 \right], \left[ \frac{4}{3}, \frac{3}{2} \right] \right], \left[ \left[ \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{3}{2} \right], \left[ -\frac{165\sqrt{10}}{t^{3/2}} \right. \right. \\
& \left. \left. - \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{3}{2} \right], \left[ -\frac{165\sqrt{10}}{t^{3/2}} - \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8}, \frac{165\sqrt{10}}{t^{3/2}} + \frac{56\sqrt{10}}{9\sqrt{t}} - \frac{5}{8} \right] \right] \\
& , \left[ [2, 0], \left[ \frac{9}{4}, 0 \right], \left[ \frac{9}{4}, 2 \right] \right], [[1, 1, 1], [1, 1, 1], [1, 2, 2], [1, 1, 1], [1, 2, 2], [1, 1, 1]] \Big] \Big]
\end{aligned}$$

**> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);**

$$\begin{aligned}
\text{info1} := & \left[ \left[ \left[ 9, x-9, \left[ -\frac{2048}{9(x-9)^2} \right], 2, \{ \operatorname{RootOf}(\_Z^2 + 2, \text{index}=1) \}, \{ \operatorname{RootOf}(\_Z^2 + 2, \right. \right. \\
& \left. \left. \text{index}=1) \} \right], \left[ \infty, \frac{1}{x}, [2x], 1, \emptyset, \emptyset \right], \left[ 12, x-12, \left[ \frac{31360}{81(x-12)} + \frac{61600}{9(x-12)^2} \right. \right. \right.
\end{aligned} \tag{72}$$

$$\begin{aligned}
& \left[ \left[ \left[ \left[ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right], \left[ \frac{1}{3}, \frac{1}{4} \right] \right], -\frac{2(x-1)^2(x-7)^3(x-3)}{(x-9)^2(x-12)^3} \right], \left[ \left[ \left[ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right], \left[ \frac{1}{3}, \frac{1}{4} \right] \right], \right. \\
& \left. \left. \frac{2(x-1)^2(x-7)^3(x-3)}{(x-9)^2(x-12)^3} \right] \right] \\
& \text{> easy1F2(L,R1,info1,x,t,ext);} \\
& \left[ \left[ \left[ \left[ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right], \left[ \frac{1}{3}, \frac{1}{4} \right] \right], \frac{2(x-1)^2(x-7)^3(x-3)}{(x-9)^2(x-12)^3} \right] \right] \\
& \text{> find1F2Rat(L,R1,info1,x,t,T,ext);} \\
& \left[ \left[ \left[ \left[ \left[ \frac{1}{2} \right], \left[ \frac{1}{3}, \frac{1}{4} \right], [0], [1] \right], \frac{2(x-1)^2(x-7)^3(x-3)}{(x-9)^2(x-12)^3} \right] \right] \right] \\
& \text{> TIME :=time();} \\
& \text{Hyp1F2Solutions(L);} \\
& \text{time() - TIME;} \\
& \text{TIME := 57.703} \\
& \left[ \left[ \left[ \left[ \left[ \frac{1}{2} \right], \left[ \frac{1}{3}, \frac{1}{4} \right], [0], [1] \right], \frac{2(x-1)^2(x-7)^3(x-3)}{(x-9)^2(x-12)^3} \right] \right] \right] \\
& 0.531
\end{aligned}
\tag{73}$$

$$\begin{aligned}
& \text{> ##### THE LOGARITHMIC CASE #####} \\
& \text{> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));} \\
& F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \\
& = 0 \\
& \text{> LA:=de2diffop(F,J(x));} \\
& LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \\
& \text{> L12:=subs({a1=1/3,b1=1,b2=1/2},LA);} \\
& L12 := x^2 Dx^3 + \frac{5 Dx^2 x}{2} + \left( \frac{1}{2} - x \right) Dx - \frac{1}{3} \\
& \text{> f:=(2*(x-7)*(x-3))/((x-12));} \\
& f := \frac{2(x-7)(x-3)}{x-12} \\
& \text{> L:=ChangeOfVariables(L12,f);} \\
& L := 6 Dx^3 (x-7)^2 (x-3)^2 (x-12)^4 (x^2 - 24 x + 99)^2 + 15 (x-3) (x-7) (x-9) (x^2 - 24 x + 99) (x^3 - 39 x^2 + 315 x - 837) (x-12)^3 Dx^2 - 3 (4 x^{10} - 425 x^9 + 19440 x^8 - 502470 x^7 + 8104140 x^6 - 85139316 x^5 + 590781600 x^4 - 2680462530 x^3 + 7632017640 x^2 - 12347944155 x + 8650389816) Dx (x-12) - 4 (x^2 - 24 x + 99)^5 \\
& \text{> ext:=indets(L,{RootOf,name}) minus {x,Dx};} \\
& \text{ext := } \emptyset \\
& \text{> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});}
\end{aligned}
\tag{76}$$

$$\tag{77}$$

$$\tag{78}$$

$$\tag{79}$$

$$\tag{80}$$

$$\tag{81}$$

$$ext := \emptyset \quad (82)$$

> extppp:={};

$$extppp := \emptyset \quad (83)$$

> E:= Singular(L,extppp);

$$E := [[\infty, \infty], [x-3, 3], [x^2-24x+99, \text{RootOf}(\_Z^2-24\_Z+99)], [x-7, 7], [x-12, 12]] \quad (84)$$

> F:=NotAppSing(L,E,ext);

$$F := [[x-3, 3], [\infty, \infty], [x-12, 12], [x-7, 7]] \quad (85)$$

> Sirr:=irrsinglF2(L,t,F,ext);

$$Sirr := \left[ [[\infty, \infty], [x-12, 12]], \left[ \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{\sqrt{2}}{\sqrt{t}}, -\frac{\sqrt{2}}{\sqrt{t}}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{3\sqrt{10}}{\sqrt{t}}, -\frac{3\sqrt{10}}{\sqrt{t}}, -\frac{6\sqrt{10}}{\sqrt{t}} \right], \left[ \frac{1}{2}, \frac{1}{2} \right], [2, 2], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right], [[\sqrt{2}t, -\sqrt{2}t, -2\sqrt{2}t], [3\sqrt{10}t, -3\sqrt{10}t, -6\sqrt{10}t], [[0, 0, 0], [0, 0, 0]], \left[ [x-3, 3], [x-7, 7], \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right], [1, 1, 1], \left[ 0, 0, \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], 3, \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right], [1, 1, 1], \left[ 0, 0, \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], 3 \right] \right] \right] \right] \quad (86)$$

> Sreg:=regsingtruelF2(L,t,Sirr[-1],ext);

$$Sreg := \left[ [[x-3, 3], [x-7, 7]], \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, 0, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right], \left[ 0, 0, \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], \left[ 0, 0, \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right] \right] \right] \quad (87)$$

> RSreg:=SregseptruelF2(L,Sreg,ext);

$$RSreg := \left[ [], [], \left[ [[x-3, 3], [x-7, 7]], \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, 0, \frac{1}{2} \right], \left[ \left[ \frac{1}{2}, \frac{1}{2} \right], [0] \right], \left[ \left[ \frac{1}{2}, \frac{1}{2} \right], [0] \right] \right] \right] \right] \quad (88)$$

> Rl:=IrrRegAppsinglF2(L,t,E,ext);

$$Rl := \left[ [[\infty, \infty], [x-12, 12]], \left[ \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{\sqrt{2}}{\sqrt{t}}, -\frac{\sqrt{2}}{\sqrt{t}}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{3\sqrt{10}}{\sqrt{t}}, -\frac{3\sqrt{10}}{\sqrt{t}}, -\frac{6\sqrt{10}}{\sqrt{t}} \right], \left[ \frac{1}{2}, \frac{1}{2} \right], [2, 2], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right], [[\sqrt{2}t, -\sqrt{2}t, -2\sqrt{2}t], [3\sqrt{10}t, -3\sqrt{10}t, -6\sqrt{10}t], [[0, 0, 0], [0, 0, 0]], \left[ [x-3, 3], [x-7, 7], \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right], [1, 1, 1], \left[ 0, 0, \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], 3, \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right], [1, 1, 1], \left[ 0, 0, \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], 3 \right] \right] \right] \right] \quad (89)$$

$$\begin{aligned}
& \left[ \left[ \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right] \right], \\
& \left[ [\sqrt{2} t, -\sqrt{2} t, -2\sqrt{2} t], [3\sqrt{10} t, -3\sqrt{10} t, -6\sqrt{10} t], [[0, 0, 0], [0, 0, 0]] \right], \left[ [[x \right. \\
& \left. - 3, 3], [x - 7, 7]], \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ 0, 0, \frac{1}{2} \right] \right], \left[ \left[ 0, \frac{1}{2}, \frac{1}{2} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right] \right], \left[ \left[ 0, 0 \right], \left[ \frac{1}{2}, \right. \right. \\
& \left. \left. 0 \right], \left[ \frac{1}{2}, 0 \right] \right], \left[ [0, 0], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right] \right], \left[ [ ], [ ], \left[ [x - 3, 3], [x - 7, 7], \left[ \left[ 0, 0, \frac{1}{2} \right], \right. \right. \right. \\
& \left. \left. \left[ 0, 0, \frac{1}{2} \right] \right], \left[ \left[ \left[ \frac{1}{2}, \frac{1}{2} \right], [0] \right], \left[ \left[ \frac{1}{2}, \frac{1}{2} \right], [0] \right] \right] \right], \left[ [x^2 - 24x + 99, \text{RootOf}(\_Z^2 \right. \\
& \left. - 24\_Z + 99) \right]], [[0, 2, 4]], [[2, 4, 2]], [[ [2, 0], [4, 0], [4, 2] ]], \left[ [[x - 3, 3], [\infty, \right. \\
& \infty], [x - 12, 12], [x - 7, 7]], \left[ \left[ 0, 0, \frac{1}{2} \right], \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3} \right], \left[ \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} \right. \right. \\
& \left. \left. + \frac{1}{3}, -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right], \left[ 0, 0, \frac{1}{2} \right] \right], \left[ \left[ 0, \frac{1}{2}, \frac{1}{2} \right], \left[ \frac{\sqrt{2}}{\sqrt{t}}, -\frac{\sqrt{2}}{\sqrt{t}}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{3\sqrt{10}}{\sqrt{t}}, \right. \right. \\
& \left. \left. -\frac{3\sqrt{10}}{\sqrt{t}}, -\frac{6\sqrt{10}}{\sqrt{t}} \right], \left[ 0, \frac{1}{2}, \frac{1}{2} \right] \right], \left[ \left[ [0, 0], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ \right. \right. \right. \\
& \left. \left. \left. -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{3} \right] \right], \left[ \left[ \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} \right. \right. \\
& \left. \left. + \frac{1}{3}, \frac{1}{3} \right], \left[ -\frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3}, \frac{3\sqrt{10}}{\sqrt{t}} + \frac{1}{3} \right] \right], \left[ [0, 0], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, 0 \right] \right], [[1, 1, 1], \\
& [1, 2, 2], [1, 2, 2], [1, 1, 1]] \right]
\end{aligned}$$

```
> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);
```

$$\text{info1} := \left[ \left[ \left[ \infty, \frac{1}{x}, [2x], 1, \emptyset, \emptyset \right], \left[ 12, x - 12, \left[ \frac{90}{x - 12} \right], 1, \emptyset, \emptyset \right] \right], 2, 4, x - 12, 1 \right] \quad (90)$$

```
> find1F2ln(L,R1,info1,x,t,ext);
```

$$\left[ \left[ \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ 1, \frac{1}{2} \right] \right], \frac{2(x-7)(x-3)}{x-12} \right], \left[ \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ 1, \frac{1}{2} \right] \right], \right. \right. \quad (91) \\
\left. \left. -\frac{2(x-7)(x-3)}{x-12} \right] \right]$$

```
> TIME :=time();
HyplF2Solutions(L);
time() - TIME;
```

$$\text{TIME} := 62.312$$

$$\left\{ \left\{ \left\{ \left[ \frac{1}{3} \right], \left[ 1, \frac{1}{2} \right], [0], [1] \right\}, \frac{2(x-7)(x-3)}{x-12} \right\} \right\}$$

(92)

```
> F:=sumdiffee(hyperterm([a1],[b1,b2],x,k),k,J(x));
```

$$F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \quad (93)$$

=0

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \quad (94)$$

```
> L12:=subs({a1=1/3,b1=1/2,b2=1/2},LA);
```

$$L12 := x^2 Dx^3 + 2 Dx^2 x + \left( \frac{1}{4} - x \right) Dx - \frac{1}{3} \quad (95)$$

```
> f:=(2*(x-7)^3)/((x-9)^2);
```

$$f := \frac{2(x-7)^3}{(x-9)^2} \quad (96)$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := 12 Dx^3 (x-13)^2 (x-9)^5 (x-7)^2 + 24 (x-19) Dx^2 (x-13) (x-9)^4 (x-7)^2 - 3 (8 x^7 - 585 x^6 + 18094 x^5 - 306679 x^4 + 3073580 x^3 - 18200551 x^2 + 58941758 x - 80537081) Dx (x-9) - 8 (x-13)^5 (x-7)^2 \quad (97)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \emptyset \quad (98)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});
```

$$ext := \emptyset \quad (99)$$

```
> extppp:={};
```

$$extppp := \emptyset \quad (100)$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-13, 13], [\infty, \infty], [x-7, 7], [x-9, 9]] \quad (101)$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[\infty, \infty], [x-9, 9], [x-7, 7]] \quad (102)$$

```
> Sirr:=irrsinglF2(L,t,F,ext);
```

$$Sirr := \left[ [[\infty, \infty], [x-9, 9]], \left[ \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12} \right], \left[ \frac{2}{3}, -\frac{8}{t} + \frac{1}{6}, \frac{8}{t} + \frac{1}{6} \right] \right], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{4}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{4}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ -\frac{8}{t} - \frac{1}{2}, \frac{8}{t} - \frac{1}{2}, \frac{16}{t} \right] \right], \left[ \frac{1}{2}, 1 \right], [2, 1], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12} \right], \left[ -\frac{8}{t} + \frac{1}{6}, \frac{2}{3} \right], \left[ \frac{8}{t} + \frac{1}{6}, \frac{2}{3} \right], \left[ \frac{8}{t} + \frac{1}{6}, -\frac{8}{t} + \frac{1}{6} \right] \right], [[\sqrt{2} t, -\sqrt{2} t, -2\sqrt{2} t], [-8 t, 8 t, 16 t]], \left[ \left[ -\frac{1}{4}, -\frac{1}{4}, 0 \right], \left[ -\frac{1}{2}, -\frac{1}{2}, 0 \right] \right], [[x-7, 7]], \left[ \left[ 0, \frac{3}{2}, \frac{3}{2} \right], \left[ \frac{3}{2}, \frac{3}{2}, \right] \right] \right] \quad (103)$$

$$0], [1, 1, 1], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, \frac{3}{2} \right], 2 \right] \right]$$

> Sreg:=regsingtrue1F2(L,t,Sirr[-1],ext);

$$Sreg := \left[ [x-7, 7], \left[ \left[ 0, \frac{3}{2}, \frac{3}{2} \right], \left[ \left[ \frac{3}{2}, \frac{3}{2}, 0 \right], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, \frac{3}{2} \right] \right] \right] \right] \right] \quad (104)$$

> RSreg:=SregseptruelF2(L,Sreg,ext);

$$RSreg := \left[ [], [], \left[ [x-7, 7], \left[ \left[ 0, \frac{3}{2}, \frac{3}{2} \right], \left[ \left[ \frac{3}{2}, \frac{3}{2} \right], [0] \right] \right] \right] \right] \quad (105)$$

> R1:=IrrRegAppsing1F2(L,t,E,ext);

$$R1 := \left[ \left[ [\infty, \infty], [x-9, 9], \left[ \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12} \right], \left[ \frac{2}{3}, -\frac{8}{t} + \frac{1}{6}, \frac{8}{t} + \frac{1}{6} \right], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{4}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{4}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ -\frac{8}{t} - \frac{1}{2}, \frac{8}{t} - \frac{1}{2}, \frac{16}{t} \right], \left[ \frac{1}{2}, 1 \right], [2, 1], \left[ \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12} \right], \left[ -\frac{8}{t} + \frac{1}{6}, \frac{2}{3} \right], \left[ \frac{8}{t} + \frac{1}{6}, \frac{2}{3} \right], \left[ \frac{8}{t} + \frac{1}{6}, -\frac{8}{t} + \frac{1}{6} \right] \right], [\sqrt{2}t, -\sqrt{2}t, -2\sqrt{2}t], [-8t, 8t, 16t], \left[ \left[ -\frac{1}{4}, -\frac{1}{4}, 0 \right], \left[ -\frac{1}{2}, -\frac{1}{2}, 0 \right] \right], \left[ [x-7, 7], \left[ \left[ 0, \frac{3}{2}, \frac{3}{2} \right], \left[ \left[ \frac{3}{2}, \frac{3}{2}, 0 \right], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, \frac{3}{2} \right] \right] \right], [ [], [], \left[ [x-7, 7], \left[ \left[ 0, \frac{3}{2}, \frac{3}{2} \right], \left[ \left[ \frac{3}{2}, \frac{3}{2}, 0 \right], [0] \right] \right] \right], \left[ [x-13, 13], [0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]] \right], \left[ [\infty, \infty], [x-9, 9], [x-7, 7], \left[ \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12} \right], \left[ \frac{2}{3}, -\frac{8}{t} + \frac{1}{6}, \frac{8}{t} + \frac{1}{6} \right], \left[ 0, \frac{3}{2}, \frac{3}{2} \right], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{4}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{4}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ -\frac{8}{t} - \frac{1}{2}, \frac{8}{t} - \frac{1}{2}, \frac{16}{t} \right], \left[ \frac{3}{2}, \frac{3}{2}, 0 \right], \left[ \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12}, \frac{\sqrt{2}}{\sqrt{t}} + \frac{1}{12} \right], \left[ -\frac{8}{t} + \frac{1}{6}, \frac{2}{3} \right], \left[ \frac{8}{t} + \frac{1}{6}, \frac{2}{3} \right], \left[ \frac{8}{t} + \frac{1}{6}, -\frac{8}{t} + \frac{1}{6} \right], \left[ \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, \frac{3}{2} \right] \right], [1, 2, 2], [1, 1, 1], [1, 1, 1] \right] \right] \right] \quad (106)$$

> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);

$$info1 := \left[ \left[ \left[ \infty, \frac{1}{x}, [2x], 1, \emptyset, \emptyset \right], \left[ 9, x-9, \left[ \frac{16}{(x-9)^2} \right], 2, \emptyset, \emptyset \right], 3, 3, (x-9)^2, x-9 \right] \right] \quad (107)$$

> find1F2ln(L,R1,info1,x,t,ext);

$$\begin{aligned}
& \left[ \left[ \left[ \left\{ \frac{2}{9}, \frac{5}{9}, \frac{8}{9} \right\}, \left[ \frac{5}{6}, \frac{5}{6} \right] \right], \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ \frac{1}{2}, \frac{1}{2} \right] \right], \left[ \left\{ \frac{1}{9}, \frac{4}{9}, \frac{7}{9} \right\}, \left[ \frac{1}{6}, \frac{1}{6} \right] \right], \right. \\
& \quad \left. \frac{2(x-7)^3}{(x-9)^2} \right], \left[ \left[ \left\{ \frac{2}{9}, \frac{5}{9}, \frac{8}{9} \right\}, \left[ \frac{5}{6}, \frac{5}{6} \right] \right], \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ \frac{1}{2}, \frac{1}{2} \right] \right], \left[ \left\{ \frac{1}{9}, \frac{4}{9}, \frac{7}{9} \right\}, \right. \right. \\
& \quad \left. \left. \left[ \frac{1}{6}, \frac{1}{6} \right] \right], -\frac{2(x-7)^3}{(x-9)^2} \right] \right] \\
& \text{> TIME :=time();} \\
& \text{Hyp1F2Solutions(L);} \\
& \text{time() - TIME;} \\
& \quad \quad \quad \text{TIME := 73.109} \\
& \quad \quad \quad \left\{ \left[ \left[ \left[ \frac{1}{3} \right], \left[ \frac{1}{2}, \frac{1}{2} \right], [0], [1] \right] \right], \frac{2(x-7)^3}{(x-9)^2} \right] \right\} \\
& \quad \quad \quad 2.297
\end{aligned} \tag{108}$$

$$\tag{109}$$

$$\begin{aligned}
& \text{> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));} \\
& F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \\
& \quad = 0
\end{aligned} \tag{110}$$

$$\begin{aligned}
& \text{> LA:=de2diffop(F,J(x));} \\
& \quad \quad \quad LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1
\end{aligned} \tag{111}$$

$$\begin{aligned}
& \text{> L12:=subs({b1=1,b2=1/2},LA);} \\
& \quad \quad \quad L12 := x^2 Dx^3 + \frac{5 Dx^2 x}{2} + \left( \frac{1}{2} - x \right) Dx - a1
\end{aligned} \tag{112}$$

$$\begin{aligned}
& \text{> f:=(2*(x-1)^2*(x-7)^3)/((x-12)^2);} \\
& \quad \quad \quad f := \frac{2(x-1)^2(x-7)^3}{(x-12)^2}
\end{aligned} \tag{113}$$

$$\begin{aligned}
& \text{> L:=ChangeOfVariables(L12,f);} \\
& L := 2 Dx^3 (x-1) (x-7)^2 (3 x^2 - 61 x + 190)^2 (x-12)^5 + (9 x^4 - 366 x^3 + 4025 x^2 \\
& \quad - 19396 x + 50576) Dx^2 (x-7) (3 x^2 - 61 x + 190) (x-12)^4 - (324 x^{12} - 33480 x^{11} \\
& \quad + 1519992 x^{10} - 39975807 x^9 + 676670581 x^8 - 7751428091 x^7 + 61522511321 x^6 \\
& \quad - 340341096906 x^5 + 1299310571074 x^4 - 3324472480752 x^3 + 5368326350576 x^2 \\
& \quad - 4831911262592 x + 1762990216384) Dx (x-12) - 4 a1 (3 x^2 - 61 x + 190)^5 (x \\
& \quad - 7)^2
\end{aligned} \tag{114}$$

$$\begin{aligned}
& \text{> ext:=indets(L,{RootOf,name}) minus {x,Dx};} \\
& \quad \quad \quad ext := \{a1\}
\end{aligned} \tag{115}$$

$$\begin{aligned}
& \text{> ext:= indets(map(s-> ReplirrRoot(s,{ } ),ext),{RootOf,name});} \\
& \quad \quad \quad ext := \{a1\}
\end{aligned} \tag{116}$$

$$\begin{aligned}
& \text{> extppp:={};} \\
& \quad \quad \quad extppp := \emptyset
\end{aligned} \tag{117}$$

$$\begin{aligned} &> \mathbf{E} := \text{Singular}(\mathbf{L}, \text{extppp}); \\ \mathbf{E} &:= \left[ \left[ x^2 - \frac{61}{3}x + \frac{190}{3}, \text{RootOf}(3\_Z^2 - 61\_Z + 190) \right], [\infty, \infty], [x-7, 7], [x-12, \right. \\ &\quad \left. 12], [x-1, 1] \right] \end{aligned} \quad (118)$$

$$\begin{aligned} &> \mathbf{F} := \text{NotAppSing}(\mathbf{L}, \mathbf{E}, \text{ext}); \\ \mathbf{F} &:= [[\infty, \infty], [x-12, 12], [x-1, 1], [x-7, 7]] \end{aligned} \quad (119)$$

$$\begin{aligned} &> \mathbf{Sirr} := \text{irrSinglF2}(\mathbf{L}, \mathbf{t}, \mathbf{F}, \text{ext}); \\ \mathbf{Sirr} &:= \left[ [[\infty, \infty], [x-12, 12]], \left[ \left[ 3al, \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} \right. \right. \right. \\ &\quad \left. \left. - \frac{3al}{2} + \frac{3}{2} \right], \left[ 2al, \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=1)}{t} - al + 1, \right. \right. \\ &\quad \left. \left. \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=2)}{t} - al + 1 \right] \right], \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{9al}{2} + \frac{3}{2}, \right. \right. \\ &\quad \left. \left. -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{9al}{2} + \frac{3}{2}, -\frac{6\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=1)}{t} \right. \right. \\ &\quad \left. \left. - 3al + 1, \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=2)}{t} - 3al + 1, \right. \right. \\ &\quad \left. \left. \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=2)}{t} - \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=1)}{t} \right] \right], \left[ \frac{3}{2}, 1 \right], [2, \\ &\quad 1], \left[ \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, 3al \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, 3al \right], \left[ \right. \right. \right. \\ &\quad \left. \left. \left. -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2} \right] \right], \right. \\ &\quad \left[ \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=1)}{t} - al + 1, 2al \right], \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=2)}{t} \right. \right. \\ &\quad \left. \left. - al + 1, 2al \right], \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=2)}{t} - al + 1, \right. \right. \\ &\quad \left. \left. \frac{110 \text{RootOf}(\_Z^2 - 10, \text{index}=1)}{t} - al + 1 \right] \right], \left[ \left[ \frac{(6t^3 + t)\sqrt{2}}{2}, -\frac{(6t^3 + t)\sqrt{2}}{2}, \right. \right. \\ &\quad \left. \left. -(6t^3 + t)\sqrt{2} \right], [110 \text{RootOf}(\_Z^2 - 10, \text{index}=1) t, 110 \text{RootOf}(\_Z^2 - 10, \text{index}=2) t, \right. \\ &\quad \left. -220 \text{RootOf}(\_Z^2 - 10, \text{index}=1) t \right], \left[ \left[ -\frac{9al}{2} + \frac{3}{2}, -\frac{9al}{2} + \frac{3}{2}, 0 \right], [-3al + 1, \right. \\ &\quad \left. -3al + 1, 0] \right], \left[ [x-1, 1], [x-7, 7], \left[ [0, 0, 1], [0, 1, 1], [1, 1, 1], [0, 0], [1, 0], \right. \right. \\ &\quad \left. \left. [1, 0], 4 \right], \left[ \left[ 0, 0, \frac{3}{2} \right], \left[ 0, \frac{3}{2}, \frac{3}{2} \right], [1, 1, 1], \left[ [0, 0], \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, 0 \right] \right], 3 \right] \right] \right] \end{aligned} \quad (120)$$

$$> \mathbf{Sreg} := \text{regSingtruelF2}(\mathbf{L}, \mathbf{t}, \mathbf{Sirr}[-1], \text{ext});$$



$$Sreg := \left[ \left[ [x-1, 1], [x-7, 7] \right], \left[ [0, 0, 1], \left[ 0, 0, \frac{3}{2} \right] \right], \left[ [0, 1, 1], \left[ 0, \frac{3}{2}, \frac{3}{2} \right] \right], \left[ [0, 0], \right. \right. \\ \left. \left. [1, 0], [1, 0], \left[ [0, 0], \left[ \frac{3}{2}, 0 \right], \left[ \frac{3}{2}, 0 \right] \right] \right] \right] \quad (121)$$

**> RSreg:=SregseptruelF2(L,Sreg,ext);**

$$RSreg := \left[ [ ], [ ], \left[ [x-1, 1], [x-7, 7] \right], \left[ [0, 0, 1], \left[ 0, 0, \frac{3}{2} \right] \right], \left[ [ ], [0, 1, 1] \right], \left[ \left[ \frac{3}{2}, \right. \right. \right. \\ \left. \left. \frac{3}{2} \right], [0] \right] \right] \quad (122)$$

**> R1:=IrrRegAppsinglF2(L,t,E,ext);**

$$R1 := \left[ \left[ [ \infty, \infty ], [x-12, 12] \right], \left[ \left[ 3al, \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} \right. \right. \right. \\ \left. \left. - \frac{3al}{2} + \frac{3}{2} \right], \left[ 2al, \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - al + 1, \right. \right. \\ \left. \left. \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - al + 1 \right] \right], \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{9al}{2} + \frac{3}{2}, \right. \right. \\ \left. \left. -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{9al}{2} + \frac{3}{2}, -\frac{6\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} \right. \right. \\ \left. \left. - 3al + 1, \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - 3al + 1, \right. \right. \\ \left. \left. \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} \right] \right], \left[ \frac{3}{2}, 1 \right], [2, \\ 1], \left[ \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, 3al \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, 3al \right], \left[ \right. \right. \right. \\ \left. \left. -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2} \right] \right], \\ \left[ \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - al + 1, 2al \right], \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} \right. \right. \\ \left. \left. - al + 1, 2al \right], \left[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - al + 1, \right. \right. \\ \left. \left. \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - al + 1 \right] \right], \left[ \left[ \frac{(6t^3+t)\sqrt{2}}{2}, -\frac{(6t^3+t)\sqrt{2}}{2}, \right. \right. \\ \left. \left. -(6t^3+t)\sqrt{2} \right], [110 \text{RootOf}(\_Z^2 - 10, index=1) t, 110 \text{RootOf}(\_Z^2 - 10, index=2) t, \right. \\ \left. -220 \text{RootOf}(\_Z^2 - 10, index=1) t] \right], \left[ \left[ -\frac{9al}{2} + \frac{3}{2}, -\frac{9al}{2} + \frac{3}{2}, 0 \right], [-3al + 1, \right. \end{array} \quad (123)$$

$$\begin{aligned}
& -3al + 1, 0] \Big] \Big], \Big[ \Big[ [x-1, 1], [x-7, 7] \Big], \Big[ [0, 0, 1], \Big[ 0, 0, \frac{3}{2} \Big] \Big], \Big[ [0, 1, 1], \Big[ 0, \frac{3}{2}, \frac{3}{2} \Big] \Big], \\
& \Big[ [0, 0], [1, 0], [1, 0] \Big], \Big[ [0, 0], \Big[ \frac{3}{2}, 0 \Big], \Big[ \frac{3}{2}, 0 \Big] \Big] \Big] \Big], \Big[ [ ], [ ], \Big[ [x-1, 1], [x-7, 7] \Big], \\
& \Big[ [0, 0, 1], \Big[ 0, 0, \frac{3}{2} \Big] \Big], \Big[ [ [ ], [0, 1, 1] ], \Big[ \Big[ \frac{3}{2}, \frac{3}{2} \Big], [0] \Big] \Big] \Big] \Big], \Big[ \Big[ \Big[ x^2 - \frac{61}{3}x + \frac{190}{3}, \\
& \text{RootOf}(3\_Z^2 - 61\_Z + 190) \Big] \Big], \Big[ [0, 2, 4] \Big], \Big[ [2, 4, 2] \Big], \Big[ [ [2, 0], [4, 0], [4, 2] ] \Big], \\
& \Big[ [ [ \infty, \infty ], [x-12, 12], [x-1, 1], [x-7, 7] ], \Big[ \Big[ 3al, \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, \\
& -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2} \Big], \Big[ 2al, \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - al + 1, \\
& \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - al + 1 \Big], [0, 0, 1], \Big[ 0, 0, \frac{3}{2} \Big] \Big], \Big[ \Big[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} \\
& - \frac{9al}{2} + \frac{3}{2}, -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{9al}{2} + \frac{3}{2}, -\frac{6\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{\sqrt{t}} \Big], \\
& \Big[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - 3al + 1, \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} \\
& - 3al + 1, \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} \Big], \\
& [0, 1, 1], \Big[ 0, \frac{3}{2}, \frac{3}{2} \Big] \Big], \Big[ \Big[ \Big[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, 3al \Big], \Big[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} \\
& - \frac{3al}{2} + \frac{3}{2}, 3al \Big], \Big[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} + \frac{3}{2}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{\sqrt{2}}{2\sqrt{t}} - \frac{3al}{2} \\
& + \frac{3}{2} \Big] \Big], \Big[ \Big[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - al + 1, 2al \Big], \\
& \Big[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} - al + 1, 2al \Big], \Big[ \frac{110 \text{RootOf}(\_Z^2 - 10, index=2)}{t} \\
& - al + 1, \frac{110 \text{RootOf}(\_Z^2 - 10, index=1)}{t} - al + 1 \Big] \Big], \Big[ [0, 0], [1, 0], [1, 0] \Big], \Big[ [0, \\
& 0], \Big[ \frac{3}{2}, 0 \Big], \Big[ \frac{3}{2}, 0 \Big] \Big], \Big[ [1, 2, 2], [1, 1, 1], [1, 1, 1], [1, 1, 1] \Big] \Big]
\end{aligned}$$

**> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);**

$$\text{info1} := \Big[ \Big[ \Big[ \infty, \frac{1}{x}, \Big[ \frac{1}{2}x + 2x^2 + 2x^3 \Big], 3, \emptyset, \{al\} \Big], \Big[ 12, x-12, \Big[ \frac{30250}{(x-12)^2} \Big], 2,$$

(124)

$$\left\{ \text{RootOf}(\_Z^2 - 10, \text{index}=1) \right\}, \left\{ a1, \text{RootOf}(\_Z^2 - 10, \text{index}=1) \right\} \right] \right], 5, 5, (x-12)^2, x - 12 \Big]$$

> find1F2ln(L,R1,info1,x,t,ext);

$$\left[ \left[ \left[ \left\{ a1, \frac{1}{3} + a1, \frac{2}{3} + a1 \right\}, \left[ 1, \frac{1}{2} \right] \right], -\frac{2(x-1)^2(x-7)^3}{(x-12)^2} \right], \left[ \left[ \left\{ a1, \frac{1}{3} + a1, \frac{2}{3} + a1 \right\}, \left[ 1, \frac{1}{2} \right] \right], \frac{2(x-1)^2(x-7)^3}{(x-12)^2} \right] \right] \quad (125)$$

> TIME :=time();  
Hyp1F2Solutions(L);  
time() - TIME;

TIME := 78.281

$$\left\{ \left[ \left[ [a1], \left[ 1, \frac{1}{2} \right], [0], [1] \right] \right], \frac{2(x-1)^2(x-7)^3}{(x-12)^2} \right\} \quad 0.531 \quad (126)$$

> ##### THE IRRATIONAL CASE #####

> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));

$$F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) = 0 \quad (127)$$

> LA:=de2diffop(F,J(x));

$$LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \quad (128)$$

> L12:=subs({a1=-1,b1=1/2,b2=RootOf(x^2+1)},LA);

$$L12 := x^2 Dx^3 + \left( \frac{3x}{2} + \text{RootOf}(\_Z^2 + 1) x \right) Dx^2 + \left( \frac{\text{RootOf}(\_Z^2 + 1)}{2} - x \right) Dx + 1 \quad (129)$$

> f:=(2\*(x-1))/x^2;

$$f := \frac{2(x-1)}{x^2} \quad (130)$$

> L:=ChangeOfVariables(L12,f);

$$L := 2 Dx^3 (x-1)^2 x^5 (x-2)^2 + \frac{1}{85} \left( (2 \text{RootOf}(\_Z^2 + 1) - 9) (-85 x^2 + 24 \text{RootOf}(\_Z^2 + 1) + 1) + 340 x - 232 \right) (x-1) (x-2) x^4 Dx^2 + \frac{1}{5} \left( (\text{RootOf}(\_Z^2 + 1) - 2) x (4 \text{RootOf}(\_Z^2 + 1) x^5 - 15 x^6 - 38 \text{RootOf}(\_Z^2 + 1) x^4 + 128 x^5 + 112 \text{RootOf}(\_Z^2 + 1) x^3 - 416 x^4 - 208 \text{RootOf}(\_Z^2 + 1) x^2 + 624 x^3 + 192 \text{RootOf}(\_Z^2 + 1) x - 576 x^2 - 64 \text{RootOf}(\_Z^2 + 1) + 384 x - 128) Dx \right) - 4 (x-2)^5 \quad (131)$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\mathbf{L}, \{\text{RootOf}, \text{name}\}) \text{ minus } \{\mathbf{x}, \mathbf{Dx}\}; \\ &\quad \text{ext} := \{\text{RootOf}(\_Z^2 + 1)\} \end{aligned} \quad (132)$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\text{map}(s \rightarrow \text{ReplirrRoot}(s, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\ &\quad \text{ext} := \{\text{RootOf}(\_Z^2 + 1)\} \end{aligned} \quad (133)$$

$$\begin{aligned} &> \text{extppp} := \{\}; \\ &\quad \text{extppp} := \emptyset \end{aligned} \quad (134)$$

$$\begin{aligned} &> \mathbf{E} := \text{Singular}(\mathbf{L}, \text{extppp}); \\ &\quad \mathbf{E} := [[x - 2, 2], [\infty, \infty], [x - 1, 1], [x, 0]] \end{aligned} \quad (135)$$

$$\begin{aligned} &> \mathbf{F} := \text{NotAppSing}(\mathbf{L}, \mathbf{E}, \text{ext}); \\ &\quad \mathbf{F} := [[\infty, \infty], [x, 0], [x - 1, 1]] \end{aligned} \quad (136)$$

$$\begin{aligned} &> \text{Sirr} := \text{irrsglF2}(\mathbf{L}, \mathbf{t}, \mathbf{F}, \text{ext}); \\ \text{Sirr} := &\left[ [[x, 0]], \left[ \left[ -2, \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \right. \right. \right. \end{aligned} \quad (137)$$

$$\begin{aligned} &\quad \left. \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1) \right], \\ &\quad \left[ \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 3 + \text{RootOf}(\_Z^2 + 1), \right. \right. \\ &\quad \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 3 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} \\ &\quad \left. \left. - \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} \right], [1], [1], \left[ \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 \right. \right. \right. \\ &\quad \left. \left. + \text{RootOf}(\_Z^2 + 1), -2 \right], \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \right. \right. \\ &\quad \left. \left. -2 \right], \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \right. \right. \\ &\quad \left. \left. \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 + 1) \right] \right], [[2 \text{RootOf}(\_Z^2 + 2, \text{index} \\ &\quad = 1) t, 2 \text{RootOf}(\_Z^2 + 2, \text{index}=2) t, -4 \text{RootOf}(\_Z^2 + 2, \text{index}=1) t]], [[3 \\ &\quad + \text{RootOf}(\_Z^2 + 1), 3 + \text{RootOf}(\_Z^2 + 1), 0]], \left[ [[\infty, \infty], [x - 1, 1]], \left[ \left[ 0, \frac{1}{2}, \right. \right. \right. \\ &\quad \left. \left. - \text{RootOf}(\_Z^2 + 1) + 1 \right], \left[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \right], [1, 1, \right. \\ &\quad 1], \left[ \left[ \frac{1}{2}, 0 \right], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \left[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \right] \right], 2 \right], \left[ \left[ 0, \frac{1}{2}, \right. \right. \\ &\quad \left. \left. - \text{RootOf}(\_Z^2 + 1) + 1 \right], \left[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \right], [1, 1, \right. \\ &\quad 1], \left[ \left[ \frac{1}{2}, 0 \right], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \left[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \right] \right], 2 \right] \right] \end{aligned}$$

$$\begin{aligned} &> \text{Sreg} := \text{regsingtrue1F2}(\mathbf{L}, \mathbf{t}, \text{Sirr}[-1], \text{ext}); \\ \text{Sreg} := &\left[ [[\infty, \infty], [x - 1, 1]], \left[ \left[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \right], \left[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) \right. \right. \right. \end{aligned} \quad (138)$$

$$+ 1 \Big] \Big], \Big[ \Big[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \Big], \Big[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \Big] \Big], \Big[ \Big[ \Big[ \frac{1}{2}, 0 \Big], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \Big[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \Big] \Big], \Big[ \Big[ \frac{1}{2}, 0 \Big], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \Big[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \Big] \Big] \Big] \Big]$$

**> RSreg:=SregseptruelF2(L,Sreg,ext);**

$$RSreg := \Big[ \Big[ [\infty, \infty], [x - 1, 1] \Big], \Big[ \Big[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \Big], \Big[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \Big] \Big], \Big[ \Big[ \Big[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \Big], [\ ] \Big], \Big[ \Big[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \Big], [\ ] \Big] \Big], [\ ], [\ ] \Big] \quad (139)$$

**> R1:=IrrRegAppsinglF2(L,t,E,ext);**

$$R1 := \Big[ \Big[ [x, 0] \Big], \Big[ \Big[ -2, \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1) \Big] \Big], \Big[ \Big[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 3 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 3 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} - \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} \Big] \Big], [1], [1], \Big[ \Big[ \Big[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), -2 \Big], \Big[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), -2 \Big], \Big[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 + 1) \Big] \Big] \Big], \Big[ [2 \text{RootOf}(\_Z^2 + 2, \text{index}=1) t, 2 \text{RootOf}(\_Z^2 + 2, \text{index}=2) t, -4 \text{RootOf}(\_Z^2 + 2, \text{index}=1) t] \Big], \Big[ [3 + \text{RootOf}(\_Z^2 + 1), 3 + \text{RootOf}(\_Z^2 + 1), 0] \Big], \Big[ [\infty, \infty], [x - 1, 1] \Big], \Big[ \Big[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \Big], \Big[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \Big] \Big], \Big[ \Big[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \Big], \Big[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \Big] \Big], \Big[ \Big[ \Big[ \frac{1}{2}, 0 \Big], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \Big[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \Big] \Big], \Big[ \Big[ \frac{1}{2}, 0 \Big], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \Big[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \Big] \Big] \Big] \Big] \quad (140)$$

$$\begin{aligned}
& -\text{RootOf}(\_Z^2 + 1) + 1, 0], \left[ -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \right] \right] \right] \right], \left[ \left[ \left[ \infty, \infty \right], [x - 1, 1] \right], \right. \\
& \left[ \left[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \right], \left[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \right] \right], \left[ \left[ \left[ \frac{1}{2}, -\text{RootOf}(\_Z^2 \right. \right. \right. \\
& \left. \left. \left. + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \right], \left[ \right] \right], \left[ \left[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1, -\text{RootOf}(\_Z^2 \right. \right. \right. \\
& \left. \left. \left. + 1) + \frac{1}{2} \right], \left[ \right] \right] \right] \right], \left[ \left[ \right], \left[ \right] \right], \left[ \left[ [x - 2, 2], [0, 2, 4], [2, 4, 2], \left[ \left[ [2, 0], [4, 0], [4, \right. \right. \right. \right. \right. \\
& \left. \left. \left. 2] \right] \right], \left[ \left[ \infty, \infty \right], [x, 0], [x - 1, 1] \right], \left[ \left[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \right], \left[ -2, \right. \right. \right. \\
& \left. \left. \left. \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} \right. \right. \right. \\
& \left. \left. \left. + 1 + \text{RootOf}(\_Z^2 + 1) \right] \right], \left[ 0, \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) + 1 \right] \right], \left[ \left[ \frac{1}{2}, -\text{RootOf}(\_Z^2 + 1) \right. \right. \right. \\
& \left. \left. \left. + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \right] \right], \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 3 + \text{RootOf}(\_Z^2 \right. \right. \\
& \left. \left. + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 3 + \text{RootOf}(\_Z^2 + 1), \right. \right. \\
& \left. \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} - \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} \right] \right], \left[ \frac{1}{2}, -\text{RootOf}(\_Z^2 \right. \\
& \left. + 1) + 1, -\text{RootOf}(\_Z^2 + 1) + \frac{1}{2} \right] \right], \left[ \left[ \left[ \frac{1}{2}, 0 \right], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \left[ \right. \right. \right. \\
& \left. \left. \left. -\text{RootOf}(\_Z^2 + 1) + 1, \frac{1}{2} \right] \right] \right], \left[ \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} + 1 + \text{RootOf}(\_Z^2 \right. \right. \right. \\
& \left. \left. \left. + 1), -2 \right] \right], \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), -2 \right] \right], \\
& \left[ \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=2)}{t} + 1 + \text{RootOf}(\_Z^2 + 1), \frac{2 \text{RootOf}(\_Z^2 + 2, \text{index}=1)}{t} \right. \\
& \left. \left. + 1 + \text{RootOf}(\_Z^2 + 1) \right] \right] \right], \left[ \left[ \frac{1}{2}, 0 \right], [-\text{RootOf}(\_Z^2 + 1) + 1, 0], \left[ -\text{RootOf}(\_Z^2 + 1) \right. \right. \\
& \left. \left. + 1, \frac{1}{2} \right] \right] \right], \left[ [1, 1, 1], [1, 1, 1], [1, 1, 1] \right] \right] \right]
\end{aligned}$$

**> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);**

$$\text{info1} := \left[ \left[ \left[ 0, x, \left[ -\frac{2}{x^2} \right], 2, \{ \text{RootOf}(\_Z^2 + 2, \text{index}=1) \}, \{ \text{RootOf}(\_Z^2 + 1), \text{RootOf}(\_Z^2 + 2, \text{index}=1) \} \right] \right], 2, 3, x^2, x \right] \quad (141)$$

**> find1F2Irr(L,R1,info1,x,t,ext);**

$$\left[ \left[ \left[ \left\{ -1, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6} \right\}, \left[ \frac{1}{2}, \text{RootOf}(\_Z^2 + 1) \right] \right], \frac{2(x-1)}{x^2} \right], \left[ \left[ \left\{ -1, 1, \frac{1}{2}, \frac{1}{3}, \right. \right. \right] \quad (142)$$

$$\left[ \frac{1}{6}, \frac{2}{3}, \frac{5}{6} \right], \left[ \frac{1}{2}, \text{RootOf}(\_Z^2 + 1) \right], -\frac{2(x-1)}{x^2} \right]$$

```

> TIME :=time();
Hyp1F2Solutions(L);
time() - TIME;

```

$$TIME := 81.484$$

$$\left\{ \left\{ \left[ -1 \right], \left[ \frac{1}{2}, \text{RootOf}(\_Z^2 + 1) \right], [0], [1] \right\}, \frac{2(x-1)}{x^2} \right\}$$

0.234 (143)

```

[> ##### THE RATIONAL CASE #####

```

```

> F:=sumdiffeq(hyperterm([a1],[b1,b2],x,k),k,J(x));

```

$$F := \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \quad (144)$$

=0

```

> LA:=de2diffop(F,J(x));

```

$$LA := x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \quad (145)$$

```

> L12:=subs({a1=1/5,b1=1/2,b2=1/3},LA);

```

$$L12 := x^2 Dx^3 + \frac{11 x Dx^2}{6} + \left( \frac{1}{6} - x \right) Dx - \frac{1}{5} \quad (146)$$

```

> f:=(2*(x-1)^3*(x-3))/((x-12));

```

$$f := \frac{2(x-1)^3(x-3)}{x-12} \quad (147)$$

```

> L:=ChangeOfVariables(L12,f);

```

$$L := 10 Dx^3 (x-1)^2 (x-3)^2 (x-12)^4 (x^2 - 18x + 39)^2 - 5 (x^4 - 36x^3 + 690x^2 - 2772x + 2601) Dx^2 (x-1) (x-3) (x-12)^3 (x^2 - 18x + 39) - 5 (36x^{12} - 2808x^{11} + 91584x^{10} - 1628137x^9 + 17358168x^8 - 116462292x^7 + 508992348x^6 - 1478623554x^5 + 2866693428x^4 - 3655809396x^3 + 2945718684x^2 - 1369913877x + 286162632) Dx (x-12) - 108 (x^2 - 18x + 39)^5 (x-1)^2 \quad (148)$$

```

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

```

$$ext := \emptyset \quad (149)$$

```

> ext:= indets(map(s-> ReplirrRoot(s,{ } ),ext),{RootOf,name});

```

$$ext := \emptyset \quad (150)$$

```

> extppp:={ };

```

$$extppp := \emptyset \quad (151)$$

```

> E:= Singular(L,extppp);

```

$$E := \left[ [\infty, \infty], [x-3, 3], [x^2 - 18x + 39, \text{RootOf}(\_Z^2 - 18\_Z + 39)], [x-12, 12], [x-1, 1] \right] \quad (152)$$

```

> F:=NotAppSing(L,E,ext);

```

(153)

$$F := [[x - 3, 3], [\infty, \infty], [x - 12, 12], [x - 1, 1]] \quad (153)$$

**> Sirr:=irrsinglF2(L,t,F,ext);**

$$\begin{aligned} Sirr := & \left[ [[\infty, \infty], [x - 12, 12]], \left[ \left[ \frac{3}{5}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5} \right], \right. \right. \\ & \left[ \frac{1}{5}, \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{3\sqrt{2}}{t^{3/2}} \right. \right. \\ & \left. \left. - \frac{3\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{6\sqrt{2}}{t^{3/2}} - \frac{6\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{33\sqrt{22}}{\sqrt{t}} - \frac{2}{15}, -\frac{33\sqrt{22}}{\sqrt{t}} - \frac{2}{15}, \right. \right. \\ & \left. \left. -\frac{66\sqrt{22}}{\sqrt{t}} \right], \left[ \frac{3}{2}, \frac{1}{2} \right], [2, 2], \left[ \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \frac{3}{5} \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} \right. \right. \right. \\ & \left. \left. + \frac{1}{5}, \frac{3}{5} \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5} \right], \left[ \left[ \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \right. \right. \right. \\ & \left. \left. \frac{1}{5} \right], \left[ -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15} \right] \right], [[3\sqrt{2}t(t^2 \\ & + 1), -3\sqrt{2}t(t^2 + 1), -6\sqrt{2}t(t^2 + 1)], [33\sqrt{22}t, -33\sqrt{22}t, -66\sqrt{22}t]], \left[ \left[ \right. \right. \\ & \left. \left. -\frac{2}{5}, -\frac{2}{5}, 0 \right], \left[ -\frac{2}{15}, -\frac{2}{15}, 0 \right], \left[ [x - 3, 3], [x - 1, 1] \right], \left[ \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{2}{3}, \frac{1}{2}, \right. \right. \right. \right. \\ & \left. \left. -\frac{1}{6} \right], [1, 1, 1], \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right], 2, \left[ \left[ \frac{3}{2}, 0, 2 \right], \left[ -\frac{3}{2}, \frac{1}{2}, 2 \right], [1, 1, 1], \right. \right. \\ & \left. \left. \left[ \left[ 0, \frac{3}{2} \right], \left[ 2, \frac{3}{2} \right], [2, 0] \right], 2 \right] \right] \right] \end{aligned} \quad (154)$$

**> Sreg:=regsingtruelF2(L,t,Sirr[-1],ext);**

$$\begin{aligned} Sreg := & \left[ [[x - 3, 3], [x - 1, 1]], \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{3}{2}, 0, 2 \right] \right], \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], \left[ -\frac{3}{2}, \frac{1}{2}, 2 \right] \right], \right. \\ & \left[ \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right], \left[ \left[ 0, \frac{3}{2} \right], \left[ 2, \frac{3}{2} \right], [2, 0] \right] \right] \end{aligned} \quad (155)$$

**> RSreg:=SregseptruelF2(L,Sreg,ext);**

$$\begin{aligned} RSreg := & \left[ [[x - 3, 3], [x - 1, 1]], \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{3}{2}, 0, 2 \right] \right], \left[ \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], [ ] \right], \left[ \left[ \right. \right. \right. \\ & \left. \left. -\frac{3}{2}, \frac{1}{2} \right], [2] \right] \right], [ ], [ ] \end{aligned} \quad (156)$$

**> R1:=IrrRegAppsinglF2(L,t,E,ext);**

$$\begin{aligned} R1 := & \left[ [[\infty, \infty], [x - 12, 12]], \left[ \left[ \frac{3}{5}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5} \right], \right. \right. \\ & \left[ \frac{1}{5}, \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{3\sqrt{2}}{t^{3/2}} \right. \right. \\ & \left. \left. - \frac{3\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{6\sqrt{2}}{t^{3/2}} - \frac{6\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{33\sqrt{22}}{\sqrt{t}} - \frac{2}{15}, -\frac{33\sqrt{22}}{\sqrt{t}} - \frac{2}{15}, \right. \right. \end{aligned} \quad (157)$$



$$\begin{aligned}
& -\frac{66\sqrt{22}}{\sqrt{t}} \Big] \Big], \left[ \frac{3}{2}, \frac{1}{2} \right], [2, 2], \left[ \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \frac{3}{5} \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} \right. \right. \right. \\
& \left. \left. \left. + \frac{1}{5}, \frac{3}{5} \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5} \right] \right], \left[ \left[ \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \right. \right. \\
& \left. \left. \frac{1}{5} \right], \left[ -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15} \right] \right], \left[ [3\sqrt{2}t(t^2 \right. \\
& \left. + 1), -3\sqrt{2}t(t^2 + 1), -6\sqrt{2}t(t^2 + 1)], [33\sqrt{22}t, -33\sqrt{22}t, -66\sqrt{22}t], \left[ \left[ \right. \right. \\
& \left. \left. -\frac{2}{5}, -\frac{2}{5}, 0 \right], \left[ -\frac{2}{15}, -\frac{2}{15}, 0 \right] \right], \left[ [[x-3, 3], [x-1, 1]], \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{3}{2}, 0, 2 \right] \right], \right. \\
& \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], \left[ -\frac{3}{2}, \frac{1}{2}, 2 \right] \right], \left[ \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right], \left[ \left[ 0, \frac{3}{2} \right], \left[ 2, \frac{3}{2} \right], [2, \right. \right. \\
& \left. \left. 0 \right] \right] \right], \left[ \left[ [[x-3, 3], [x-1, 1]], \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{3}{2}, 0, 2 \right] \right], \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], [ ] \right], \left[ \left[ \right. \right. \right. \\
& \left. \left. -\frac{3}{2}, \frac{1}{2} \right], [2] \right] \right], [ ], [ ] \Big], \left[ [[x^2 - 18x + 39, \text{RootOf}(\_Z^2 - 18\_Z + 39)], [0, 2, 4]], \right. \\
& \left. [[2, 4, 2]], [[2, 0], [4, 0], [4, 2]] \right], \left[ [[x-3, 3], [\infty, \infty], [x-12, 12], [x-1, 1]], \right. \\
& \left[ \left[ 0, \frac{2}{3}, \frac{1}{2} \right], \left[ \frac{3}{5}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5} \right], \left[ \frac{1}{5}, \frac{33\sqrt{22}}{\sqrt{t}} \right. \right. \\
& \left. \left. + \frac{1}{15}, -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15} \right], \left[ \frac{3}{2}, 0, 2 \right] \right], \left[ \left[ \frac{2}{3}, \frac{1}{2}, -\frac{1}{6} \right], \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, \right. \right. \\
& \left. \left. -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{6\sqrt{2}}{t^{3/2}} - \frac{6\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{33\sqrt{22}}{\sqrt{t}} - \frac{2}{15}, -\frac{33\sqrt{22}}{\sqrt{t}} - \frac{2}{15}, \right. \right. \\
& \left. \left. -\frac{66\sqrt{22}}{\sqrt{t}} \right], \left[ -\frac{3}{2}, \frac{1}{2}, 2 \right] \right], \left[ \left[ \left[ \frac{2}{3}, 0 \right], \left[ \frac{1}{2}, 0 \right], \left[ \frac{1}{2}, \frac{2}{3} \right] \right], \left[ \left[ \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \right. \right. \\
& \left. \left. \frac{3}{5} \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \frac{3}{5} \right], \left[ -\frac{3\sqrt{2}}{t^{3/2}} - \frac{3\sqrt{2}}{\sqrt{t}} + \frac{1}{5}, \frac{3\sqrt{2}}{t^{3/2}} + \frac{3\sqrt{2}}{\sqrt{t}} \right. \right. \\
& \left. \left. + \frac{1}{5} \right] \right], \left[ \left[ \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \frac{1}{5} \right], \left[ -\frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15}, \right. \right. \\
& \left. \left. \frac{33\sqrt{22}}{\sqrt{t}} + \frac{1}{15} \right] \right], \left[ \left[ 0, \frac{3}{2} \right], \left[ 2, \frac{3}{2} \right], [2, 0] \right], [[1, 1, 1], [1, 2, 2], [1, 2, 2], [1, 1, 1]] \Big] \Big]
\end{aligned}$$

**> info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);**

$$\text{info1} := \left[ \left[ \left[ \infty, \frac{1}{x}, [2x^3 + 12x^2 + 18x], 3, \emptyset, \emptyset \right], \left[ 12, x-12, \left[ \frac{23958}{x-12} \right], 1, \emptyset, \emptyset \right] \right], 4, 5, x \right] \quad (158)$$

$$-12, 1]$$

$$\begin{aligned} &> \text{find1F2Rat}(L, R1, \text{info1}, x, t, T, \text{ext}); \\ &\left[ \left[ \left[ \left\{ \frac{1}{5}, \frac{8}{15}, \frac{13}{15} \right\}, \left[ \frac{1}{2}, \frac{1}{3} \right] \right], \frac{2(x-1)^3(x-3)}{x-12} \right] \right] \end{aligned} \quad (159)$$

$$\begin{aligned} &> \text{TIME} := \text{time}(); \\ &\text{Hyp1F2Solutions}(L); \\ &\text{time}() - \text{TIME}; \\ &\text{TIME} := 85.109 \\ &\left[ \left[ \left[ \left[ \left[ \frac{1}{5} \right], \left[ \frac{1}{2}, \frac{1}{3} \right], [0], [1] \right] \right], \frac{2(x-1)^3(x-3)}{x-12} \right] \right] \\ &0.484 \end{aligned} \quad (160)$$

$$\begin{aligned} &> F := \text{sumdiffeq}(\text{hyperterm}([a1], [b1, b2], x, k), k, J(x)); \\ F &:= \left( \frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 + 1) \left( \frac{d^2}{dx^2} J(x) \right) x + (b2 b1 - x) \left( \frac{d}{dx} J(x) \right) - a1 J(x) \\ &= 0 \end{aligned} \quad (161)$$

$$\begin{aligned} &> LA := \text{de2diffop}(F, J(x)); \\ LA &:= x^2 Dx^3 + (b1 x + b2 x + x) Dx^2 + (b2 b1 - x) Dx - a1 \end{aligned} \quad (162)$$

$$\begin{aligned} &> L12 := \text{subs}(\{a1=1/3, b1=1/2, b2=1/5\}, LA); \\ L12 &:= x^2 Dx^3 + \frac{17x Dx^2}{10} + \left( \frac{1}{10} - x \right) Dx - \frac{1}{3} \end{aligned} \quad (163)$$

$$\begin{aligned} &> f := (2*(x-7)^2)/((x-1)); \\ f &:= \frac{2(x-7)^2}{x-1} \end{aligned} \quad (164)$$

$$\begin{aligned} &> L := \text{ChangeOfVariables}(L12, f); \\ L &:= 30 Dx^3 (x-1)^4 (x-7) (x+5)^2 + 3 (17x^2 + 170x - 1735) Dx^2 (x-1)^3 (x+5) \\ &\quad - 3 (20x^5 + 259x^4 + 174x^3 - 12248x^2 - 49090x - 94635) Dx (x-1) - 20 (x+5)^5 \end{aligned} \quad (165)$$

$$\begin{aligned} &> \text{ext} := \text{indets}(L, \{\text{RootOf}, \text{name}\}) \text{ minus } \{x, Dx\}; \\ \text{ext} &:= \emptyset \end{aligned} \quad (166)$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\text{map}(s \rightarrow \text{ReplirrRoot}(s, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\ \text{ext} &:= \emptyset \end{aligned} \quad (167)$$

$$\begin{aligned} &> \text{extppp} := \{\}; \\ \text{extppp} &:= \emptyset \end{aligned} \quad (168)$$

$$\begin{aligned} &> E := \text{Singular}(L, \text{extppp}); \\ E &:= [[\infty, \infty], [x+5, -5], [x-7, 7], [x-1, 1]] \end{aligned} \quad (169)$$

$$\begin{aligned} &> F := \text{NotAppSing}(L, E, \text{ext}); \\ F &:= [[x-7, 7], [\infty, \infty], [x-1, 1]] \end{aligned} \quad (170)$$

$$\begin{aligned} &> \text{Sirr} := \text{irrsglF2}(L, t, F, \text{ext}); \\ \text{Sirr} &:= \left[ [[\infty, \infty], [x-1, 1]], \left[ \left[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \right], \left[ \frac{1}{3}, \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \right. \right. \right] \end{aligned} \quad (171)$$

$$\begin{aligned}
& -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big] \Big], \Big[ \Big[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{2\sqrt{2}}{\sqrt{t}} \Big], \Big[ \frac{6\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{6\sqrt{2}}{\sqrt{t}} \\
& -\frac{2}{5}, -\frac{12\sqrt{2}}{\sqrt{t}} \Big] \Big], \Big[ \frac{1}{2}, \frac{1}{2} \Big], [2, 2], \Big[ \Big[ \Big[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{\sqrt{2}}{\sqrt{t}} \\
& -\frac{1}{15}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big] \Big], \Big[ \Big[ \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \\
& \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big] \Big], \Big[ [\sqrt{2}t, -\sqrt{2}t, -2\sqrt{2}t], [6\sqrt{2}t, -6\sqrt{2}t, -12\sqrt{2}t] \Big], \Big[ \Big[ -\frac{2}{5}, \\
& -\frac{2}{5}, 0 \Big], \Big[ -\frac{2}{5}, -\frac{2}{5}, 0 \Big] \Big], \Big[ [x-7, 7], \Big[ \Big[ \Big[ \frac{8}{5}, 0, 1 \Big], \Big[ -\frac{8}{5}, -\frac{3}{5}, 1 \Big], [1, 1, 1], \Big[ \Big[ 0, \frac{8}{5} \Big], \\
& \Big[ 1, \frac{8}{5} \Big], [1, 0] \Big], 2 \Big] \Big] \Big] \Big]
\end{aligned}$$

**> Sreg:=regsingtrue1F2(L,t,Sirr[-1],ext);**

$$Sreg := \Big[ [x-7, 7], \Big[ \Big[ \frac{8}{5}, 0, 1 \Big], \Big[ \Big[ -\frac{8}{5}, -\frac{3}{5}, 1 \Big], \Big[ \Big[ \Big[ 0, \frac{8}{5} \Big], \Big[ 1, \frac{8}{5} \Big], [1, 0] \Big] \Big] \Big] \Big] \quad (172)$$

**> RSreg:=Sregseptrue1F2(L,Sreg,ext);**

$$RSreg := \Big[ \Big[ [x-7, 7], \Big[ \Big[ \frac{8}{5}, 0, 1 \Big], \Big[ \Big[ \Big[ -\frac{8}{5}, -\frac{3}{5} \Big], [1] \Big] \Big] \Big], [ ], [ ] \Big] \quad (173)$$

**> R1:=IrrRegAppsing1F2(L,t,E,ext);**

$$\begin{aligned}
RI := & \Big[ \Big[ [ \infty, \infty ], [x-1, 1] \Big], \Big[ \Big[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big], \Big[ \frac{1}{3}, \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \\
& -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big] \Big], \Big[ \Big[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{2\sqrt{2}}{\sqrt{t}} \Big], \Big[ \frac{6\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{6\sqrt{2}}{\sqrt{t}} \\
& -\frac{2}{5}, -\frac{12\sqrt{2}}{\sqrt{t}} \Big] \Big], \Big[ \frac{1}{2}, \frac{1}{2} \Big], [2, 2], \Big[ \Big[ \Big[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{\sqrt{2}}{\sqrt{t}} \\
& -\frac{1}{15}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big] \Big], \Big[ \Big[ \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \Big], \Big[ -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \\
& \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big] \Big], \Big[ [\sqrt{2}t, -\sqrt{2}t, -2\sqrt{2}t], [6\sqrt{2}t, -6\sqrt{2}t, -12\sqrt{2}t] \Big], \Big[ \Big[ -\frac{2}{5}, \\
& -\frac{2}{5}, 0 \Big], \Big[ -\frac{2}{5}, -\frac{2}{5}, 0 \Big] \Big], \Big[ [x-7, 7], \Big[ \Big[ \frac{8}{5}, 0, 1 \Big], \Big[ \Big[ -\frac{8}{5}, -\frac{3}{5}, 1 \Big], \Big[ \Big[ \Big[ 0, \frac{8}{5} \Big], \Big[ 1, \\
& \frac{8}{5} \Big], [1, 0] \Big] \Big], \Big[ \Big[ [x-7, 7], \Big[ \Big[ \frac{8}{5}, 0, 1 \Big], \Big[ \Big[ \Big[ -\frac{8}{5}, -\frac{3}{5} \Big], [1] \Big] \Big] \Big], [ ], [ ] \Big], \Big[ [x+5, \\
& -5], [[0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]]], \Big[ [x-7, 7], [ \infty, \infty ], [x-1, \\
& 1], \Big[ \Big[ \frac{8}{5}, 0, 1 \Big], \Big[ \frac{1}{3}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \Big], \Big[ \frac{1}{3}, \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, -\frac{6\sqrt{2}}{\sqrt{t}}
\end{aligned}$$

$$\left[ \left[ -\frac{1}{15} \right], \left[ \left[ -\frac{8}{5}, -\frac{3}{5}, 1 \right], \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{2\sqrt{2}}{\sqrt{t}} \right], \left[ \frac{6\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{2}{5}, -\frac{12\sqrt{2}}{\sqrt{t}} \right], \left[ \left[ 0, \frac{8}{5} \right], \left[ 1, \frac{8}{5} \right], [1, 0] \right], \left[ \left[ \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \right], \left[ -\frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \right], \left[ \left[ \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \right], \left[ -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{1}{3} \right], \left[ -\frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15}, \frac{6\sqrt{2}}{\sqrt{t}} - \frac{1}{15} \right] \right], [[1, 1, 1], [1, 2, 2], [1, 2, 2]] \right]$$

> **info1:=Sirr1F2info1(L,R1[1],R1[2],x,t,ext);**

$$info1 := \left[ \left[ \left[ \infty, \frac{1}{x}, [2x], 1, \emptyset, \emptyset \right], \left[ 1, x-1, \left[ \frac{72}{x-1} \right], 1, \emptyset, \emptyset \right], 2, 3, x-1, 1 \right] \right] \quad (175)$$

> **find1F2Rat(L,R1,info1,x,t,T,ext);**

$$\left[ \left[ \left[ \left\{ -1, 1, \frac{1}{3}, \frac{2}{3} \right\}, \left[ \frac{1}{2}, \frac{1}{5} \right] \right], \left[ \left\{ \frac{1}{2}, \frac{1}{6}, \frac{5}{6} \right\}, \left[ \frac{1}{2}, \frac{7}{10} \right] \right], \frac{2(x-7)^2}{x-1} \right] \right] \quad (176)$$

> **TIME :=time();**  
**Hyp1F2Solutions(L);**  
**time() - TIME;**

$$TIME := 95.328$$

$$\left\{ \left[ \left[ \left[ \frac{1}{3} \right], \left[ \frac{1}{2}, \frac{1}{5} \right], [0], [1] \right], \frac{2(x-7)^2}{x-1} \right] \right\}$$

$$2.265$$

(177)