Rechnen mit Zahlen

Taschenrechner

In[2]:=
$$\frac{1.23 + 2.25}{3.67}$$

Out[2]= 0.948229

In[3]:=
```
N[π, 500]
```

Out[3]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803
82534211706798214808651328230664709384460955058223172535940812848111745028410270193852
10555964462294895493038196442881097566593344612847564823378678316527120190914564856692
46034861045432664821339360726024914127372458700660631558817488152092096282925409171536
36789259036001133053054882046652138414695194151160943305727036575959195309218611738193
61179310511854807446237996274956735188575272489122793818301194 91

Ganzzahlarithmetik

- Graphics -

In[4]:=
```
100!
```

Out[4]=
```
933262154439441526816992388562667004907159682643816214685929638952175999932299156089414697615651828625369792082722375825118521091686400000000000000000000000
```

In[5]:=
```
Binomial[100, 90]
```

Out[5]=
```
17310309456440
```

In[6]:=
```
2^16 - 1
```

Out[6]=
```
65535
```

In[7]:=
```
2^32 - 1
```

Out[7]=
```
4294967295
```

In[8]:=
```
N[%]
```

Out[8]=
$$4.29497 \times 10^9$$

In[9]:=
$$\frac{100!}{2^{100}}$$

Out[9]=
```
588971222367687651371627846346807888288472382883312574253249804256440585603406374176100603020409333040832764576077461242675781258
```

In[10]:=
$$\sum_{k=1}^{100} \frac{1}{k}$$

Out[10]=
$$\frac{14466636279520351160221518043104131447711}{2788815009188499086581352357412492142272}$$

In[11]:=
```
% // N
```

Out[11]=
```
5.18738
```

In[12]:=
```
FactorInteger[100!]
```

Out[12]=

2	97
3	48
5	24
7	16
11	9
13	7
17	5
19	5
23	4
29	3
31	3
37	2
41	2
43	2
47	2
53	1
59	1
61	1
67	1
71	1
73	1
79	1
83	1
89	1
97	1

In[13]:=
```
PrimeQ[1234567]
```

Out[13]= False

In[14]:=
```
FactorInteger[1234567]
```

Out[14]=

127	1
9721	1

In[15]:=
```
NextPrime[n_] := n + 1 /; PrimeQ[n + 1]
NextPrime[n_] := NextPrime[n + 1]
```

In[17]:=
```
NextPrime[1234567]
```

Out[17]= 1234577

In[18]:=
```
PrimeQ[%]
```

Out[18]= True

Rechnen mit Symbolen

Polynome und rationale Funktionen

In[19]:=
```
pol = (x + y)^10 - (x - y)^10
```

Out[19]= $(x + y)^{10} - (x - y)^{10}$

In[20]:=
```
Expand[pol]
```

Out[20]= $20\, y\, x^9 + 240\, y^3\, x^7 + 504\, y^5\, x^5 + 240\, y^7\, x^3 + 20\, y^9\, x$

In[21]:=
```
Factor[pol]
```

Out[21]= $4\, x\, y\, (5\, x^4 + 10\, y^2\, x^2 + y^4)\, (x^4 + 10\, y^2\, x^2 + 5\, y^4)$

In[22]:=
```
rat = (1 - x^10)/(1 - x^4)
```

Out[22]= $\frac{1 - x^{10}}{1 - x^4}$

In[23]:=
```
Together[rat]
```

Out[23]= $\frac{x^8 + x^6 + x^4 + x^2 + 1}{x^2 + 1}$

In[24]:=
```
Factor[rat]
```

Out[24]= $\frac{(x^4 - x^3 + x^2 - x + 1)\, (x^4 + x^3 + x^2 + x + 1)}{x^2 + 1}$

In[25]:=
```
Factor[x^6 + x^2 + 1]
```

Out[25]= $x^6 + x^2 + 1$

In[26]:=
```
Factor[x^4 + 1]
```

Out[26]= $x^4 + 1$

In[27]:=
```
Factor[x^4 + 1, GaussianIntegers → True]
```

Out[27]= $(x^2 - i)\,(x^2 + i)$

In[28]:=
```
Factor[x^4 + 1, Extension → {Sqrt[2]}]
```

Out[28]= $-\left(-x^2 + \sqrt{2}\,x - 1\right)\left(x^2 + \sqrt{2}\,x + 1\right)$

Gleichungen

In[29]:=
```
s = Solve[x^2 - 3 x - 1 == 0, x]
```

Out[29]= $\left\{\left\{x \to \frac{1}{2}\left(3 - \sqrt{13}\right)\right\}, \left\{x \to \frac{1}{2}\left(3 + \sqrt{13}\right)\right\}\right\}$

In[30]:=
```
x /. s
```

Out[30]= $\left\{\frac{1}{2}\left(3 - \sqrt{13}\right), \frac{1}{2}\left(3 + \sqrt{13}\right)\right\}$

In[31]:=
```
s = Solve[x^3 - 3 x - 1 == 0, x]
```

Out[31]=
$$\left\{\left\{x \to \frac{1}{\sqrt[3]{\frac{1}{2}\left(1 + i\sqrt{3}\right)}} + \sqrt[3]{\frac{1}{2}\left(1 + i\sqrt{3}\right)}\right\},\right.$$
$$\left.\left\{x \to -\frac{1}{2}\left(1 - i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(1 + i\sqrt{3}\right)} - \left(\frac{1}{2}\left(1 + i\sqrt{3}\right)\right)^{2/3}\right\}, \left\{x \to -\frac{1 - i\sqrt{3}}{2^{2/3}\sqrt[3]{1 + i\sqrt{3}}} - \frac{\left(1 + i\sqrt{3}\right)^{4/3}}{2\sqrt[3]{2}}\right\}\right\}$$

In[32]:=
```
s // N
```

Out[32]= $\{\{x \to 1.87939 + 0.\,i\}, \{x \to -1.53209 + 0.\,i\}, \{x \to -0.347296 + 3.33067 \times 10^{-16}\,i\}\}$

In[33]:=
```
NSolve[x^3 - 3 x - 1 == 0, x]
```

Out[33]= $\{\{x \to -1.53209\}, \{x \to -0.347296\}, \{x \to 1.87939\}\}$

In[34]:=
```
s = Solve[x^4 - 3 x - 1 == 0, x]
```

Out[34]=

$$\left\{\left\{x \to -\frac{1}{2}\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}-\frac{1}{2}\sqrt{\left(4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}-\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}-\frac{6}{\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}}\right)}\right\},\right.$$

$$\left\{x \to -\frac{1}{2}\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}+\frac{1}{2}\sqrt{\left(4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}-\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}-\frac{6}{\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}}\right)}\right\},$$

$$\left\{x \to \frac{1}{2}\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}-\frac{1}{2}\sqrt{\left(4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}-\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}+\frac{6}{\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}}\right)}\right\},$$

$$\left.\left\{x \to \frac{1}{2}\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}+\frac{1}{2}\sqrt{\left(4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}-\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}+\frac{6}{\sqrt{-4\sqrt[3]{\frac{2}{3\left(81+\sqrt{7329}\right)}}+\frac{\sqrt[3]{\frac{1}{2}\left(81+\sqrt{7329}\right)}}{3^{2/3}}}}\right)}\right\}\right\}$$

In[35]:=
```
s // N
```

Out[35]=
$$\{\{x \to -0.605102 - 1.26713\, i\},\ \{x \to -0.605102 + 1.26713\, i\},\ \{x \to -0.329409\},\ \{x \to 1.53961\}\}$$

In[36]:=
```
s = Solve[x^5 - 3 x - 1 == 0, x]
```

Out[36]=
$$\{\{x \to \text{Root}[\#1^5 - 3\,\#1 - 1\ \&,\ 1]\},\ \{x \to \text{Root}[\#1^5 - 3\,\#1 - 1\ \&,\ 2]\},\ \{x \to \text{Root}[\#1^5 - 3\,\#1 - 1\ \&,\ 3]\},\ \{x \to \text{Root}[\#1^5 - 3\,\#1 - 1\ \&,\ 4]\},\ \{x \to \text{Root}[\#1^5 - 3\,\#1 - 1\ \&,\ 5]\}\}$$

In[37]:=
```
s // N
```

Out[37]=
$$\{\{x \to -1.21465\},\ \{x \to -0.334734\},\ \{x \to 1.38879\},\ \{x \to 0.0802951 - 1.32836\, i\},\ \{x \to 0.0802951 + 1.32836\, i$$

In[38]:=
```
Product[(x /. s[[k]]), {k, 1, 5}] // RootReduce
```

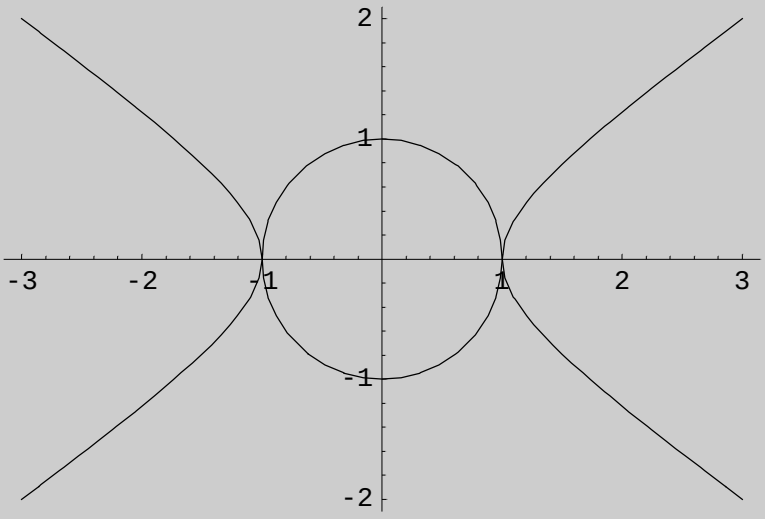
Out[38]=
$$1$$

In[39]:=
```
Solve[{x^2 + y^2 == 1, -x^2 + 2 y^2 + 1 == 0}, {x, y}]
```

Out[39]=
$$\{\{x \to -1,\ y \to 0\},\ \{x \to -1,\ y \to 0\},\ \{x \to 1,\ y \to 0\},\ \{x \to 1,\ y \to 0\}\}$$

In[40]:=
```
Needs["Graphics`ImplicitPlot`"]
```

In[41]:=
```
ImplicitPlot[{x^2 + y^2 == 1, -x^2 + 2 y^2 + 1 == 0}, {x, -3, 3}, {y, -3, 3}]
```

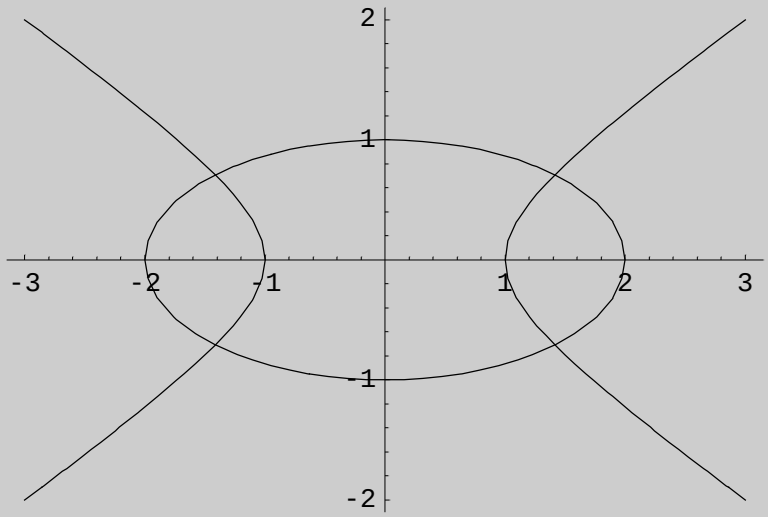


Out[41]= - Graphics -

In[42]:=
```
Solve[{x^2/4 + y^2 == 1, -x^2 + 2 y^2 + 1 == 0}, {x, y}]
```

Out[42]=
$$\left\{\left\{x \to -\sqrt{2},\ y \to -\frac{1}{\sqrt{2}}\right\},\ \left\{x \to -\sqrt{2},\ y \to \frac{1}{\sqrt{2}}\right\},\ \left\{x \to \sqrt{2},\ y \to -\frac{1}{\sqrt{2}}\right\},\ \left\{x \to \sqrt{2},\ y \to \frac{1}{\sqrt{2}}\right\}\right\}$$

In[43]:=
```
ImplicitPlot[{x^2/4 + y^2 == 1, -x^2 + 2 y^2 + 1 == 0}, {x, -3, 3}, {y, -3, 3}]
```

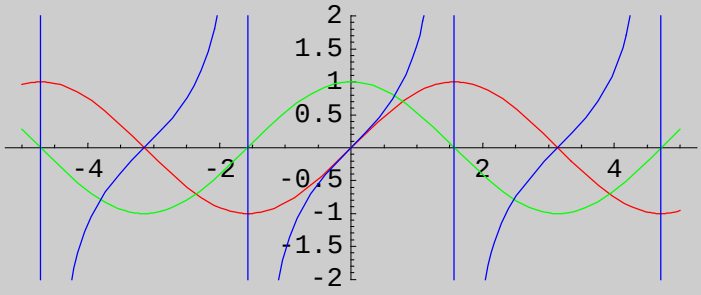


Out[43]= - Graphics -

Graphische Darstellungen

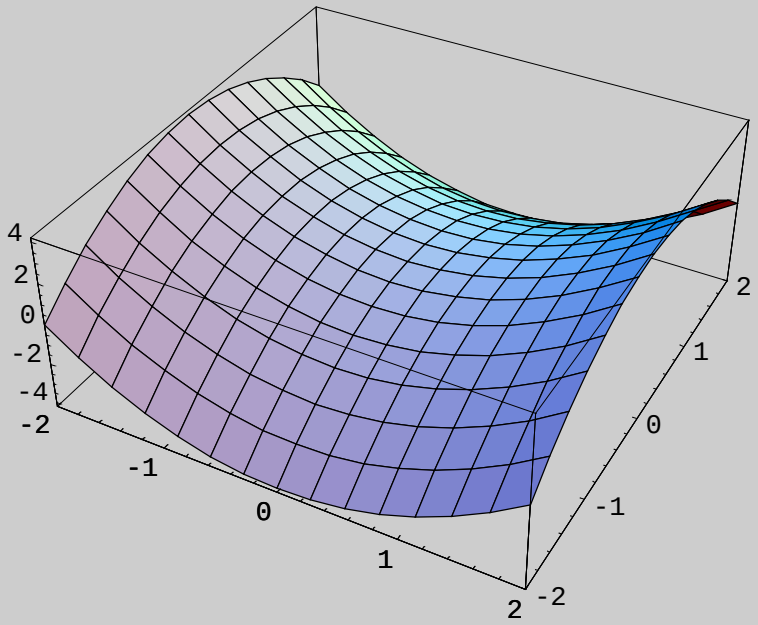
Funktionen $\mathbb{R}^2 \to \mathbb{R}$

In[44]:=
```
Plot[{Sin[x], Cos[x], Tan[x]}, {x, -5, 5},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
 PlotRange → {-2, 2}, AspectRatio → Automatic]
```



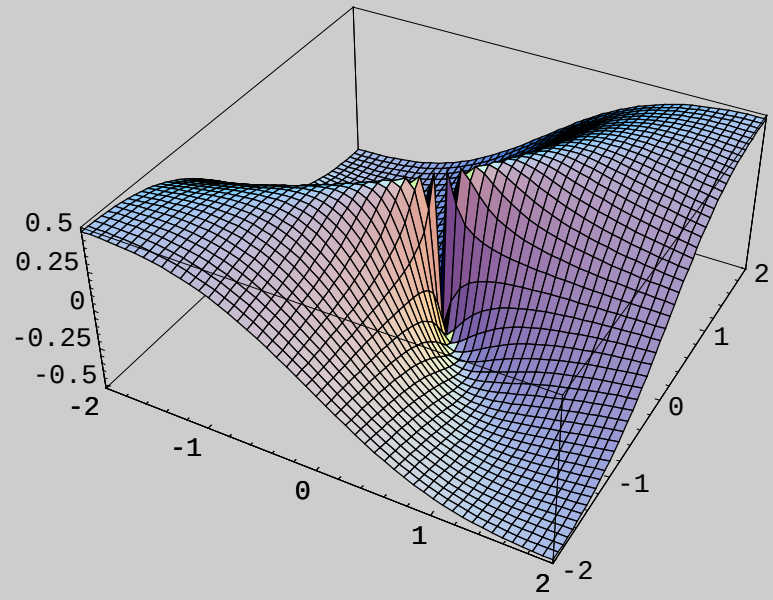
Out[44]= - Graphics -

In[45]:=
```
Plot3D[x^2 - y^2, {x, -2, 2}, {y, -2, 2}]
```



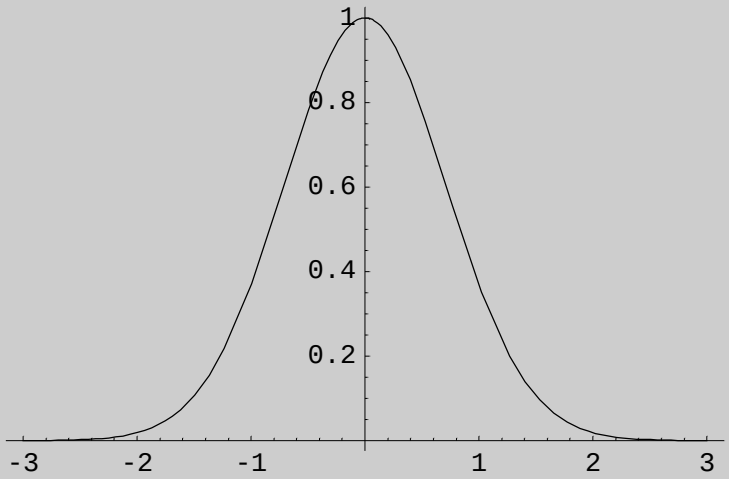
Out[45]= - SurfaceGraphics -

In[46]:=
```
Plot3D[x y / (x^2 + y^2), {x, -2, 2}, {y, -2, 2}, PlotPoints → 50]
```



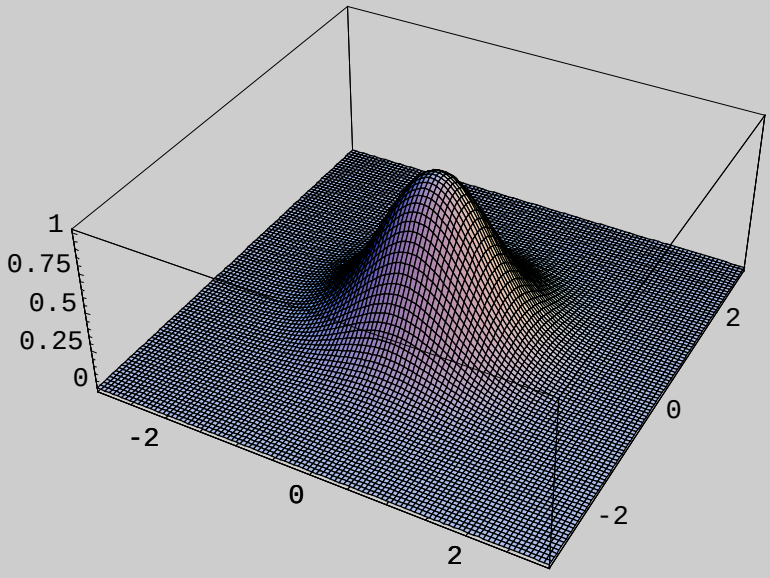
Out[46]= - SurfaceGraphics -

In[47]:=
```
Plot[e^-x^2, {x, -3, 3}]
```



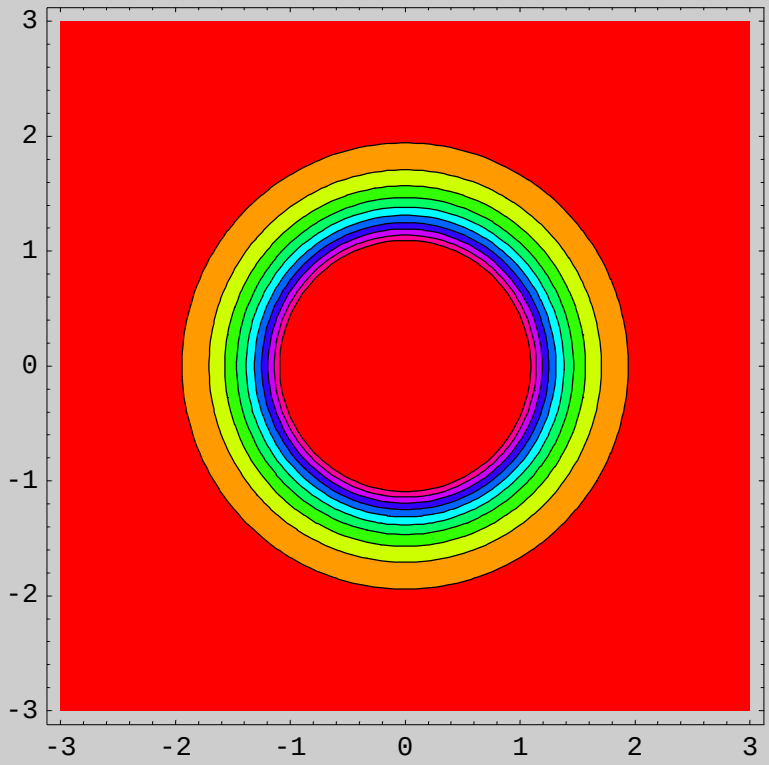
Out[47]= - Graphics -

In[48]:=
```
Plot3D[e^-(x^2+y^2), {x, -3, 3}, {y, -3, 3}, PlotRange → All, PlotPoints → 100]
```



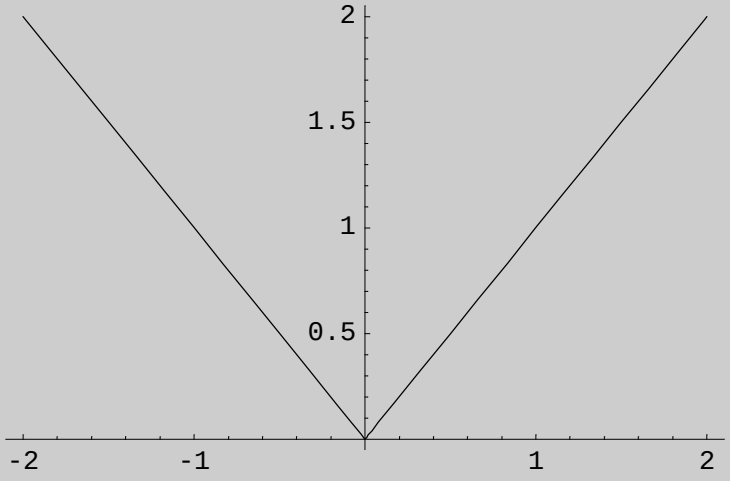
Out[48]= - SurfaceGraphics -

In[49]:=
```
ContourPlot[e^-(x^2+y^2), {x, -3, 3},
 {y, -3, 3}, ColorFunction → Hue, PlotPoints → 100]
```



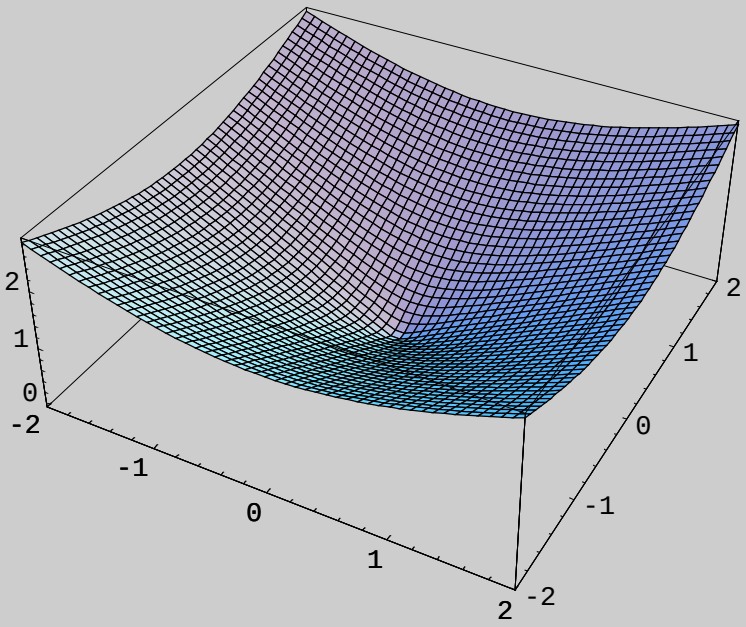
Out[49]= - ContourGraphics -

In[50]:=
```
Plot[√x^2, {x, -2, 2}]
```



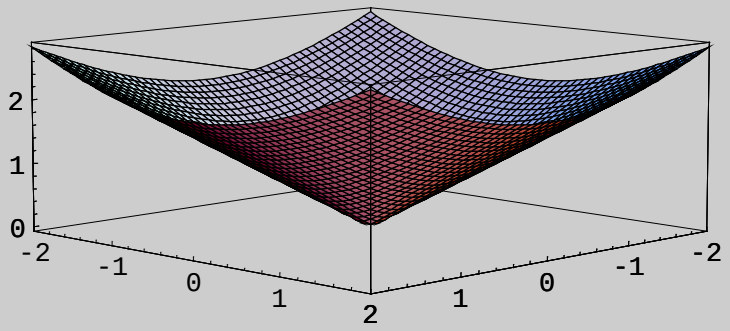
Out[50]= - Graphics -

In[51]:=
```
Plot3D[Sqrt[x^2 + y^2], {x, -2, 2}, {y, -2, 2}, PlotPoints → 50]
```



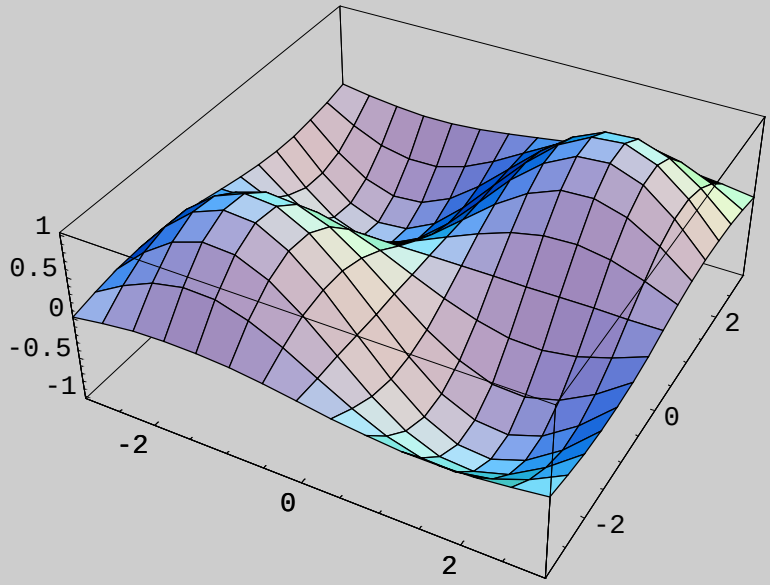
Out[51]= - SurfaceGraphics -

In[52]:=
```
Plot3D[Sqrt[x^2 + y^2], {x, -2, 2}, {y, -2, 2}, PlotPoints → 50, ViewPoint → {5, 5, 1}]
```



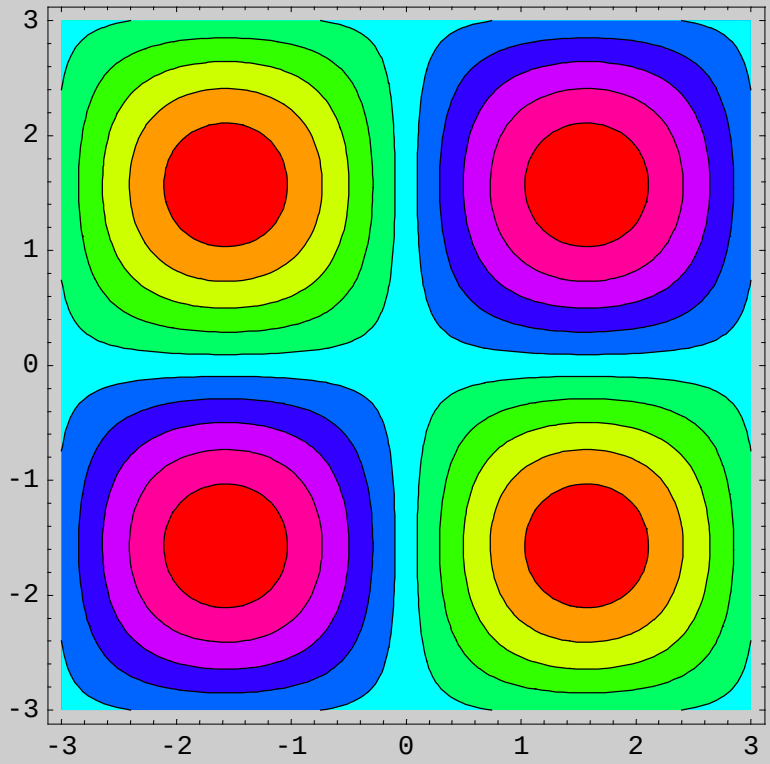
Out[52]= - SurfaceGraphics -

In[53]:=
```
Plot3D[Sin[x] Sin[y], {x, -3, 3}, {y, -3, 3}]
```



Out[53]= - SurfaceGraphics -

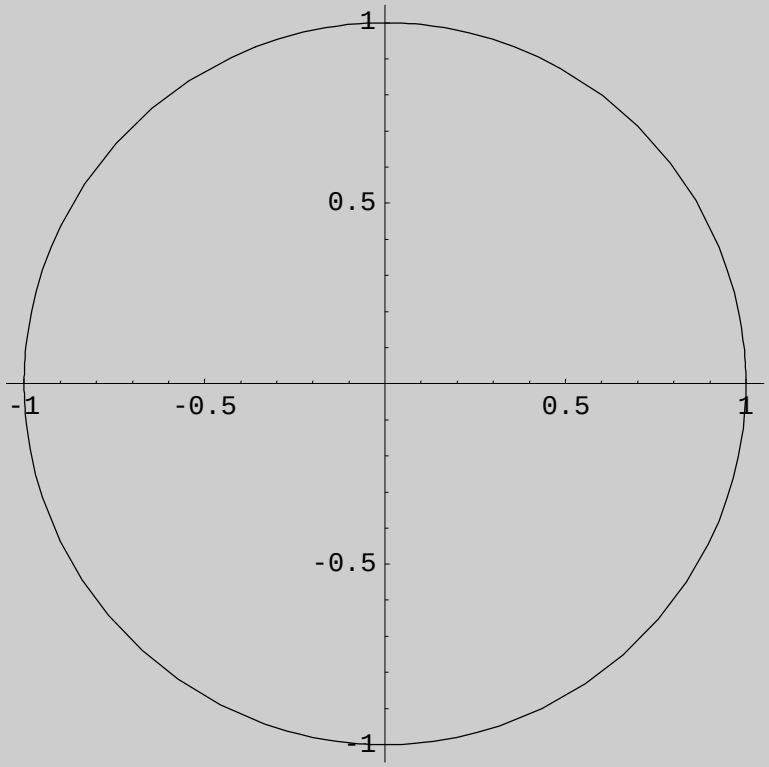
In[54]:=
```
ContourPlot[Sin[x] Sin[y], {x, -3, 3},
 {y, -3, 3}, ColorFunction → Hue, PlotPoints → 100]
```



Out[54]= - ContourGraphics -

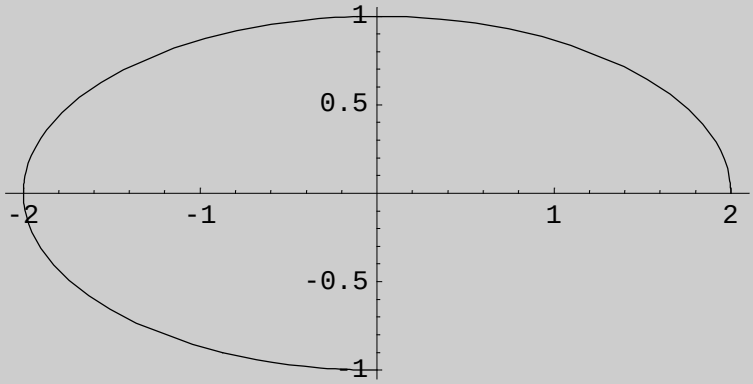
Funktionen $\mathbb{R} \to \mathbb{R}^2$: Kurven

In[55]:=
```
ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 π}, AspectRatio → Automatic]
```



Out[55]= - Graphics -

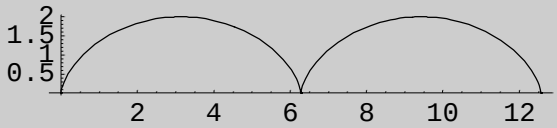
In[56]:=
```
ParametricPlot[{2 Cos[t], Sin[t]}, {t, 0, 3/2 π}, AspectRatio → Automatic]
```



Out[56]= - Graphics -

■ Zykloide

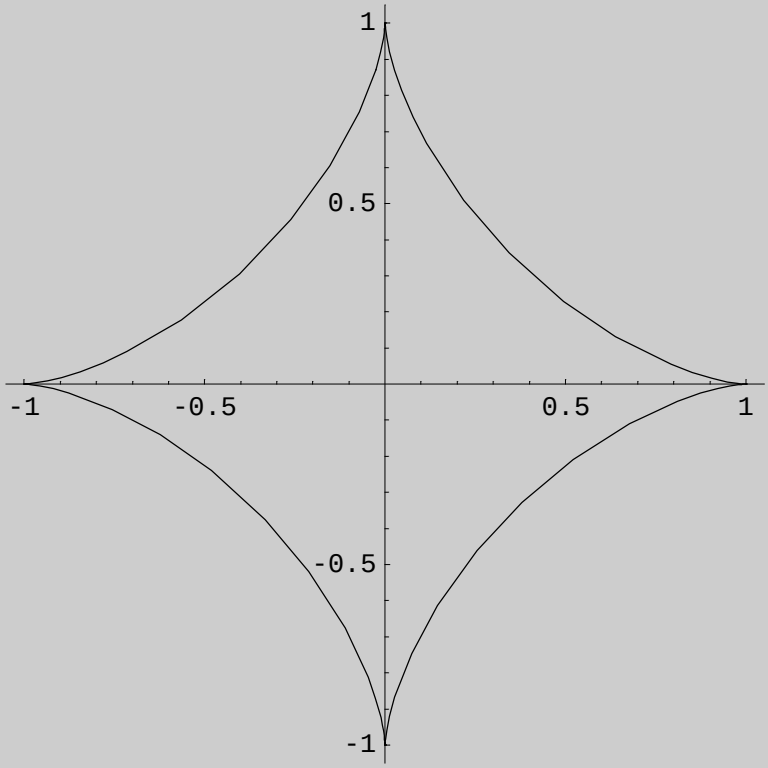
In[57]:=
```
ParametricPlot[{t - Sin[t], 1 - Cos[t]}, {t, 0, 4 π}, AspectRatio → Automatic]
```



Out[57]= - Graphics -

■ Astroide

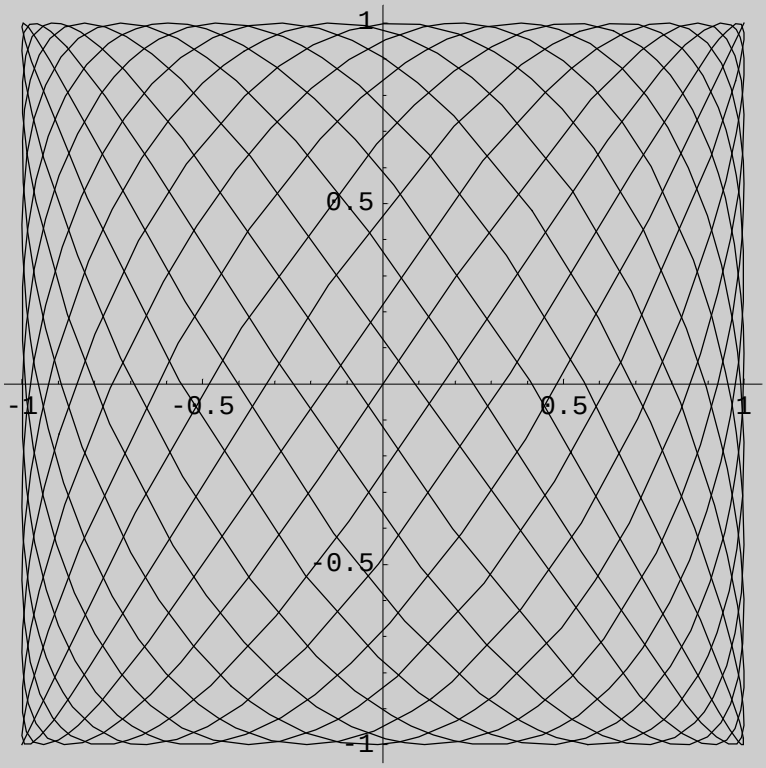
In[58]:=
```
ParametricPlot[{Cos[t]^3, Sin[t]^3}, {t, 0, 2 π}, AspectRatio → Automatic]
```



Out[58]= - Graphics -

■ Lissajous-Kurve

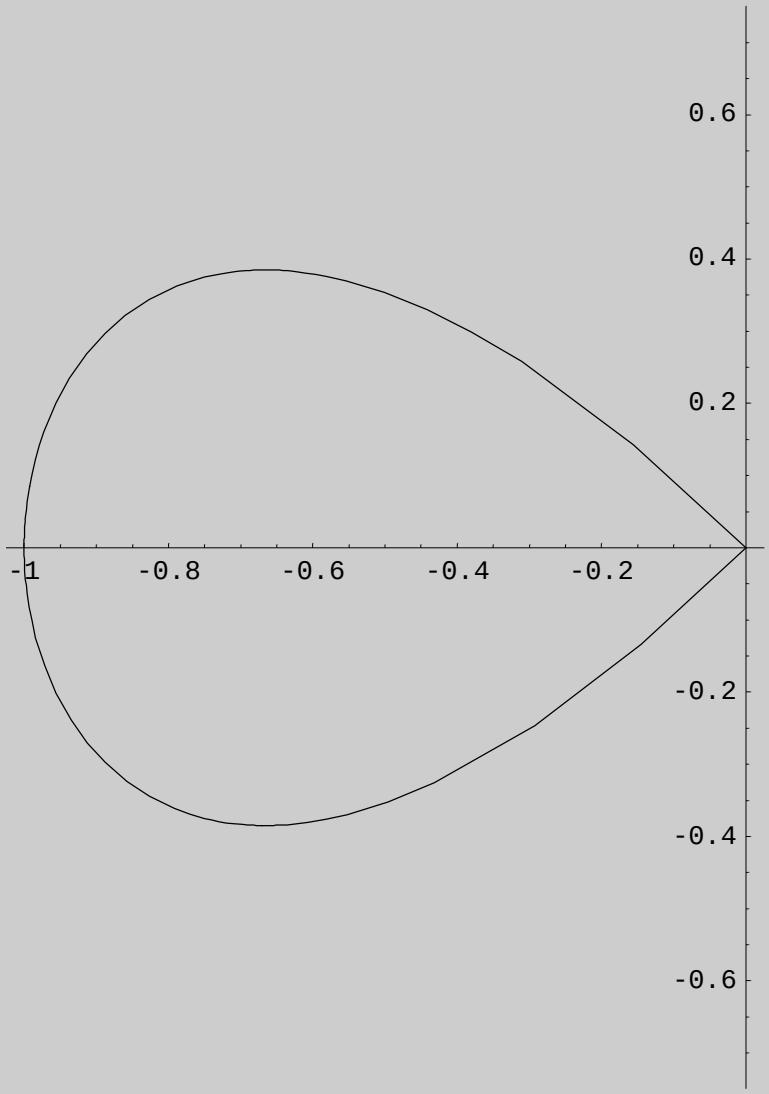
In[59]:=
```
ParametricPlot[{Cos[25 t], Cos[37 t]}, {t, 0, π}, AspectRatio → Automatic]
```



Out[59]= - Graphics -

■ Kurve mit Doppelpunkt

In[60]:=
```
ParametricPlot[{t^2 - 1, t^3 - t}, {t, -1, 1},
 AspectRatio → Automatic, PlotRange → {-0.75, 0.75}]
```



Out[60]= - Graphics -

Fakultäten und Binomialkoeffizienten

In[61]:=
```
1000!
```

Out[61]=
```
4023872600770937735437024339230039857193748642107146325437999104299385123986290205920442
8486969404800479988610197196058631666872994808558901323829669944590997424504087073759
8823627727188732519779505950995276120874975462497043601418278094646496291056393887437
6487337119181045825783647849977012476632889835955735432513185323958463075557409114262
7474349347553428646576611667797396668820291207379143853719588249808126867838374559731
6136085379534524221586593201928090878297308431392844403281231558611036976801357304216
8747609675871348312025478589320767169132448426236131412508780208000261683151027341827
7704784635868170164365024153691398281264810213092761244896359928705114964975419909342
1566832572080821333186116811553615836546984046708975602900950537616475847728421889679
6244945160765353408198901385442487984959953319101723355556602139450399736280750137837
5307127761926849034352625200015888535147331611702103968175921510907788019393178114194
5257223865541461062892187960223838971476088506276862967146674697562911234082439208160
3780889893964518263243671616762179168909779911903754031274622289988005195444414282012
7361745992642956581746628302955570299024324153181617210465832036786906117260158783520
1516284225540265170483304226143974286933061690897968482590125458327168226458066526769
8652682272807075781391858178889652208164348344825993266043367660176999612831860788386
0279465955131156552036093988180612138558600301435694527224206344631797460594682573103
0084024432438465657245014402821885252470935190620929023136493273497565513958720559654
8749774011413346962715422845862377387538230483865688976461927383814900140767310446640
9899490222221765904339901886018566526485061799702356193897017860040811889729918311021
1229845901641921068884387121855646124960798722908519296819372388642614839657382291123
5024186649353143970137428531926649875337218940694281434118520158014123344828015051399
4290153483077644569099073152433278288269864602789864321139083506217095002597389863554
7196742822248757586765752344220207573630569498825087968928162753848863396909959826280
6121450994871701244516461260379029309120889086942028510640182154399457156805941872748
8094254742173582401063677404595741785160829230135358081840096996372524230560855903700
4271243416909004153690105933983835777939410970027753472000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000
```

In[62]:=
```
?Binomial
```

```
Binomial[n, m] gives the binomial coefficient. Mehr…
```

In[63]:=
```
Binomial[100, 50]
```

Out[63]=
```
100891344545564193334812497256
```

In[64]:=
```
Clear[binomial1]
binomial1[n_, k_] := n! / (k! (n - k)!)
```

In[66]:=
```
binomial1[100, 50]
```

Out[66]=
```
100891344545564193334812497256
```

In[67]:=
```
100!
```

Out[67]=
```
933262154439441526816992388562667004907159682643816214685929638952175999932299156089414697615651828625369792082722375825118521091686400000000000000000000000
```

In[68]:=
```
Clear[binomial2]
binomial2[n_, k_] := Product[(n - j + 1) / j, {j, 1, k}]
```

In[70]:=
```
binomial2[100, 50]
```

Out[70]=
```
100891344545564193334812497256
```

In[6]:=
```
$RecursionLimit = ∞;
Clear[binomial3]
binomial3[n_, 0] := 1;
binomial3[n_, n_] := 1;
binomial3[n_, k_] :=
 (binomial3[n, k] = binomial3[n - 1, k - 1] + binomial3[n - 1, k])
```

In[11]:=
```
binomial3[100, 50]
```

Out[11]=
```
100891344545564193334812497256
```

In[25]:=
```
binomial3[5, 2]
```

Out[25]=
```
10
```

In[15]:=
```
Table[binomial3[n, k], {n, 0, 5}, {k, 0, n}]
```

Out[15]=
```
{{1}, {1, 1}, {1, 2, 1}, {1, 3, 3, 1}, {1, 4, 6, 4, 1}, {1, 5, 10, 10, 5, 1}}
```

In[16]:=
```
Multinomial[1, 2, 3, 4]
```

Out[16]= 12600