MR3394757 33C20 42C05
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On structure formulas for Wilson polynomials. (English summary)
Integral Transforms Spec. Funct. **26** (2015), *no. 12,* 1000–1014.

The Wilson polynomials

$$\frac{W_n(x^2;a,b,c,d)}{(a+b)_n(a+c)_n(a+d)_n} = {}_4F_3\left(\begin{matrix} -n, a+b+c+d+n-1, a-ix, a+ix \\ a+b, a+c, a+d \end{matrix}; 1\right)$$

are orthogonal with respect to the weight function

$$\omega(x) = \left|\frac{\Gamma(a+ix)\Gamma(b+ix)\Gamma(c+ix)\Gamma(d+ix)}{\Gamma(2if)}\right|^2$$

on $\mathbb{R}^+$, with

$$\int_0^\infty W_n\left(x^2;a,b,c,d\right) W_m\left(x^2;a,b,c,d\right)\omega\left(x\right)dx =$$

$$\delta_{nm} 2\pi\left(n+a+b+c+d-1\right)_n n!\Gamma\left(n+a+b\right)\Gamma\left(n+a+c\right) \times\frac{\Gamma\left(n+a+d\right)\Gamma\left(n+b+c\right)\Gamma\left(n+b+d\right)\Gamma\left(n+c+d\right)}{\Gamma\left(2n+a+b+c+d\right)}.$$

Here, $a,b,c,d>0$, $(a)_n$ is the Pochhammer symbol defined by

$$(a)_n = a(a+1)\dots(a+n-1), \quad n\geq 1,$$
$$(a)_0 = 1,$$

and ${}_pF_q$ is the generalized hypergeometric function.

In this paper, first, the authors show that the Wilson polynomials are solutions of the second-order difference equation

$$\phi\left(x^2\right)\mathbf{D}^2 y\left(x\right) + \psi\left(x^2\right)\mathbf{S}\mathbf{D}y\left(x\right) + \lambda_n y\left(x\right) = 0,$$

where ϕ and ψ are polynomials of degree 2 and 1, respectively, λ_n is a constant and

$$\mathbf{D}f\left(x\right) = \frac{f\left(x+i/2\right) - f\left(x-i/2\right)}{2ix}, \quad \mathbf{S}f\left(x\right) = \frac{f\left(x+i/2\right)+f\left(x-i/2\right)}{2}.$$

Then, the three-term recurrence relation satisfied by the Wilson polynomials is obtained and some structural relations for these polynomials are given. Moreover, the inversion and connection problems are solved for the Wilson polynomials and also for the continuous dual Hahn polynomials.
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.