■ Permutationen

In[1]:=
```
perm = Permutations[{1, 2, 3, 4}]
```

Out[1]=

$$\begin{pmatrix}
1 & 2 & 3 & 4 \\
1 & 2 & 4 & 3 \\
1 & 3 & 2 & 4 \\
1 & 3 & 4 & 2 \\
1 & 4 & 2 & 3 \\
1 & 4 & 3 & 2 \\
2 & 1 & 3 & 4 \\
2 & 1 & 4 & 3 \\
2 & 3 & 1 & 4 \\
2 & 3 & 4 & 1 \\
2 & 4 & 1 & 3 \\
2 & 4 & 3 & 1 \\
3 & 1 & 2 & 4 \\
3 & 1 & 4 & 2 \\
3 & 2 & 1 & 4 \\
3 & 2 & 4 & 1 \\
3 & 4 & 1 & 2 \\
3 & 4 & 2 & 1 \\
4 & 1 & 2 & 3 \\
4 & 1 & 3 & 2 \\
4 & 2 & 1 & 3 \\
4 & 2 & 3 & 1 \\
4 & 3 & 1 & 2 \\
4 & 3 & 2 & 1
\end{pmatrix}$$

In[2]:=
```
Length[perm]
```

Out[2]= 24

In[3]:=
```
sign = Map[Signature, Permutations[{1, 2, 3, 4}]]
```

Out[3]= {1, −1, −1, 1, 1, −1, −1, 1, 1, −1, −1, 1, 1, −1, −1, 1, 1, −1, −1, 1, 1, −1, −1, 1}

In[4]:=
```
Transpose[Join[Transpose[perm], {sign}]]
```

Out[4]=
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1 \\ 1 & 2 & 4 & 3 & -1 \\ 1 & 3 & 2 & 4 & -1 \\ 1 & 3 & 4 & 2 & 1 \\ 1 & 4 & 2 & 3 & 1 \\ 1 & 4 & 3 & 2 & -1 \\ 2 & 1 & 3 & 4 & -1 \\ 2 & 1 & 4 & 3 & 1 \\ 2 & 3 & 1 & 4 & 1 \\ 2 & 3 & 4 & 1 & -1 \\ 2 & 4 & 1 & 3 & -1 \\ 2 & 4 & 3 & 1 & 1 \\ 3 & 1 & 2 & 4 & 1 \\ 3 & 1 & 4 & 2 & -1 \\ 3 & 2 & 1 & 4 & -1 \\ 3 & 2 & 4 & 1 & 1 \\ 3 & 4 & 1 & 2 & 1 \\ 3 & 4 & 2 & 1 & -1 \\ 4 & 1 & 2 & 3 & -1 \\ 4 & 1 & 3 & 2 & 1 \\ 4 & 2 & 1 & 3 & 1 \\ 4 & 2 & 3 & 1 & -1 \\ 4 & 3 & 1 & 2 & -1 \\ 4 & 3 & 2 & 1 & 1 \end{pmatrix}$$

■ Determinanten

In[5]:=
```
A = {{0, 2, 1}, {1/2, 3, 0}, {5, 0, 2}}
```

Out[5]=
$$\begin{pmatrix} 0 & 2 & 1 \\ \frac{1}{2} & 3 & 0 \\ 5 & 0 & 2 \end{pmatrix}$$

In[6]:=
```
Det[A]
```

Out[6]= -17

In[7]:=
```
A = Array[a, {2, 2}]
```

Out[7]=
$$\begin{pmatrix} a(1,1) & a(1,2) \\ a(2,1) & a(2,2) \end{pmatrix}$$

In[8]:=
```
Det[A]
```

Out[8]= $a(1,1)\,a(2,2) - a(1,2)\,a(2,1)$

In[9]:=
```
A = Array[a, {3, 3}]
```

Out[9]=
$$\begin{pmatrix} a(1,1) & a(1,2) & a(1,3) \\ a(2,1) & a(2,2) & a(2,3) \\ a(3,1) & a(3,2) & a(3,3) \end{pmatrix}$$

In[10]:=
```
Det[A]
```

Out[10]=
$$-a(1,3)\,a(2,2)\,a(3,1)+a(1,2)\,a(2,3)\,a(3,1)+a(1,3)\,a(2,1)\,a(3,2)-a(1,1)\,a(2,3)\,a(3,2)-a(1,2)\,a(2,1)\,a(3,3)+a(1,1)\,a(2,2)\,a(3,3)$$

■ Entwicklungssatz für Determinanten

In[11]:=
```
Untermatrix[A_, {p_, q_}] := Delete[Transpose[Delete[Transpose[A], q]], p]

Clear[Determinante]
Determinante[{{a_}}] := a;
Determinante[A_] :=
 Sum[(-1)^(1+k) A[[k, 1]] * Determinante[Untermatrix[A, {k, 1}]], {k, 1, Length[A]}]
```

In[15]:=
```
A = {{1, 2, -1}, {1, 3, 2}, {2, 0, 1}}
```

Out[15]=
$$\begin{pmatrix}1 & 2 & -1\\ 1 & 3 & 2\\ 2 & 0 & 1\end{pmatrix}$$

In[16]:=
```
Det[A]
```

Out[16]=
$$15$$

In[17]:=
```
Determinante[A]
```

Out[17]=
$$15$$

In[18]:=
```
A = {{5, -8, 7, 9}, {2, 0, 4, 6}, {3, 1, 7, 7}, {2, 8, 2, 5}}
```

Out[18]=
$$\begin{pmatrix}5 & -8 & 7 & 9\\ 2 & 0 & 4 & 6\\ 3 & 1 & 7 & 7\\ 2 & 8 & 2 & 5\end{pmatrix}$$

In[19]:=
```
Det[A]
```

Out[19]=
$$-290$$

In[20]:=
```
Untermatrix[A, {2, 3}]
```

Out[20]=
$$\begin{pmatrix}5 & -8 & 9\\ 3 & 1 & 7\\ 2 & 8 & 5\end{pmatrix}$$

In[21]:=
```
Determinante[A]
```

Out[21]=
$$-290$$

In[22]:=
```
A = Array[a, {4, 4}]
```

Out[22]=
$$\begin{pmatrix}a(1,1) & a(1,2) & a(1,3) & a(1,4)\\ a(2,1) & a(2,2) & a(2,3) & a(2,4)\\ a(3,1) & a(3,2) & a(3,3) & a(3,4)\\ a(4,1) & a(4,2) & a(4,3) & a(4,4)\end{pmatrix}$$

In[23]:=
```
det = Det[A]
```

Out[23]=
$$a(1,4)\,a(2,3)\,a(3,2)\,a(4,1) - a(1,3)\,a(2,4)\,a(3,2)\,a(4,1) - a(1,4)\,a(2,2)\,a(3,3)\,a(4,1) +$$
$$a(1,2)\,a(2,4)\,a(3,3)\,a(4,1) + a(1,3)\,a(2,2)\,a(3,4)\,a(4,1) - a(1,2)\,a(2,3)\,a(3,4)\,a(4,1) -$$
$$a(1,4)\,a(2,3)\,a(3,1)\,a(4,2) + a(1,3)\,a(2,4)\,a(3,1)\,a(4,2) + a(1,4)\,a(2,1)\,a(3,3)\,a(4,2) -$$
$$a(1,1)\,a(2,4)\,a(3,3)\,a(4,2) - a(1,3)\,a(2,1)\,a(3,4)\,a(4,2) + a(1,1)\,a(2,3)\,a(3,4)\,a(4,2) +$$
$$a(1,4)\,a(2,2)\,a(3,1)\,a(4,3) - a(1,2)\,a(2,4)\,a(3,1)\,a(4,3) - a(1,4)\,a(2,1)\,a(3,2)\,a(4,3) +$$
$$a(1,1)\,a(2,4)\,a(3,2)\,a(4,3) + a(1,2)\,a(2,1)\,a(3,4)\,a(4,3) - a(1,1)\,a(2,2)\,a(3,4)\,a(4,3) -$$
$$a(1,3)\,a(2,2)\,a(3,1)\,a(4,4) + a(1,2)\,a(2,3)\,a(3,1)\,a(4,4) + a(1,3)\,a(2,1)\,a(3,2)\,a(4,4) -$$
$$a(1,1)\,a(2,3)\,a(3,2)\,a(4,4) - a(1,2)\,a(2,1)\,a(3,3)\,a(4,4) + a(1,1)\,a(2,2)\,a(3,3)\,a(4,4)$$

In[24]:=
```
Length[det]
```

Out[24]= 24

In[25]:=
```
det2 = Determinante[A]
```

Out[25]=
$$-((a(1,3)\,a(2,4) - a(1,4)\,a(2,3))\,a(3,2) -$$
$$a(2,2)\,(a(1,3)\,a(3,4) - a(1,4)\,a(3,3)) + a(1,2)\,(a(2,3)\,a(3,4) - a(2,4)\,a(3,3)))\,a(4,1) +$$
$$a(3,1)\,((a(1,3)\,a(2,4) - a(1,4)\,a(2,3))\,a(4,2) - a(2,2)\,(a(1,3)\,a(4,4) - a(1,4)\,a(4,3)) +$$
$$a(1,2)\,(a(2,3)\,a(4,4) - a(2,4)\,a(4,3))) -$$
$$a(2,1)\,((a(1,3)\,a(3,4) - a(1,4)\,a(3,3))\,a(4,2) - a(3,2)\,(a(1,3)\,a(4,4) - a(1,4)\,a(4,3)) +$$
$$a(1,2)\,(a(3,3)\,a(4,4) - a(3,4)\,a(4,3))) +$$
$$a(1,1)\,((a(2,3)\,a(3,4) - a(2,4)\,a(3,3))\,a(4,2) - a(3,2)\,(a(2,3)\,a(4,4) - a(2,4)\,a(4,3)) +$$
$$a(2,2)\,(a(3,3)\,a(4,4) - a(3,4)\,a(4,3)))$$

In[26]:=
```
Expand[det2]
```

Out[26]=
$$a(1,4)\,a(2,3)\,a(3,2)\,a(4,1) - a(1,3)\,a(2,4)\,a(3,2)\,a(4,1) - a(1,4)\,a(2,2)\,a(3,3)\,a(4,1) +$$
$$a(1,2)\,a(2,4)\,a(3,3)\,a(4,1) + a(1,3)\,a(2,2)\,a(3,4)\,a(4,1) - a(1,2)\,a(2,3)\,a(3,4)\,a(4,1) -$$
$$a(1,4)\,a(2,3)\,a(3,1)\,a(4,2) + a(1,3)\,a(2,4)\,a(3,1)\,a(4,2) + a(1,4)\,a(2,1)\,a(3,3)\,a(4,2) -$$
$$a(1,1)\,a(2,4)\,a(3,3)\,a(4,2) - a(1,3)\,a(2,1)\,a(3,4)\,a(4,2) + a(1,1)\,a(2,3)\,a(3,4)\,a(4,2) +$$
$$a(1,4)\,a(2,2)\,a(3,1)\,a(4,3) - a(1,2)\,a(2,4)\,a(3,1)\,a(4,3) - a(1,4)\,a(2,1)\,a(3,2)\,a(4,3) +$$
$$a(1,1)\,a(2,4)\,a(3,2)\,a(4,3) + a(1,2)\,a(2,1)\,a(3,4)\,a(4,3) - a(1,1)\,a(2,2)\,a(3,4)\,a(4,3) -$$
$$a(1,3)\,a(2,2)\,a(3,1)\,a(4,4) + a(1,2)\,a(2,3)\,a(3,1)\,a(4,4) + a(1,3)\,a(2,1)\,a(3,2)\,a(4,4) -$$
$$a(1,1)\,a(2,3)\,a(3,2)\,a(4,4) - a(1,2)\,a(2,1)\,a(3,3)\,a(4,4) + a(1,1)\,a(2,2)\,a(3,3)\,a(4,4)$$

■ Beispiel 7.13

In[27]:=
```
A = {{3, 2, 1}, {2, 0, 2}, {1, 3, 4}}
```

Out[27]=
$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 3 & 4 \end{pmatrix}$$

In[28]:=
```
Det[A]
```

Out[28]= −24

■ Beispiel 7.13

In[29]:=
```
A = Table[a_j^k, {j, 1, 3}, {k, 0, 2}]
```

Out[29]=
$$\begin{pmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{pmatrix}$$

In[30]:=
```
Det[A]
```

Out[30]= $-a_2\,a_1^2 + a_3\,a_1^2 + a_2^2\,a_1 - a_3^2\,a_1 + a_2\,a_3^2 - a_2^2\,a_3$

In[31]:=
```
Factor[Det[A]]
```

Out[31]= $-(a_1 - a_2)\,(a_1 - a_3)\,(a_2 - a_3)$

In[32]:=
```
A = Table[a_j^k, {j, 1, 8}, {k, 0, 7}]
```

Out[32]=
$$\begin{pmatrix} 1 & a_1 & a_1^2 & a_1^3 & a_1^4 & a_1^5 & a_1^6 & a_1^7 \\ 1 & a_2 & a_2^2 & a_2^3 & a_2^4 & a_2^5 & a_2^6 & a_2^7 \\ 1 & a_3 & a_3^2 & a_3^3 & a_3^4 & a_3^5 & a_3^6 & a_3^7 \\ 1 & a_4 & a_4^2 & a_4^3 & a_4^4 & a_4^5 & a_4^6 & a_4^7 \\ 1 & a_5 & a_5^2 & a_5^3 & a_5^4 & a_5^5 & a_5^6 & a_5^7 \\ 1 & a_6 & a_6^2 & a_6^3 & a_6^4 & a_6^5 & a_6^6 & a_6^7 \\ 1 & a_7 & a_7^2 & a_7^3 & a_7^4 & a_7^5 & a_7^6 & a_7^7 \\ 1 & a_8 & a_8^2 & a_8^3 & a_8^4 & a_8^5 & a_8^6 & a_8^7 \end{pmatrix}$$

In[33]:=
```
Factor[Det[A]]
```

Out[33]=
$$(a_1 - a_2)\,(a_1 - a_3)\,(a_2 - a_3)\,(a_1 - a_4)\,(a_2 - a_4)\,(a_3 - a_4)\,(a_1 - a_5)\,(a_2 - a_5)\,(a_3 - a_5)$$
$$(a_4 - a_5)\,(a_1 - a_6)\,(a_2 - a_6)\,(a_3 - a_6)\,(a_4 - a_6)\,(a_5 - a_6)\,(a_1 - a_7)\,(a_2 - a_7)\,(a_3 - a_7)$$
$$(a_4 - a_7)\,(a_5 - a_7)\,(a_6 - a_7)\,(a_1 - a_8)\,(a_2 - a_8)\,(a_3 - a_8)\,(a_4 - a_8)\,(a_5 - a_8)\,(a_6 - a_8)\,(a_7 - a_8)$$

■ Übung 7.4

In[34]:=
```
Clear[A]
```

In[35]:=
```
A[n_] := Table[If[j == k, b, If[j - k == 1, b, If[j - k == -1, 1, 0]]], {j, n}, {k, n}]
```

In[36]:=
```
A[5]
```

Out[36]=
$$\begin{pmatrix} b & 1 & 0 & 0 & 0 \\ b & b & 1 & 0 & 0 \\ 0 & b & b & 1 & 0 \\ 0 & 0 & b & b & 1 \\ 0 & 0 & 0 & b & b \end{pmatrix}$$

In[37]:=
```
tab = Table[Det[A[n]], {n, 10}]
```

Out[37]=
$$\{b,\ b^2 - b,\ b^3 - 2\,b^2,\ b^4 - 3\,b^3 + b^2,\ b^5 - 4\,b^4 + 3\,b^3,\ b^6 - 5\,b^5 + 6\,b^4 - b^3,\ b^7 - 6\,b^6 + 10\,b^5 - 4\,b^4,$$
$$b^8 - 7\,b^7 + 15\,b^6 - 10\,b^5 + b^4,\ b^9 - 8\,b^8 + 21\,b^7 - 20\,b^6 + 5\,b^5,\ b^{10} - 9\,b^9 + 28\,b^8 - 35\,b^7 + 15\,b^6 - b^5\}$$