MR1995218 (2004h:33017)

Foupouagnigni, M. (D-KSSL-MI); **Koepf, W.** (D-KSSL-MI); **Ronveaux, A.** (B-NDP)

On fourth-order difference equations for orthogonal polynomials of a discrete variable: derivation, factorization and solutions. (English summary)

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For a classical discrete orthogonal polynomial system the authors consider a class of systems of polynomials which includes the associated polynomials, the generalized co-recursive, co-recursive associated, co-dilated and co-modified polynomials. They derive a 4-th order difference equation (which acts on the argument of the polynomial) satisfied by polynomials of this class. This equation is factored as a product of two second-order difference relations. Using Maple 8 the authors obtained explicit formulas for the factors in the case of associated polynomials. For all systems mentioned above they obtain the basic sets of solutions of the 4-th order difference equation. The results for the associated polynomials with integer order of association are extended to those with real order of association. For the case of semi-classical discrete orthogonal polynomials a factored 4-th order difference equation for modified polynomials is also obtained.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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