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**On linearization coefficients of Jacobi polynomials. (English summary)**

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In this article the authors explore the standard linearization problem (or Clebsch-Gordan type problem) for Jacobi polynomials  $P_k^{(\alpha,\beta)}(x)$ . They show that the linearization coefficients  $L_{ij}(k)$  of

$$P_i^{(\lambda,\delta)}(x)P_j^{(\mu,\gamma)}(x) = \sum_{k=0}^{i+j} L_{ij}(k)P_k^{(\alpha,\beta)}(x)$$

can be expressed in terms of double hypergeometric functions (Kampé de Fériet functions). Specializing to the case  $\alpha = \mu + \lambda$  and  $\beta = \delta + \gamma$ , these coefficients can be expressed by a product of two  ${}_3F_1$  hypergeometric functions, and by setting  $\lambda = \delta = \mu = \nu$  one of these hypergeometric sums can be simplified further to hypergeometric terms. This last simplification is obtained with Zeilberger's summation algorithm.

Reviewed by *Carsten Schneider*

## References

1. E.D. Rainville, *Special Functions*, The Macmillan Company, New York, 1960. [MR0107725 \(21 #6447\)](#)
2. E. Hylleraas, Linearization of products of Jacobi polynomials, *Math. Scand.* 10 (1962) 189–200. [MR0145123 \(26 #2658\)](#)
3. R. Askey, Linearization of the product of Jacobi polynomials II, *Canad. J. Math.* 22(1970) 582–593. [MR0264136 \(41 #8732\)](#)
4. G. Gasper, Linearization of the product of Jacobi polynomials I, *Canad. J. Math.* 22 (1970) 171–175. [MR0257433 \(41 #2084\)](#)
5. T. Koornwinder, Positivity proofs for linearization and connection coefficients for orthogonal polynomials satisfying an addition formula, *J. Lond. Math. Soc.* 18 (1978) 101–114. [MR0493144 \(58 #12175\)](#)
6. M. Rahman, The linearization of the product of continuous  $q$ -Jacobi-polynomials, *Canad. J. Math.* 33 (1981) 961–987. [MR0634153 \(83i:33007b\)](#)
7. M. Rahman, A non-negative representation of the linearization coefficients of the product of Jacobi polynomials, *Canad. J. Math.* 33 (1981) 915–928. [MR0634149 \(83i:33007a\)](#)
8. H.M. Srivastava, H.L. Manocha, *A Treatise on Generating Functions*, John Willey and Sons, New York, Chichester, Brisbane, Toronto, 1984. [MR0750112 \(85m:33016\)](#)
9. H. Chaggara, I. Lamiri, Linearization coefficients for Boas-Buck polynomial sets, *Appl. Math. Comput.* 189 (2007) 1533–1549. [MR2332108 \(2008b:33018\)](#)
10. R.P. Boas Jr, R.C. Buck, *Polynomial Expansions of Analytic Functions*, Springer Verlag, Berlin,

Göttingen, Heidelberg, 1964.

11. R. Koekoek, R.F. Swarttouw, The Askey-scheme of hypergeometric orthogonal polynomials and its  $q$ -analogue, Tech. Report 98-17, Faculty of the Technical Mathematics and Informatics, Delft University of Technology, Delft, 1998.
12. G. Gasper, Nonnegativity of a discrete Poisson kernel for the Hahn polynomials, *J. Math. Anal. Appl.* 42 (1973) 438–451. [MR0320392 \(47 #8931\)](#)
13. W. Koepf, *Hypergeometric Summation*, Vieweg, Braunschweig-Wiesbaden, 1998. [MR1644447 \(2000c:33002\)](#)
14. M. van Hoeij, Finite singularities and hypergeometric solutions of linear recurrence equations, *J. Pure Appl. Algebra* 139 (1999) 109–131. [MR1700540 \(2001h:39023\)](#)
15. G. Gasper, M. Rahman, Basic hypergeometric series, in: *Encyclopedia of Mathematics and its Applications*, vol. 35, Cambridge University Press, 1990. [MR1052153 \(91d:33034\)](#)
16. R. Askey, Orthogonal polynomials and special functions, in: *CBMS Regional Conference Series*, vol. 21, Society for Industrial and Applied Mathematics, Philadelphia, 1975. [MR0481145 \(58 #1288\)](#)
17. W. Koepf, D. Schmersau, Representations of orthogonal polynomials, *J. Comput. Appl. Math.* 90 (1998) 57–94. [MR1627168 \(2000d:33005\)](#)

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