■ Charakteristisches Polynom

In[1]:=
```
A = {{1, 0, 1}, {2, 1, 0}, {0, 2, 1}}
```

Out[1]= $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$

In[2]:=
```
χ = Det[A - λ IdentityMatrix[3]]
```

Out[2]= $-\lambda^3 + 3\,\lambda^2 - 3\,\lambda + 5$

In[3]:=
```
CharacteristicPolynomial[A, λ]
```

Out[3]= $-\lambda^3 + 3\,\lambda^2 - 3\,\lambda + 5$

■ Beispiel 8.3

In[4]:=
```
A = {{-1, -3}, {2, -1}}
```

Out[4]= $\begin{pmatrix} -1 & -3 \\ 2 & -1 \end{pmatrix}$

In[5]:=
```
χ = CharacteristicPolynomial[A, λ]
```

Out[5]= $\lambda^2 + 2\,\lambda + 7$

In[6]:=
```
χ /. {λ → A}
```

Out[6]= $\begin{pmatrix} 6 & 10 \\ 15 & 6 \end{pmatrix}$

In[7]:=
```
MatrixPower[A, 2] + 2 A + 7 IdentityMatrix[2]
```

Out[7]= $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

■ Beispiel 8.4

In[8]:=
```
A = {{2, 4}, {1, 2}}
```

Out[8]= $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$

In[9]:=
```
χ = CharacteristicPolynomial[A, λ]
```

Out[9]= $\lambda^2 - 4\,\lambda$

```
In[10]:= Table[{MatrixPower[A, n]}, {n, 0, 10}]
```

Out[10]=
$$\begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \\ \begin{pmatrix} 8 & 16 \\ 4 & 8 \end{pmatrix} \\ \begin{pmatrix} 32 & 64 \\ 16 & 32 \end{pmatrix} \\ \begin{pmatrix} 128 & 256 \\ 64 & 128 \end{pmatrix} \\ \begin{pmatrix} 512 & 1024 \\ 256 & 512 \end{pmatrix} \\ \begin{pmatrix} 2048 & 4096 \\ 1024 & 2048 \end{pmatrix} \\ \begin{pmatrix} 8192 & 16\,384 \\ 4096 & 8192 \end{pmatrix} \\ \begin{pmatrix} 32\,768 & 65\,536 \\ 16\,384 & 32\,768 \end{pmatrix} \\ \begin{pmatrix} 131\,072 & 262\,144 \\ 65\,536 & 131\,072 \end{pmatrix} \\ \begin{pmatrix} 524\,288 & 1\,048\,576 \\ 262\,144 & 524\,288 \end{pmatrix} \end{pmatrix}$$

■ Beispiel 8.5

```
In[11]:= A = {{1, 0, 2}, {3, 0, 0}, {0, 0, ⅈ}}
```

Out[11]=
$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}$$

```
In[12]:= χ = CharacteristicPolynomial[A, λ]
```

Out[12]= $-\lambda^3 + (1 + i)\,\lambda^2 - i\,\lambda$

```
In[13]:= Factor[χ]
```

Out[13]= $-(\lambda - 1)\,\lambda\,(\lambda - i)$

```
In[14]:= Solve[χ == 0, λ]
```

Out[14]= $\{\{\lambda \to 0\}, \{\lambda \to i\}, \{\lambda \to 1\}\}$

```
In[15]:= Eigenvalues[A]
```

Out[15]= $\{i, 1, 0\}$

```
In[16]:= Eigensystem[A]
```

Out[16]=
$$\begin{pmatrix} i & 1 & 0 \\ \{-1 - i, -3 + 3\,i, 1\} & \{1, 3, 0\} & \{0, 1, 0\} \end{pmatrix}$$

Beispiel 8.6

In[17]:=
```
A = {{0, 1, 1}, {1, 0, 1}, {1, 1, 0}}
```

Out[17]= $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

In[18]:=
```
χ = CharacteristicPolynomial[A, λ]
```

Out[18]= $-\lambda^3 + 3\lambda + 2$

In[19]:=
```
Factor[χ]
```

Out[19]= $-(\lambda - 2)(\lambda + 1)^2$

In[20]:=
```
Solve[χ == 0, λ]
```

Out[20]= $\{\{\lambda \to -1\}, \{\lambda \to -1\}, \{\lambda \to 2\}\}$

In[21]:=
```
Eigenvalues[A]
```

Out[21]= $\{2, -1, -1\}$

In[22]:=
```
Eigensystem[A]
```

Out[22]= $\begin{pmatrix} 2 & -1 & -1 \\ \{1, 1, 1\} & \{-1, 0, 1\} & \{-1, 1, 0\} \end{pmatrix}$

Beispiel 8.7

In[23]:=
```
A = {{1, 1, 1}, {0, 1, 2}, {0, 0, 1}}
```

Out[23]= $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

In[24]:=
```
χ = CharacteristicPolynomial[A, λ]
```

Out[24]= $-\lambda^3 + 3\lambda^2 - 3\lambda + 1$

In[25]:=
```
Factor[χ]
```

Out[25]= $-(\lambda - 1)^3$

In[26]:=
```
Eigensystem[A]
```

Out[26]= $\begin{pmatrix} 1 & 1 & 1 \\ \{1, 0, 0\} & \{0, 0, 0\} & \{0, 0, 0\} \end{pmatrix}$

Beispiel 8.8

In[27]:=
```
A = {{0, 1, 1}, {1, 0, 1}, {1, 1, 0}}
```

Out[27]= $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

In[28]:=
```
sys = Eigensystem[A]
```

Out[28]= $\begin{pmatrix} 2 & -1 & -1 \\ \{1, 1, 1\} & \{-1, 0, 1\} & \{-1, 1, 0\} \end{pmatrix}$

In[29]:=
```
B = Transpose[sys[[2]]]
```

Out[29]=

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

In[30]:=
```
Inverse[B]
```

Out[30]=

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$

In[31]:=
```
Inverse[B].A.B
```

Out[31]=

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$