## ■ Linearkombination

### ■ Beispiel 5.6

In[1]:= `a1 = {1, 2}`

Out[1]= $\{1, 2\}$

In[2]:= `a2 = {-2, 4}`

Out[2]= $\{-2, 4\}$

In[3]:= `b = {3, 5}`

Out[3]= $\{3, 5\}$

In[4]:= `Solve[b == λ1 a1 + λ2 a2, {λ1, λ2}]`

Out[4]= $\left\{\left\{\lambda_1 \to \frac{11}{4}, \lambda_2 \to -\frac{1}{8}\right\}\right\}$

### ■ Beispiel 5.8

In[5]:= `a1 = {1, i}`

Out[5]= $\{1, i\}$

In[6]:= `a2 = {1 + i, 1}`

Out[6]= $\{1 + i, 1\}$

In[7]:= `b = {x, y}`

Out[7]= $\{x, y\}$

In[8]:= `Solve[b == λ1 a1 + λ2 a2, {λ1, λ2}]`

Out[8]= $\left\{\left\{\lambda_1 \to \left(\frac{2}{5} + \frac{i}{5}\right)(x - (1 + i)\, y), \lambda_2 \to \left(\frac{1}{5} - \frac{2\, i}{5}\right) x + \left(\frac{2}{5} + \frac{i}{5}\right) y\right\}\right\}$

### ■ Übung 5.1

In[9]:= `a1 = {3, 5, 2}`

Out[9]= $\{3, 5, 2\}$

In[10]:= `a2 = {1, 1, -1}`

Out[10]= $\{1, 1, -1\}$

In[11]:= `a3 = {2, 4, 1}`

Out[11]= $\{2, 4, 1\}$

In[12]:= `b = {1, 2, 0}`

Out[12]= $\{1, 2, 0\}$

In[13]:= `Solve[b == λ1 a1 + λ2 a2 + λ3 a3, {λ1, λ2, λ3}]`

Out[13]= $\left\{\left\{\lambda_1 \to -\frac{1}{4}, \lambda_2 \to \frac{1}{4}, \lambda_3 \to \frac{3}{4}\right\}\right\}$

## Lineare Unabhängigkeit

### Beispiel 5.10 (a)

In[14]:= $a_1 = \{1, 0, 0\}$

Out[14]= $\{1, 0, 0\}$

In[15]:= $a_2 = \{0, 1, 0\}$

Out[15]= $\{0, 1, 0\}$

In[16]:= $a_3 = \{0, 0, 1\}$

Out[16]= $\{0, 0, 1\}$

In[17]:= $b = \{0, 0, 0\}$

Out[17]= $\{0, 0, 0\}$

In[18]:= Solve[$b == \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3$, $\{\lambda_1, \lambda_2, \lambda_3\}$]

Out[18]= $\{\{\lambda_1 \to 0, \lambda_2 \to 0, \lambda_3 \to 0\}\}$

### Beispiel 5.10 (b)

In[19]:= $a_1 = \{1, 1, 2\}$

Out[19]= $\{1, 1, 2\}$

In[20]:= $a_2 = \{3, 1, 0\}$

Out[20]= $\{3, 1, 0\}$

In[21]:= $a_3 = \{2, 2, 2\}$

Out[21]= $\{2, 2, 2\}$

In[22]:= $b = \{0, 0, 0\}$

Out[22]= $\{0, 0, 0\}$

In[23]:= Solve[$b == \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3$, $\{\lambda_1, \lambda_2, \lambda_3\}$]

Out[23]= $\{\{\lambda_1 \to 0, \lambda_2 \to 0, \lambda_3 \to 0\}\}$

### Beispiel 5.10 (c)

In[24]:= $a_1 = \{1, 3, 1\}$

Out[24]= $\{1, 3, 1\}$

In[25]:= $a_2 = \{2, 6, 1\}$

Out[25]= $\{2, 6, 1\}$

In[26]:= $a_3 = \{0, 0, 1\}$

Out[26]= $\{0, 0, 1\}$

In[27]:= $b = \{0, 0, 0\}$

Out[27]= $\{0, 0, 0\}$

In[28]:= `Solve[b == λ1 a1 + λ2 a2 + λ3 a3, {λ1, λ2, λ3}]`

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[28]= $\left\{\left\{\lambda_2 \to -\frac{\lambda_1}{2}, \lambda_3 \to -\frac{\lambda_1}{2}\right\}\right\}$

- **Beispiel 5.11**

In[29]:= `Clear[b]`

In[30]:= `a1 = {a, b, c}`

Out[30]= $\{a, b, c\}$

In[31]:= `a2 = {1 - i, 1/2, 2 i}`

Out[31]= $\left\{1 - i, \frac{1}{2}, 2\, i\right\}$

In[32]:= `a3 = {0, 1 + i, i}`

Out[32]= $\{0, 1 + i, i\}$

In[33]:= `d = {0, 0, 0}`

Out[33]= $\{0, 0, 0\}$

$\{0, 0, 0\}$

In[34]:= `Solve[d == λ1 a1 + λ2 a2 + λ3 a3, {λ1, λ2, λ3}]`

Out[34]= $\{\{\lambda_1 \to 0, \lambda_2 \to 0, \lambda_3 \to 0\}\}$

In[35]:= `A = Join[{a1}, {a2}, {a3}]`

Out[35]= $\begin{pmatrix} a & b & c \\ 1 - i & \frac{1}{2} & 2\, i \\ 0 & 1 + i & i \end{pmatrix}$

In[36]:= `RowReduce[A]`

Out[36]= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

In[37]:= `Inverse[A]`

Out[37]= $\begin{pmatrix} \frac{2-\frac{3i}{2}}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} & \frac{(1+i)c-i\,b}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} & \frac{2\,i\,b-\frac{c}{2}}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} \\ -\frac{1+i}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} & \frac{i\,a}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} & \frac{(1-i)c-2\,i\,a}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} \\ \frac{2}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} & -\frac{(1+i)a}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} & \frac{\frac{a}{2}-(1-i)b}{\left(2-\frac{3i}{2}\right)a-(1+i)b+2c} \end{pmatrix}$

Hieran sieht man, dass für $\left(2 - \frac{3\,i}{2}\right) a - (1 + i)\, b + 2\, c = 0$ lineare Abhängigkeit vorliegt.

- **Beispiel 5.14 (b)**

In[38]:= $b_1$ = {1, 0}

Out[38]= {1, 0}

In[39]:= $b_2$ = {1, 1}

Out[39]= {1, 1}

In[40]:= RowReduce[Join[{$b_1$}, {$b_2$}]]

Out[40]= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- **Beispiel 5.14 (c)**

In[41]:= $b_1$ = {-1, 0}

Out[41]= {−1, 0}

In[42]:= $b_2$ = {2, -3}

Out[42]= {2, −3}

In[43]:= RowReduce[Join[{$b_1$}, {$b_2$}]]

Out[43]= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- **Übung 5.3**

In[44]:= $b_1$ = {1, -1, 2}

Out[44]= {1, −1, 2}

In[45]:= $b_2$ = {2, 3, -1}

Out[45]= {2, 3, −1}

In[46]:= $b_3$ = {4, -1, 0}

Out[46]= {4, −1, 0}

In[47]:= RowReduce[Join[{$b_1$}, {$b_2$}, {$b_3$}]]

Out[47]= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- **Übung 5.3,2**

In[48]:= $b_1$ = {1, -1, 2}

Out[48]= {1, −1, 2}

In[49]:= $b_2$ = {2, 3, -1}

Out[49]= {2, 3, −1}

In[50]:= $b_3$ = {4, 1, 3}

Out[50]= {4, 1, 3}

In[51]:=
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RowReduce[Join[{b1}, {b2}, {b3}]]
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Out[51]=
$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

In[52]:=
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Solve[b3 == λ1 b1 + λ2 b2, {λ1, λ2}]
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Out[52]= $\{\{\lambda_1 \to 2, \lambda_2 \to 1\}\}$