

■ Orientierungstreue lineare Abbildungen

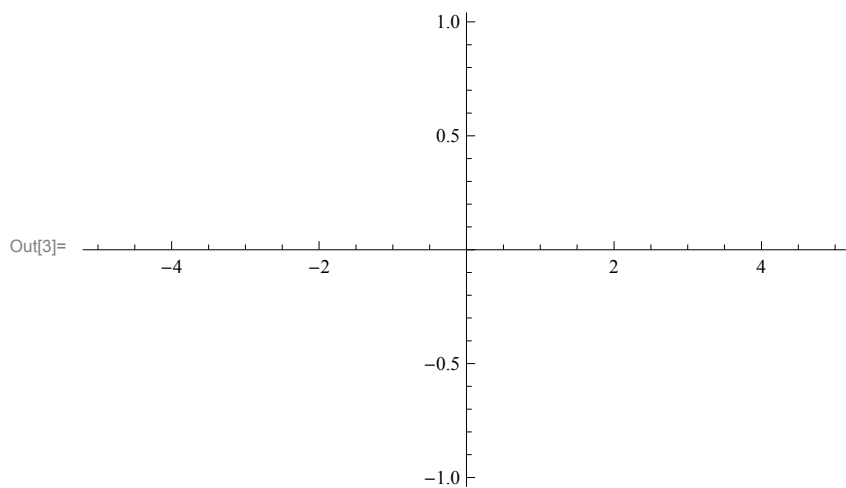
```
In[1]:= F = {{1, 0}, {1, 3}, {2, 3}, {2, 2.5}, {1.5, 2.5},  
            {1.5, 2}, {2, 2}, {2, 1.5}, {1.5, 1.5}, {1.5, 0}, {1, 0}}
```

```
Out[1]= 
$$\begin{pmatrix} 1 & 0 \\ 1 & 3 \\ 2 & 3 \\ 2 & 2.5 \\ 1.5 & 2.5 \\ 1.5 & 2 \\ 2 & 2 \\ 2 & 1.5 \\ 1.5 & 1.5 \\ 1.5 & 0 \\ 1 & 0 \end{pmatrix}$$

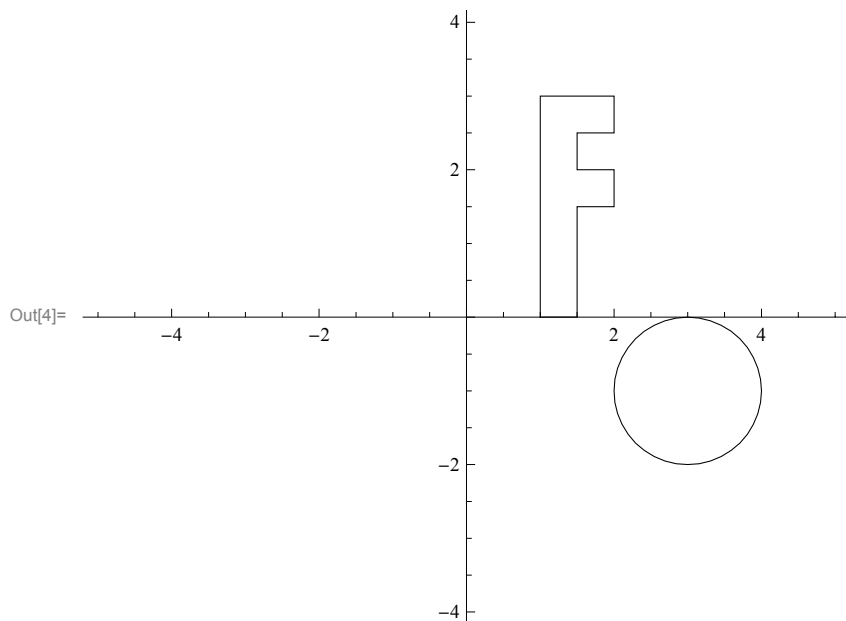
```

```
In[2]:= kreis = Table[{3 + Cos[phi], -1 + Sin[phi]}, {phi, 0, 2 pi, pi / 20}];
```

```
In[3]:= achsenkreuz = Plot[{}, {x, -5, 5}]
```



```
In[4]:= Show[achsenkreuz, Graphics[Line[F]], Graphics[Line[kreis]],
  PlotRange -> {{-5, 5}, {-4, 4}}, AspectRatio -> Automatic]
```



```
In[5]:= A = {{-1, -1/4}, {1, -1/2}}
```

Out[5]= $\begin{pmatrix} -1 & -\frac{1}{4} \\ 1 & -\frac{1}{2} \end{pmatrix}$

```
In[6]:= Det[A]
```

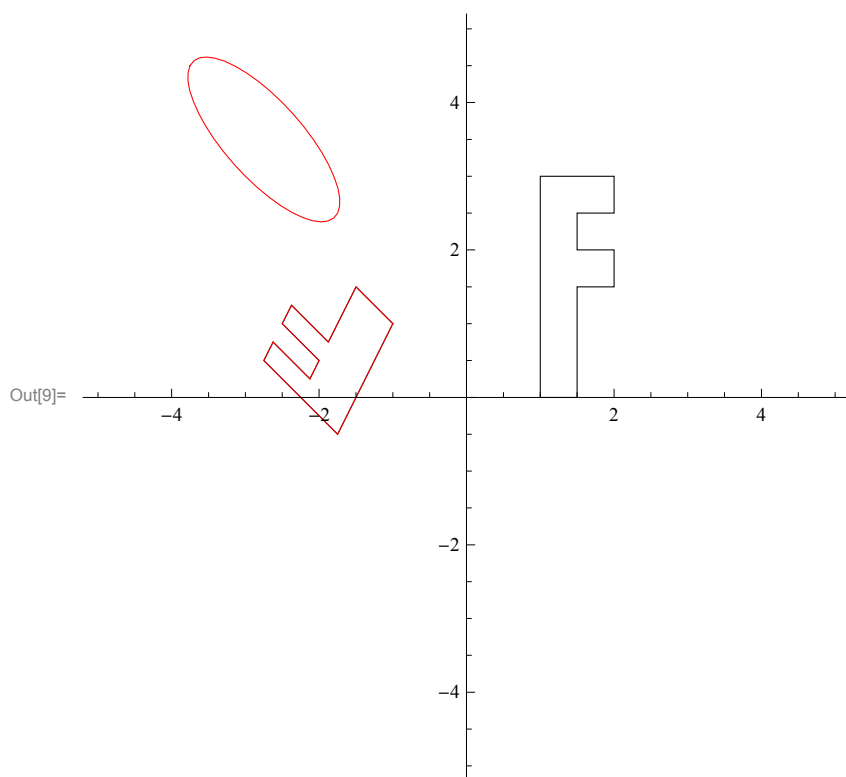
Out[6]= $\frac{3}{4}$

```
In[7]:= F2 = Map[A.# &, F]
```

Out[7]= $\begin{pmatrix} -1 & 1 \\ -\frac{7}{4} & -\frac{1}{2} \\ -\frac{11}{4} & \frac{1}{2} \\ -2.625 & 0.75 \\ -2.125 & 0.25 \\ -2. & 0.5 \\ -\frac{5}{2} & 1 \\ -2.375 & 1.25 \\ -1.875 & 0.75 \\ -1.5 & 1.5 \\ -1 & 1 \end{pmatrix}$

```
In[8]:= kreis2 = Map[A.# &, kreis];
```

```
In[9]:= Show[achsenkreuz, Graphics[Line[F]],
Graphics[Line[F2]], Graphics[{RGBColor[1, 0, 0], Line[F2]}],
Graphics[{RGBColor[1, 0, 0], Line[kreis2]}],
PlotRange -> {{-5, 5}, {-5, 5}}, AspectRatio -> Automatic]
```



```
In[10]:= B = {{-1/4, 1}, {1, 1/2}}
```

```
Out[10]=  $\begin{pmatrix} -\frac{1}{4} & 1 \\ 1 & \frac{1}{2} \end{pmatrix}$ 
```

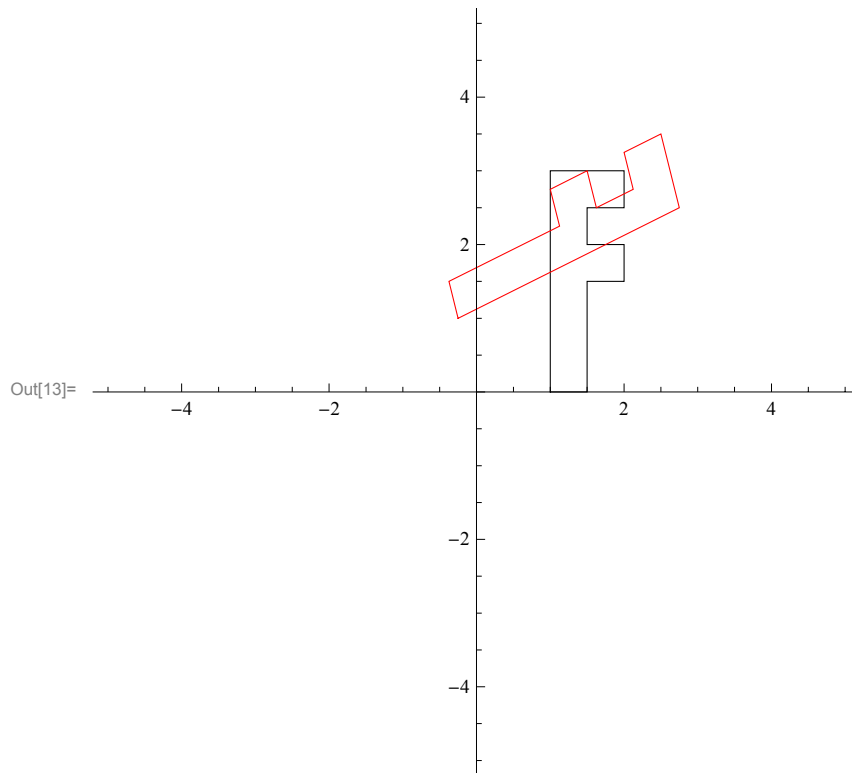
```
In[11]:= Det[B]
```

```
Out[11]=  $-\frac{9}{8}$ 
```

```
In[12]:= F3 = Map[B.# &, F]
```

```
Out[12]=  $\begin{pmatrix} -\frac{1}{4} & 1 \\ \frac{11}{4} & \frac{5}{2} \\ \frac{5}{2} & \frac{7}{2} \\ 2. & 3.25 \\ 2.125 & 2.75 \\ 1.625 & 2.5 \\ \frac{3}{2} & 3 \\ 1. & 2.75 \\ 1.125 & 2.25 \\ -0.375 & 1.5 \\ -\frac{1}{4} & 1 \end{pmatrix}$ 
```

```
In[13]:= Show[achsenkreuz, Graphics[Line[F]], Graphics[{RGBColor[1, 0, 0], Line[F3]}],
  PlotRange -> {{-5, 5}, {-5, 5}}, AspectRatio -> Automatic]
```



■ Drehungen und Spiegelungen

```
In[14]:= A = {{Cos[α], -Sin[α]}, {Sin[α], Cos[α]}}
```

```
Out[14]=  $\begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$ 
```

```
In[15]:= Det[A]
```

```
Out[15]=  $\sin^2(\alpha) + \cos^2(\alpha)$ 
```

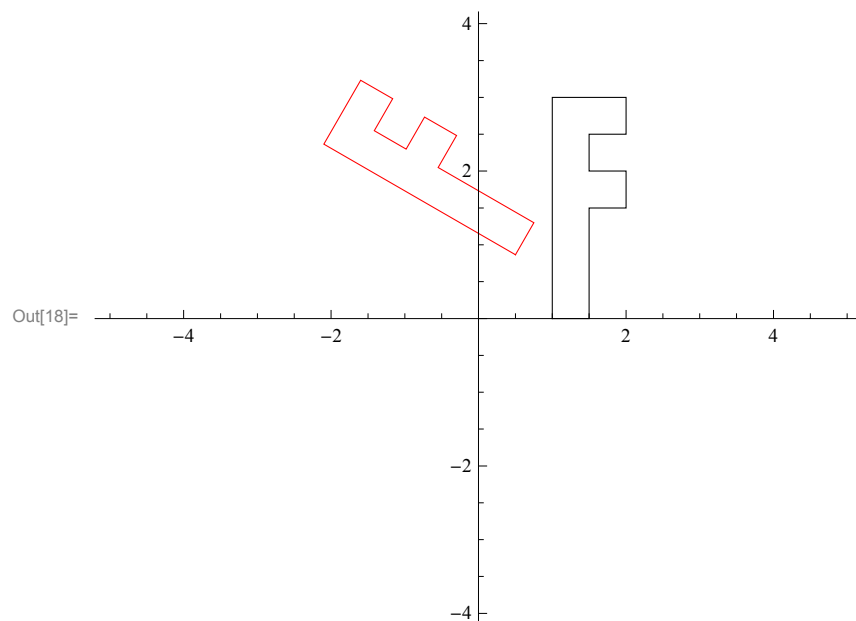
```
In[16]:= Det[A] // Simplify
```

```
Out[16]= 1
```

In[17]:= **F4 = Map[A.# &, F] /. {α → $\frac{\pi}{3}$ }**

Out[17]=
$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} - \frac{3\sqrt{3}}{2} & \frac{3}{2} + \frac{\sqrt{3}}{2} \\ 1 - \frac{3\sqrt{3}}{2} & \frac{3}{2} + \sqrt{3} \\ -1.16506 & 2.98205 \\ -1.41506 & 2.54904 \\ -0.982051 & 2.29904 \\ 1 - \sqrt{3} & 1 + \sqrt{3} \\ -0.299038 & 2.48205 \\ -0.549038 & 2.04904 \\ 0.75 & 1.29904 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

In[18]:= **Show[achsenkreuz, Graphics[Line[F]], Graphics[{RGBColor[1, 0, 0], Line[F4]}], PlotRange → {{-5, 5}, {-4, 4}}, AspectRatio → Automatic]**



In[19]:= **Eigensystem[A]**

Out[19]=
$$\begin{pmatrix} \cos(\alpha) - i \sin(\alpha) & \cos(\alpha) + i \sin(\alpha) \\ \{-i, 1\} & \{i, 1\} \end{pmatrix}$$

In[20]:= **A = {{Cos[α], Sin[α]}, {Sin[α], -Cos[α]}}**

Out[20]=
$$\begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & -\cos(\alpha) \end{pmatrix}$$

In[21]:= **Det[A]**

Out[21]= $-\sin^2(\alpha) - \cos^2(\alpha)$

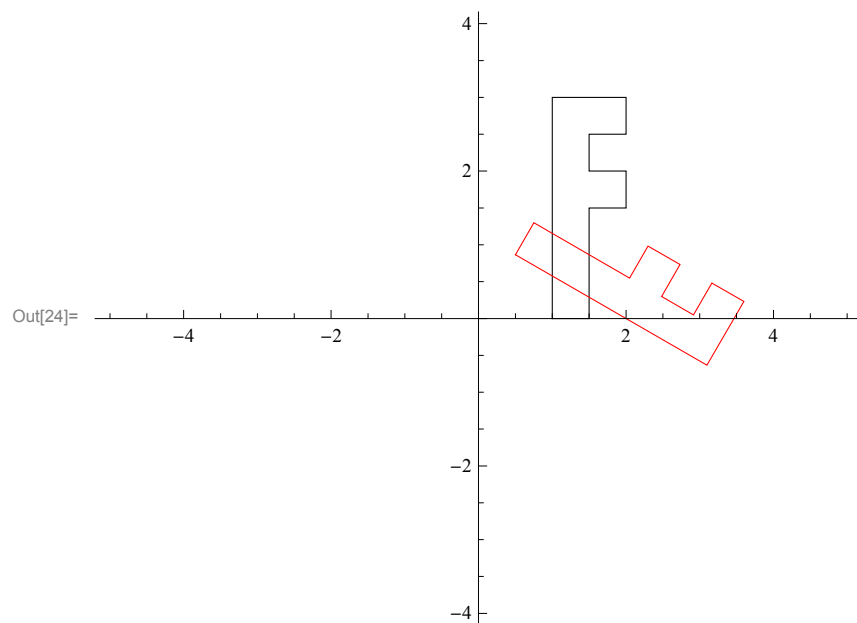
In[22]:= **Det[A] // Simplify**

Out[22]= -1

In[23]:= **F5 = Map[A.# &, F] /. { $\alpha \rightarrow \frac{\pi}{3}$ }**

Out[23]=
$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} + \frac{3\sqrt{3}}{2} & -\frac{3}{2} + \frac{\sqrt{3}}{2} \\ 1 + \frac{3\sqrt{3}}{2} & -\frac{3}{2} + \sqrt{3} \\ 3.16506 & 0.482051 \\ 2.91506 & 0.0490381 \\ 2.48205 & 0.299038 \\ 1 + \sqrt{3} & -1 + \sqrt{3} \\ 2.29904 & 0.982051 \\ 2.04904 & 0.549038 \\ 0.75 & 1.29904 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

In[24]:= **Show[achsenkreuz, Graphics[Line[F]], Graphics[{RGBColor[1, 0, 0], Line[F5]}], PlotRange → {{-5, 5}, {-4, 4}}, AspectRatio → Automatic]**



In[25]:= **Eigensystem[A]**

Out[25]=
$$\left(\begin{array}{cc} -1 & 1 \\ \{(\cos(\alpha) - 1) \csc(\alpha), 1\} & \{(\cos(\alpha) + 1) \csc(\alpha), 1\} \end{array} \right)$$