■ Matrizen

In[1]:= `A = Array[a, {4, 6}]`

Out[1]=
$$\begin{pmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) & a(1,5) & a(1,6) \\ a(2,1) & a(2,2) & a(2,3) & a(2,4) & a(2,5) & a(2,6) \\ a(3,1) & a(3,2) & a(3,3) & a(3,4) & a(3,5) & a(3,6) \\ a(4,1) & a(4,2) & a(4,3) & a(4,4) & a(4,5) & a(4,6) \end{pmatrix}$$

In[2]:= `Transpose[A]`

Out[2]=
$$\begin{pmatrix} a(1,1) & a(2,1) & a(3,1) & a(4,1) \\ a(1,2) & a(2,2) & a(3,2) & a(4,2) \\ a(1,3) & a(2,3) & a(3,3) & a(4,3) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \\ a(1,5) & a(2,5) & a(3,5) & a(4,5) \\ a(1,6) & a(2,6) & a(3,6) & a(4,6) \end{pmatrix}$$

In[3]:= `3 A`

Out[3]=
$$\begin{pmatrix} 3\,a(1,1) & 3\,a(1,2) & 3\,a(1,3) & 3\,a(1,4) & 3\,a(1,5) & 3\,a(1,6) \\ 3\,a(2,1) & 3\,a(2,2) & 3\,a(2,3) & 3\,a(2,4) & 3\,a(2,5) & 3\,a(2,6) \\ 3\,a(3,1) & 3\,a(3,2) & 3\,a(3,3) & 3\,a(3,4) & 3\,a(3,5) & 3\,a(3,6) \\ 3\,a(4,1) & 3\,a(4,2) & 3\,a(4,3) & 3\,a(4,4) & 3\,a(4,5) & 3\,a(4,6) \end{pmatrix}$$

In[4]:= `X = {{x1}, {x2}, {x3}, {x4}, {x5}, {x6}}`

Out[4]=
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}$$

In[5]:= `A.X`

Out[5]=
$$\begin{pmatrix} a(1,1)\,x_1 + a(1,2)\,x_2 + a(1,3)\,x_3 + a(1,4)\,x_4 + a(1,5)\,x_5 + a(1,6)\,x_6 \\ a(2,1)\,x_1 + a(2,2)\,x_2 + a(2,3)\,x_3 + a(2,4)\,x_4 + a(2,5)\,x_5 + a(2,6)\,x_6 \\ a(3,1)\,x_1 + a(3,2)\,x_2 + a(3,3)\,x_3 + a(3,4)\,x_4 + a(3,5)\,x_5 + a(3,6)\,x_6 \\ a(4,1)\,x_1 + a(4,2)\,x_2 + a(4,3)\,x_3 + a(4,4)\,x_4 + a(4,5)\,x_5 + a(4,6)\,x_6 \end{pmatrix}$$

In[6]:= `B = Array[b, {4, 6}]`

Out[6]=
$$\begin{pmatrix} b(1,1) & b(1,2) & b(1,3) & b(1,4) & b(1,5) & b(1,6) \\ b(2,1) & b(2,2) & b(2,3) & b(2,4) & b(2,5) & b(2,6) \\ b(3,1) & b(3,2) & b(3,3) & b(3,4) & b(3,5) & b(3,6) \\ b(4,1) & b(4,2) & b(4,3) & b(4,4) & b(4,5) & b(4,6) \end{pmatrix}$$

In[7]:= `A + B`

Out[7]=
$$\begin{pmatrix} a(1,1)+b(1,1) & a(1,2)+b(1,2) & a(1,3)+b(1,3) & a(1,4)+b(1,4) & a(1,5)+b(1,5) & a(1,6)+b(1,6) \\ a(2,1)+b(2,1) & a(2,2)+b(2,2) & a(2,3)+b(2,3) & a(2,4)+b(2,4) & a(2,5)+b(2,5) & a(2,6)+b(2,6) \\ a(3,1)+b(3,1) & a(3,2)+b(3,2) & a(3,3)+b(3,3) & a(3,4)+b(3,4) & a(3,5)+b(3,5) & a(3,6)+b(3,6) \\ a(4,1)+b(4,1) & a(4,2)+b(4,2) & a(4,3)+b(4,3) & a(4,4)+b(4,4) & a(4,5)+b(4,5) & a(4,6)+b(4,6) \end{pmatrix}$$

In[8]:= **A.B**

Dot::dotsh : Tensors $\begin{pmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) & a(1,5) & a(1,6) \\ a(2,1) & a(2,2) & a(2,3) & a(2,4) & a(2,5) & a(2,6) \\ a(3,1) & a(3,2) & a(3,3) & a(3,4) & a(3,5) & a(3,6) \\ a(4,1) & a(4,2) & a(4,3) & a(4,4) & a(4,5) & a(4,6) \end{pmatrix}$ and $\begin{pmatrix} b(1,1) & b(1,2) & b(1,3) & b(1,4) & b(1,5) & b(1,6) \\ b(2,1) & b(2,2) & b(2,3) & b(2,4) & b(2,5) & b(2,6) \\ b(3,1) & b(3,2) & b(3,3) & b(3,4) & b(3,5) & b(3,6) \\ b(4,1) & b(4,2) & b(4,3) & b(4,4) & b(4,5) & b(4,6) \end{pmatrix}$ have incompatible shapes. ≫

Out[8]= $$\begin{pmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) & a(1,5) & a(1,6) \\ a(2,1) & a(2,2) & a(2,3) & a(2,4) & a(2,5) & a(2,6) \\ a(3,1) & a(3,2) & a(3,3) & a(3,4) & a(3,5) & a(3,6) \\ a(4,1) & a(4,2) & a(4,3) & a(4,4) & a(4,5) & a(4,6) \end{pmatrix} . \begin{pmatrix} b(1,1) & b(1,2) & b(1,3) & b(1,4) & b(1,5) & b(1,6) \\ b(2,1) & b(2,2) & b(2,3) & b(2,4) & b(2,5) & b(2,6) \\ b(3,1) & b(3,2) & b(3,3) & b(3,4) & b(3,5) & b(3,6) \\ b(4,1) & b(4,2) & b(4,3) & b(4,4) & b(4,5) & b(4,6) \end{pmatrix}$$

In[9]:= **Transpose[A].B**

Out[9]= $$\begin{pmatrix} a(1,1)\,b(1,1)+a(2,1)\,b(2,1)+a(3,1)\,b(3,1)+a(4,1)\,b(4,1) & a(1,1)\,b(1,2)+a(2,1)\,b(2,2)+a(3,1)\,b(3,2)+a(\\ a(1,2)\,b(1,1)+a(2,2)\,b(2,1)+a(3,2)\,b(3,1)+a(4,2)\,b(4,1) & a(1,2)\,b(1,2)+a(2,2)\,b(2,2)+a(3,2)\,b(3,2)+a(\\ a(1,3)\,b(1,1)+a(2,3)\,b(2,1)+a(3,3)\,b(3,1)+a(4,3)\,b(4,1) & a(1,3)\,b(1,2)+a(2,3)\,b(2,2)+a(3,3)\,b(3,2)+a(\\ a(1,4)\,b(1,1)+a(2,4)\,b(2,1)+a(3,4)\,b(3,1)+a(4,4)\,b(4,1) & a(1,4)\,b(1,2)+a(2,4)\,b(2,2)+a(3,4)\,b(3,2)+a(\\ a(1,5)\,b(1,1)+a(2,5)\,b(2,1)+a(3,5)\,b(3,1)+a(4,5)\,b(4,1) & a(1,5)\,b(1,2)+a(2,5)\,b(2,2)+a(3,5)\,b(3,2)+a(\\ a(1,6)\,b(1,1)+a(2,6)\,b(2,1)+a(3,6)\,b(3,1)+a(4,6)\,b(4,1) & a(1,6)\,b(1,2)+a(2,6)\,b(2,2)+a(3,6)\,b(3,2)+a(\end{pmatrix}$$

In[10]:= **A = {{2, 1}, {-3, -2}}**

Out[10]= $\begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}$

In[11]:= **B =** $\left\{\{-1, 2, 1\}, \left\{-5, 0, \frac{3}{2}\right\}\right\}$

Out[11]= $\begin{pmatrix} -1 & 2 & 1 \\ -5 & 0 & \frac{3}{2} \end{pmatrix}$

In[12]:= **A.B**

Out[12]= $\begin{pmatrix} -7 & 4 & \frac{7}{2} \\ 13 & -6 & -6 \end{pmatrix}$

In[13]:= **B.A**

Dot::dotsh : Tensors $\begin{pmatrix} -1 & 2 & 1 \\ -5 & 0 & \frac{3}{2} \end{pmatrix}$ and $\begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}$ have incompatible shapes. ≫

Out[13]= $\begin{pmatrix} -1 & 2 & 1 \\ -5 & 0 & \frac{3}{2} \end{pmatrix} . \begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}$

In[14]:= **A = {{1, 1, 1 + i, 0}, {-1, 2, 0, 0}, {1, 0, 3, -i}}**

Out[14]= $\begin{pmatrix} 1 & 1 & 1+i & 0 \\ -1 & 2 & 0 & 0 \\ 1 & 0 & 3 & -i \end{pmatrix}$

In[15]:= `B = {{5, 9, 2}, {0, 0, 1}, {1, -1, -2}, {-2, 3, 4}}`

Out[15]= $\begin{pmatrix} 5 & 9 & 2 \\ 0 & 0 & 1 \\ 1 & -1 & -2 \\ -2 & 3 & 4 \end{pmatrix}$

In[16]:= `A.B`

Out[16]= $\begin{pmatrix} 6+i & 8-i & 1-2i \\ -5 & -9 & 0 \\ 8+2i & 6-3i & -4-4i \end{pmatrix}$

In[17]:= `A = {{2, -1, -3}}`

Out[17]= $\begin{pmatrix} 2 & -1 & -3 \end{pmatrix}$

In[18]:= `B = {{1}, {i}, {-1}}`

Out[18]= $\begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix}$

In[19]:= `A.B`

Out[19]= $\begin{pmatrix} 5-i \end{pmatrix}$

In[20]:= `B.A`

Out[20]= $\begin{pmatrix} 2 & -1 & -3 \\ 2i & -i & -3i \\ -2 & 1 & 3 \end{pmatrix}$

In[21]:= `A = {{a, 1}, {3, 0}}`

Out[21]= $\begin{pmatrix} a & 1 \\ 3 & 0 \end{pmatrix}$

■ Inerse Matrizen

In[22]:= `Inverse[A]`

Out[22]= $\begin{pmatrix} 0 & \frac{1}{3} \\ 1 & -\frac{a}{3} \end{pmatrix}$

In[23]:= `Inverse[A].A`

Out[23]= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

In[24]:= `A.Inverse[A]`

Out[24]= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

In[25]:= `X = {{x_1}, {x_2}}`

Out[25]= $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

In[26]:= `B = {{b1}, {b2}}`

Out[26]= $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

In[27]:= `Solve[A.X == B, {x1, x2}]`

Out[27]= $\left\{\left\{x_1 \to \frac{b_2}{3}, x_2 \to \frac{1}{3}(3\,b_1 - a\,b_2)\right\}\right\}$

In[28]:= `Inverse[A].B`

Out[28]= $\begin{pmatrix} \frac{b_2}{3} \\ b_1 - \frac{a\,b_2}{3} \end{pmatrix}$

In[29]:= `A = {{i, 0, 1}, {1, 0, i}, {0, 1, 0}}`

Out[29]= $\begin{pmatrix} i & 0 & 1 \\ 1 & 0 & i \\ 0 & 1 & 0 \end{pmatrix}$

In[30]:= `Inverse[A]`

Out[30]= $\begin{pmatrix} -\frac{i}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & -\frac{i}{2} & 0 \end{pmatrix}$

In[31]:= `Aerw = Transpose[Join[Transpose[A], IdentityMatrix[3]]]`

Out[31]= $\begin{pmatrix} i & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & i & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

In[32]:= `RowReduce[Aerw]`

Out[32]= $\begin{pmatrix} 1 & 0 & 0 & -\frac{i}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{i}{2} & 0 \end{pmatrix}$

In[33]:= `A = {{Cos[ϕ], -Sin[ϕ]}, {Sin[ϕ], Cos[ϕ]}}`

Out[33]= $\begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix}$

In[34]:= `Inverse[A]`

Out[34]= $\begin{pmatrix} \frac{\cos(\phi)}{\cos^2(\phi)+\sin^2(\phi)} & \frac{\sin(\phi)}{\cos^2(\phi)+\sin^2(\phi)} \\ -\frac{\sin(\phi)}{\cos^2(\phi)+\sin^2(\phi)} & \frac{\cos(\phi)}{\cos^2(\phi)+\sin^2(\phi)} \end{pmatrix}$

In[35]:= `Inverse[A] // Simplify`

Out[35]= $\begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix}$

In[36]:=
```
Aerw = Transpose[Join[Transpose[A], IdentityMatrix[2]]]
```

Out[36]= $$\begin{pmatrix} \cos(\phi) & -\sin(\phi) & 1 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 1 \end{pmatrix}$$

In[37]:=
```
RowReduce[Aerw]
```

Out[37]= $$\begin{pmatrix} 1 & 0 & \frac{\cos(\phi)}{\cos^2(\phi)+\sin^2(\phi)} & \frac{\sin(\phi)}{\cos^2(\phi)+\sin^2(\phi)} \\ 0 & 1 & -\frac{\sin(\phi)}{\cos^2(\phi)+\sin^2(\phi)} & \frac{\cos(\phi)}{\cos^2(\phi)+\sin^2(\phi)} \end{pmatrix}$$

In[38]:=
```
RowReduce[Aerw] // Simplify
```

Out[38]= $$\begin{pmatrix} 1 & 0 & \cos(\phi) & \sin(\phi) \\ 0 & 1 & -\sin(\phi) & \cos(\phi) \end{pmatrix}$$

In[39]:=
```
A = {{a, 1}, {3, 0}}
```

Out[39]= $$\begin{pmatrix} a & 1 \\ 3 & 0 \end{pmatrix}$$

In[40]:=
```
Inverse[A]
```

Out[40]= $$\begin{pmatrix} 0 & \frac{1}{3} \\ 1 & -\frac{a}{3} \end{pmatrix}$$

■ Lösen von Gleichungssystemen mit Matrizen

In[41]:=
```
A = {{0, 2, 1}, {1/2, 3, 0}, {5, 0, 2}}
```

Out[41]= $$\begin{pmatrix} 0 & 2 & 1 \\ \frac{1}{2} & 3 & 0 \\ 5 & 0 & 2 \end{pmatrix}$$

In[42]:=
```
b = {{2}, {2}, {1}}
```

Out[42]= $$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

In[43]:=
```
Aerw = Transpose[Join[Transpose[A], Transpose[b]]]
```

Out[43]= $$\begin{pmatrix} 0 & 2 & 1 & 2 \\ \frac{1}{2} & 3 & 0 & 2 \\ 5 & 0 & 2 & 1 \end{pmatrix}$$

In[44]:=
```
RowReduce[Aerw]
```

Out[44]= $$\begin{pmatrix} 1 & 0 & 0 & -\frac{1}{17} \\ 0 & 1 & 0 & \frac{23}{34} \\ 0 & 0 & 1 & \frac{11}{17} \end{pmatrix}$$

In[45]:= `Inverse[A].b`

Out[45]= $\begin{pmatrix} -\frac{1}{17} \\ \frac{23}{34} \\ \frac{11}{17} \end{pmatrix}$

In[46]:= `A = {{2, 3, 3}, {1, 5, 5/2}, {7, 21, 27/2}}`

Out[46]= $\begin{pmatrix} 2 & 3 & 3 \\ 1 & 5 & \frac{5}{2} \\ 7 & 21 & \frac{27}{2} \end{pmatrix}$

In[47]:= `b = {{-5}, {0}, {-10}}`

Out[47]= $\begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix}$

In[48]:= `Aerw = Transpose[Join[Transpose[A], Transpose[b]]]`

Out[48]= $\begin{pmatrix} 2 & 3 & 3 & -5 \\ 1 & 5 & \frac{5}{2} & 0 \\ 7 & 21 & \frac{27}{2} & -10 \end{pmatrix}$

In[49]:= `RowReduce[Aerw]`

Out[49]= $\begin{pmatrix} 1 & 0 & \frac{15}{14} & -\frac{25}{7} \\ 0 & 1 & \frac{2}{7} & \frac{5}{7} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

In[50]:= `Inverse[A].b`

Inverse::sing : Matrix $\begin{pmatrix} 2 & 3 & 3 \\ 1 & 5 & \frac{5}{2} \\ 7 & 21 & \frac{27}{2} \end{pmatrix}$ is singular. ≫

Out[50]= $\begin{pmatrix} 2 & 3 & 3 \\ 1 & 5 & \frac{5}{2} \\ 7 & 21 & \frac{27}{2} \end{pmatrix}^{-1} . \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix}$

In[51]:= `A = {{2/3, 4, -3/2}, {1/2, -2, 3}, {7/6, 26/3, -4}}`

Out[51]= $\begin{pmatrix} \frac{2}{3} & 4 & -\frac{3}{2} \\ \frac{1}{2} & -2 & 3 \\ \frac{7}{6} & \frac{26}{3} & -4 \end{pmatrix}$

In[52]:= `b = {{5}, {-3}, {-1}}`

Out[52]= $\begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix}$

In[53]:= `Aerw = Transpose[Join[Transpose[A], Transpose[b]]]`

Out[53]=
$$\begin{pmatrix} \frac{2}{3} & 4 & -\frac{3}{2} & 5 \\ \frac{1}{2} & -2 & 3 & -3 \\ \frac{7}{6} & \frac{26}{3} & -4 & -1 \end{pmatrix}$$

In[54]:= `RowReduce[Aerw]`

Out[54]=
$$\begin{pmatrix} 1 & 0 & \frac{27}{10} & 0 \\ 0 & 1 & -\frac{33}{40} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[55]:= `Inverse[A].b`

Inverse::sing : Matrix $\begin{pmatrix} \frac{2}{3} & 4 & -\frac{3}{2} \\ \frac{1}{2} & -2 & 3 \\ \frac{7}{6} & \frac{26}{3} & -4 \end{pmatrix}$ is singular. ≫

Out[55]=
$$\begin{pmatrix} \frac{2}{3} & 4 & -\frac{3}{2} \\ \frac{1}{2} & -2 & 3 \\ \frac{7}{6} & \frac{26}{3} & -4 \end{pmatrix}^{-1} . \begin{pmatrix} 5 \\ -3 \\ -1 \end{pmatrix}$$