

■ Komplexe Zahlen

In[53]:= **I**

Out[53]= i

In[54]:= **i**

Out[54]= i

In[55]:= **z = x + i y**

Out[55]= $x + i y$

In[56]:= **? Re**

Re[z] gives the real part of the complex number z. >>

In[57]:= **ComplexExpand[Re[z]]**

Out[57]= x

In[58]:= **ComplexExpand[Im[z]]**

Out[58]= y

In[59]:= **$\frac{1}{z}$**

Out[59]= $\frac{1}{x + i y}$

In[60]:= **ComplexExpand[$\frac{1}{z}$]**

Out[60]= $\frac{x}{x^2 + y^2} - \frac{i y}{x^2 + y^2}$

In[61]:= **Abs[z]**

Out[61]= $|x + i y|$

In[62]:= **ComplexExpand[Abs[z]]**

Out[62]= $\sqrt{x^2 + y^2}$

In[63]:= **Conjugate[z]**

Out[63]= $x - i y$

In[64]:= **Solve[ComplexExpand[Im[$\frac{z-2}{z-i}$]] == 0, {x, y}]**

Solve::svars : Equations may not give solutions for all "solve" variables. >>

Out[64]= $\left\{ \left\{ y \rightarrow 1 - \frac{x}{2} \right\} \right\}$

In[65]:= **Solve**[**ComplexExpand**[**Abs**[$\frac{z+2}{z-i}$]] == 2, {x, y}]

Solve::svars : Equations may not give solutions for all "solve" variables. >>

Out[65]= $\left\{ \left\{ x \rightarrow \frac{1}{3} \left(2 - \sqrt{-9y^2 + 24y + 4} \right) \right\}, \left\{ x \rightarrow \frac{1}{3} \left(\sqrt{-9y^2 + 24y + 4} + 2 \right) \right\} \right\}$

In[66]:= **Arg**[-1 + i]

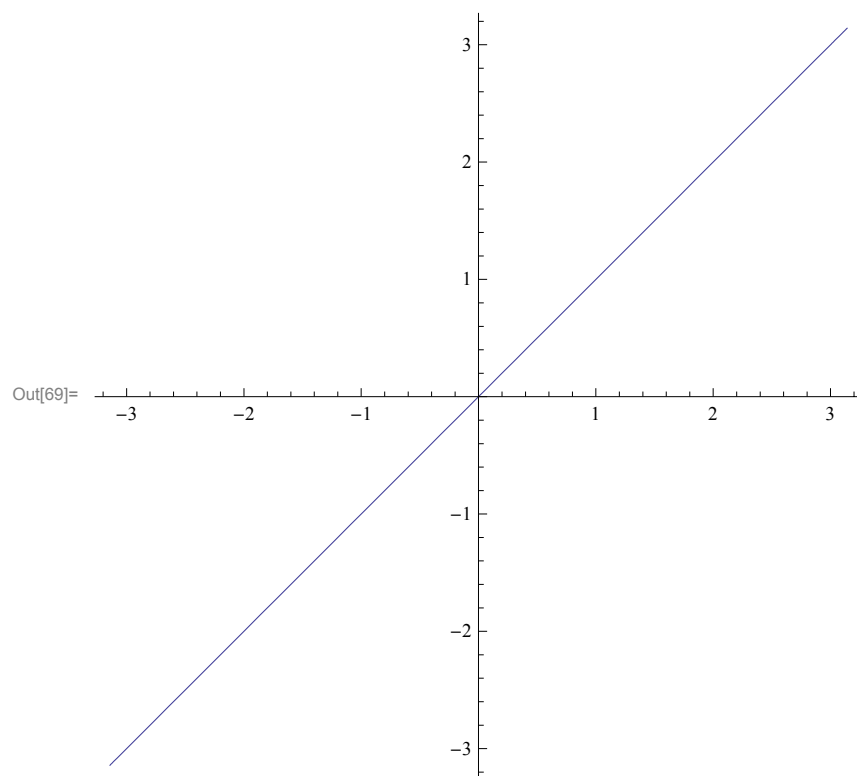
Out[66]= $\frac{3\pi}{4}$

In[67]:= **ArcCos** $\left[\frac{-1}{\sqrt{2}}\right]$

Out[67]= $\frac{3\pi}{4}$

In[68]:= **Arg1**[ϕ] := 2 **ArcTan** $\left[\frac{\text{Sin}[\phi]}{1 + \text{Cos}[\phi]}\right]$

In[69]:= **Plot**[**Arg1**[ϕ], { ϕ , - π , π }, **AspectRatio** → **Automatic**]



In[70]:= **ComplexExpand**[**Exp**[i x]]

Out[70]= $\cos(x) + i \sin(x)$

■ Umrechnen der Zahldarstellungen

In[71]:= $z = 3 - 3\sqrt{3} \text{ I}$

Out[71]= $3 - 3i\sqrt{3}$

In[72]:= $\left\{ \text{Abs}[z], 2 \text{ArcTan}\left[\frac{\text{Im}[z]}{\text{Abs}[z] + \text{Re}[z]}\right], \text{Arg}[z] \right\} // \text{FullSimplify}$

Out[72]= $\left\{ 6, -\frac{\pi}{3}, -\frac{\pi}{3} \right\}$

In[73]:= $\text{ComplexExpand}\left[6 e^{\frac{-i \pi}{3}}\right]$

Out[73]= $3 - 3 i \sqrt{3}$

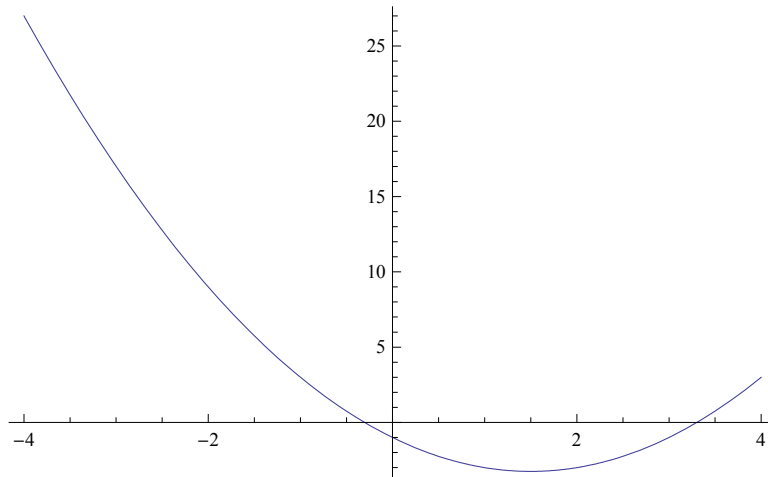
In[74]:= $\text{ComplexExpand}\left[\sqrt{2} e^{\frac{i \pi}{4}}\right]$

Out[74]= $1 + i$

■ Cardanische Formeln

In[75]:= $\text{Plot}\left[x^2 - 3x - 1, \{x, -4, 4\}\right]$

Out[75]=

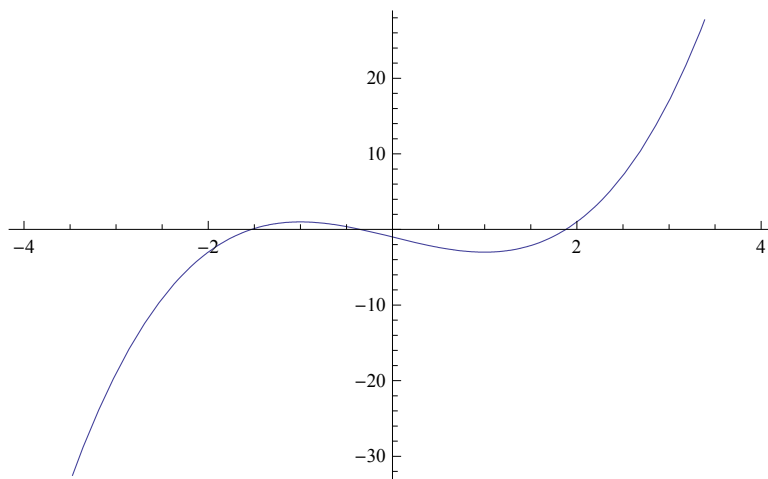


In[76]:= $\text{Solve}\left[x^2 - 3x - 1 == 0, x\right]$

Out[76]= $\left\{ \left\{ x \rightarrow \frac{1}{2} \left(3 - \sqrt{13} \right) \right\}, \left\{ x \rightarrow \frac{1}{2} \left(3 + \sqrt{13} \right) \right\} \right\}$

In[77]:= $\text{Plot}\left[x^3 - 3x - 1, \{x, -4, 4\}\right]$

Out[77]=



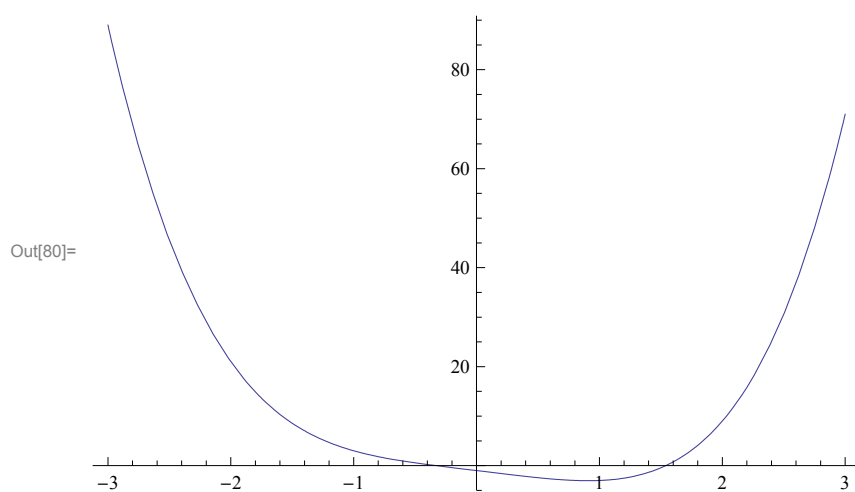
In[78]:= **sol = Solve** $\left[\mathbf{x}^3 - 3 \mathbf{x} - 1 == 0, \mathbf{x}\right]$

Out[78]= $\left\{ \left\{ x \rightarrow \frac{1}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} \right\}, \right.$
 $\left. \left\{ x \rightarrow -\frac{1}{2}(1-i\sqrt{3}) \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} - \left(\frac{1}{2}(1+i\sqrt{3}) \right)^{2/3} \right\}, \left\{ x \rightarrow -\frac{1-i\sqrt{3}}{2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(1+i\sqrt{3})^{4/3}}{2 \sqrt[3]{2}} \right\} \right\}$

In[79]:= **N[sol]**

Out[79]= $\left\{ \{x \rightarrow 1.87939 + 0. i\}, \{x \rightarrow -1.53209 + 1.11022 \times 10^{-16} i\}, \{x \rightarrow -0.347296 + 1.11022 \times 10^{-16} i\} \right\}$

In[80]:= **Plot** $\left[\mathbf{x}^4 - 3 \mathbf{x} - 1, \{\mathbf{x}, -3, 3\}\right]$



In[81]:= **sol = Solve** $\left[\mathbf{x}^4 - 3 \mathbf{x} - 1 == 0, \mathbf{x}\right]$

Out[81]= $\left\{ \left\{ x \rightarrow -\frac{1}{2} \sqrt{\frac{\sqrt[3]{\frac{1}{2}(81+\sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81+\sqrt{7329})}}} - \right. \right.$
 $\left. \frac{1}{2} \sqrt{4 \sqrt[3]{\frac{2}{3(81+\sqrt{7329})}} - \frac{\sqrt[3]{\frac{1}{2}(81+\sqrt{7329})}}{3^{2/3}} - \frac{6}{\sqrt{\frac{\sqrt[3]{\frac{1}{2}(81+\sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81+\sqrt{7329})}}}}} \right\},$

$$\left\{x \rightarrow \frac{1}{2} \sqrt{4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}} - \frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - \frac{6}{\sqrt{\frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}}}} - \right.$$

$$\left. \frac{1}{2} \sqrt{\frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}}} \right\},$$

$$\left\{x \rightarrow \frac{1}{2} \sqrt{\frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}}} - \right.$$

$$\left. \frac{1}{2} \sqrt{4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}} - \frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} + \frac{6}{\sqrt{\frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}}}} \right\},$$

$$\left\{x \rightarrow \frac{1}{2} \sqrt{\frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}}} + \right.$$

$$\left. \frac{1}{2} \sqrt{4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}} - \frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} + \frac{6}{\sqrt{\frac{\sqrt[3]{\frac{1}{2}(81 + \sqrt{7329})}}{3^{2/3}} - 4 \sqrt[3]{\frac{2}{3(81 + \sqrt{7329})}}}} \right\}}$$

In[82]:= **N[sol]**

Out[82]= $\{\{x \rightarrow -0.605102 - 1.26713 i\}, \{x \rightarrow -0.605102 + 1.26713 i\}, \{x \rightarrow -0.329409\}, \{x \rightarrow 1.53961\}\}$

■ Multiplikation

In[83]:= **z = 1 + i**

Out[83]= **1 + i**

In[84]:= **w = -1 + i**

Out[84]= $-1 + i$

In[85]:= **z⁵ w⁷**

Out[85]= $64 i$

In[86]:= **z⁵**

Out[86]= $-4 - 4 i$

In[87]:= **w⁷**

Out[87]= $-8 - 8 i$

In[88]:= **Clear[z]**

In[89]:= **Solve[z⁵ == 1, z]**

Out[89]= $\left\{ \{z \rightarrow 1\}, \left\{ z \rightarrow -\sqrt[5]{-1} \right\}, \left\{ z \rightarrow (-1)^{2/5} \right\}, \left\{ z \rightarrow -(-1)^{3/5} \right\}, \left\{ z \rightarrow (-1)^{4/5} \right\} \right\}$

In[90]:= **sol = Solve[z⁵ == 1, z] // ComplexExpand**

Out[90]= $\left\{ \{z \rightarrow 1\}, \left\{ z \rightarrow -\frac{1}{4} - \frac{\sqrt{5}}{4} - i \sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}} \right\}, \left\{ z \rightarrow -\frac{1}{4} + \frac{\sqrt{5}}{4} + i \sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}} \right\}, \right.$
 $\left. \left\{ z \rightarrow -\frac{1}{4} + \frac{\sqrt{5}}{4} - i \sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}} \right\}, \left\{ z \rightarrow -\frac{1}{4} - \frac{\sqrt{5}}{4} + i \sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}} \right\} \right\}$