MR2326190 (2008f:33010) 33C45 (05A15 05E35 33C20)
Chaggara, Hamza; **Koepf, Wolfram** (D-KSSL)
Duplication coefficients via generating functions. (English summary)
Complex Var. Elliptic Equ. **52** (2007), *no. 6,* 537–549.

A polynomial sequence $\{P_n\}_{n\geq 0}$ with complex coefficients is called a polynomial set if $\deg P_n = n$ for all nonnegative integers n. For a given polynomial set $\{P_n\}_{n\geq 0}$, it is very interesting to find the coefficients $C_m(n,a)$ in the expansion

$$P_n(ax)=\sum_{m=0}^{n} C_m(n,a)P_m(x),$$

where $a\neq 0$; this is called a duplication problem.

In this paper, the authors give a theorem to express explicitly the coefficients $C_m(n,a)$ in the case of Boas-Buck type, where the definition of Boas-Buck type is as follows: a polynomial set $\{P_n\}_{n\geq 0}$ is said to be of Boas-Buck type if there exists a sequence of nonzero numbers $(\lambda_n)_{n\geq 0}$ such that

$$\sum_{n=0}^{\infty}\lambda_n P_n(x)t^n = A(t)B(xC(t)),$$

where A, B, C are three formal power series such that

$$A(0)C'(0)\neq 0,\quad C(0)=0 \quad\text{and}\quad B^{(k)}(0)\neq 0,\quad k\in\mathbf{N}.$$

The proof of this theorem is based on Corollary 3.9 in [Y. Ben Cheikh and H. Chaggara, J. Comput. Appl. Math. **178** (2005), no. 1-2, 45–61; MR2127869 (2006f:33004)].

The authors also develop applications to classical orthogonal polynomials and classical discrete orthogonal polynomials.

Reviewed by *Shigeru Watanabe*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.