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Factorization of fourth-order differential equations for perturbed classical orthogonal polynomials. (English summary)

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The paper investigates five classes of perturbed classical orthogonal polynomials. They are called the rth associated, the general co-recursive, the general co-recursive associated, the general co-dilated and the general co-modified classical polynomials. For example, the general co-recursive orthogonal polynomials are generated from the three-term recursion $P_{n+1}(x)=(x-\beta_n)P_n(x)-\gamma_nP_{n-1}(x)$, where the coefficients β_n,γ_n are the same as those appearing in the recursion formula for a sequence of classical polynomials except with β_k changed to $\beta_k+\mu$ for a specific k. The perturbed classical orthogonal polynomials satisfy a fourth-order linear differential equation with polynomial coefficients whose degrees do not depend on the degree of the orthogonal polynomials. It is shown that the corresponding linear differential operators of order four can be written as products of linear differential operators of order two. Some of the factors are given explicitly but others turn out to be too complicated to be presented. Moreover, fundamental systems of solutions of the differential equations of order four are computed. The solutions are expressed in terms of classical orthogonal polynomials and the corresponding functions of the second kind. The perturbed classical orthogonal polynomials are then expressed as linear combinations of the computed fundamental systems. The last section of the paper adds various applications and extensions.

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