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Orthogonal polynomials and recurrence equations, operator equations and factorization. (English summary)

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Given a linear operator L in the dual algebraic space of polynomials with real coefficients, a quasi-definite linear functional u is said to be L-classical if there exist a monic polynomial σ of degree at most 2 and a polynomial τ of degree 1 such that the functional equation $L(\sigma u) = \tau u$ holds. Three cases have been studied in the literature [cf., e.g., A. F. Nikiforov and V. B. Uvarov, *Special functions of mathematical physics*, Translated from the Russian and with a preface by Ralph P. Boas, Birkhäuser, Basel, 1988; MR0922041 (89h:33001); A. F. Nikiforov, S. K. Suslov and V. B. Uvarov, *Classical orthogonal polynomials of a discrete variable*, Translated from the Russian, Springer, Berlin, 1991; MR1149380 (92m:33019)] when L is either the standard derivative operator D, the difference operator Δ, or the q-difference operator D_q.

In the paper under review and using a computer algebra package like Maple, the author computes how the coefficients of the three-term recurrence relation in which the sequence (P_n) of monic polynomials orthogonal with respect to u can be expressed in terms of the coefficients of the polynomials σ and τ. Furthermore, the converse problem is also analyzed.

Finally, from the fourth-order L-difference equation satisfied by the rth associated monic polynomials corresponding to an L-classical linear functional and using computer algebra, a factorization in terms of two second-order L-difference operators is obtained.

The author implements the above algorithms for some examples of L-classical linear functionals.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.