Koordinaten in Vektorräumen

Beispiel 5.21

Man bestimme die Koordinaten des Vektors a = $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ **bezüglich der Basis** $b[1] = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $b[2] = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$, $b[3] = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$.

In[1]:= $b[1] = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$; $b[2] = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$; $b[3] = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$; $a = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$;

Wir zeigen zuerst, dass wir eine Basis haben:

In[2]:= $\text{Solve}\left[\sum_{k=1}^{3} \lambda[k]\, b[k] == 0, \text{Table}[\lambda[k], \{k, 1, 3\}]\right]$

Out[2]= $\{\{\lambda(1) \to 0, \lambda(2) \to 0, \lambda(3) \to 0\}\}$

Nun soll die Linearkombination gleich a sein

In[3]:= $\text{sol} = \text{Solve}\left[\sum_{k=1}^{3} \lambda[k]\, b[k] == a, \text{Table}[\lambda[k], \{k, 1, 3\}]\right]$

Out[3]= $\left\{\left\{\lambda(1) \to x, \lambda(2) \to x - y, \lambda(3) \to \frac{z}{3}\right\}\right\}$

Übergangsmatrix

In[4]:=
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B = Transpose[Join[Transpose[b[1]], Transpose[b[2]], Transpose[b[3]]]]
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Out[4]= $\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

In[5]:=
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Inverse[B].a
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Out[5]= $\begin{pmatrix} x \\ x - y \\ \frac{z}{3} \end{pmatrix}$

Beispiel 5.23

In[6]:= $b[1] = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$; $b[2] = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$; $b[3] = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$;

In[7]:=
```
M1 = Transpose[Join[Transpose[b[1]], Transpose[b[2]], Transpose[b[3]]]]
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Out[7]= $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 2 \\ -1 & 3 & 0 \end{pmatrix}$

In[8]:=
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4 Inverse[M1]
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Out[8]= $\begin{pmatrix} 6 & -3 & -4 \\ 2 & -1 & 0 \\ -6 & 5 & 4 \end{pmatrix}$

In[9]:=
```
btilde[1] = (2, 0, 1); btilde[2] = (0, 3, 0); btilde[3] = (1, 2, 1);
```

$\text{btilde}[1] = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix};\ \text{btilde}[2] = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix};\ \text{btilde}[3] = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix};$

In[10]:=
```
M2 = Transpose[
  Join[Transpose[btilde[1]], Transpose[btilde[2]], Transpose[btilde[3]]]]
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Out[10]= $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

In[11]:=
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Btilde = Inverse[M1].M2
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Out[11]= $\begin{pmatrix} 2 & -\frac{9}{4} & -1 \\ 1 & -\frac{3}{4} & 0 \\ -2 & \frac{15}{4} & 2 \end{pmatrix}$

In[12]:=
```
Table[btilde[j] == Sum[βtilde[k, j] b[k], {k, 1, 3}], {j, 1, 3}]
```

Out[12]= $\left\{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} == \begin{pmatrix} \beta\text{tilde}(1, 1) + 2\,\beta\text{tilde}(2, 1) + \beta\text{tilde}(3, 1) \\ 2\,\beta\text{tilde}(1, 1) + 2\,\beta\text{tilde}(3, 1) \\ 3\,\beta\text{tilde}(2, 1) - \beta\text{tilde}(1, 1) \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} == \begin{pmatrix} \beta\text{tilde}(1, 2) + 2\,\beta\text{tilde}(2, 2) + \beta\text{tilde}(3, 2) \\ 2\,\beta\text{tilde}(1, 2) + 2\,\beta\text{tilde}(3, 2) \\ 3\,\beta\text{tilde}(2, 2) - \beta\text{tilde}(1, 2) \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} == \begin{pmatrix} \beta\text{tilde}(1, 3) + 2\,\beta\text{tilde}(2, 3) + \beta\text{tilde}(3, 3) \\ 2\,\beta\text{tilde}(1, 3) + 2\,\beta\text{tilde}(3, 3) \\ 3\,\beta\text{tilde}(2, 3) - \beta\text{tilde}(1, 3) \end{pmatrix}\right\}$

In[13]:=
```
Solve[Table[btilde[j] == Sum[βtilde[k, j] b[k], {k, 1, 3}], {j, 1, 3}],
 Flatten[Table[βtilde[k, j], {k, 1, 3}, {j, 1, 3}]]]
```

Out[13]= $\left\{\left\{\beta\text{tilde}(1, 1) \to 2,\ \beta\text{tilde}(1, 2) \to -\frac{9}{4},\ \beta\text{tilde}(1, 3) \to -1,\ \beta\text{tilde}(2, 1) \to 1,\ \beta\text{tilde}(2, 2) \to -\frac{3}{4},\ \beta\text{tilde}(2, 3) \to 0,\ \beta\text{tilde}(3, 1) \to -2,\ \beta\text{tilde}(3, 2) \to \frac{15}{4},\ \beta\text{tilde}(3, 3) \to 2\right\}\right\}$

In[14]:=
```
Solve[Table[b[k] == Sum[β[k, j] btilde[j], {j, 1, 3}], {k, 1, 3}],
 Flatten[Table[β[k, j], {j, 1, 3}, {k, 1, 3}]]]
```

Out[14]= $\left\{\left\{\beta(1, 1) \to 2,\ \beta(2, 1) \to -1,\ \beta(3, 1) \to 1,\ \beta(1, 2) \to \frac{8}{3},\ \beta(2, 2) \to -\frac{8}{3},\ \beta(3, 2) \to \frac{4}{3},\ \beta(1, 3) \to -3,\ \beta(2, 3) \to 4,\ \beta(3, 3) \to -1\right\}\right\}$

In[15]:=
```
Inverse[M2].M1
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Out[15]= $\begin{pmatrix} 2 & -1 & 1 \\ \frac{8}{3} & -\frac{8}{3} & \frac{4}{3} \\ -3 & 4 & -1 \end{pmatrix}$

In[16]:=
```
B = Inverse[Btilde]
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Out[16]=
$$\begin{pmatrix} 2 & -1 & 1 \\ \frac{8}{3} & -\frac{8}{3} & \frac{4}{3} \\ -3 & 4 & -1 \end{pmatrix}$$