```
> restart;
> read "hsum19.mpl";
```

Package "Hypergeometric Summation", Maple V - Maple 2019

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```
> RecurrenceNormalForm2:=proc(P,k,pn)
  local p,n;
  p:=op(0,pn);
  n:=op(1,pn);
  [sumrecursion(P,k,p(n)),normal(subs[eval]({k=0,n=0},P)),normal
  (subs[eval]({k=0,n=1},P)+subs[eval]({k=1,n=1},P))]
  end proc:
```

Jacobi polynomials

```
> term1:=pochhammer(alpha+1,n)/n!*hyperterm([-n,n+alpha+beta+1],
  [alpha+1],(1-x)/2,k);
```

$$term1 := \frac{\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(n+\alpha+\beta+1,k)\left(\frac{1}{2}-\frac{x}{2}\right)^k}{n!\,\mathrm{pochhammer}(\alpha+1,k)\,k!} \quad (2)$$

```
> RE1:=RecurrenceNormalForm2(term1,k,p(n));
```

$$RE1 := \left[2\,(n+2)\,(2+2\,n+\alpha+\beta)\,(n+2+\alpha+\beta)\,p(n+2)-(2\,n+3+\alpha+\beta)\,(\alpha^2\,x + 2\,\alpha\,\beta\,x+4\,\alpha\,n\,x+\beta^2\,x+4\,\beta\,n\,x+4\,n^2\,x+\alpha^2+6\,\alpha\,x-\beta^2+6\,\beta\,x+12\,n\,x+8\,x)\,p(n+1)+2\,(1+n+\beta)\,(\alpha+1+n)\,(\alpha+\beta+2\,n+4)\,p(n)=0, 1, \frac{1}{2}\,\alpha+x+\frac{1}{2}\,\alpha\,x-\frac{1}{2}\,\beta+\frac{1}{2}\,\beta\,x\right] \quad (3)$$

```
> term2:=(-1)^n*binomial(n+beta,n)*hyperterm([-n,n+alpha+beta+1],
  [beta+1],(1+x)/2,k);
```

$$term2 := \frac{(-1)^n\binom{n+\beta}{n}\,\mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(n+\alpha+\beta+1,k)\left(\frac{1}{2}+\frac{x}{2}\right)^k}{\mathrm{pochhammer}(\beta+1,k)\,k!} \quad (4)$$

```
> RE2:=RecurrenceNormalForm2(term2,k,p(n));
```

$$RE2 := \left[2\,(n+2)\,(2+2\,n+\alpha+\beta)\,(n+2+\alpha+\beta)\,p(n+2)-(2\,n+3+\alpha+\beta)\,(\alpha^2\,x + 2\,\alpha\,\beta\,x+4\,\alpha\,n\,x+\beta^2\,x+4\,\beta\,n\,x+4\,n^2\,x+\alpha^2+6\,\alpha\,x-\beta^2+6\,\beta\,x+12\,n\,x+8\,x)\,p(n+1)+2\,(1+n+\beta)\,(\alpha+1+n)\,(\alpha+\beta+2\,n+4)\,p(n)=0, 1, \frac{1}{2}\,\alpha+x+\frac{1}{2}\,\alpha\,x-\frac{1}{2}\,\beta+\frac{1}{2}\,\beta\,x\right] \quad (5)$$

```
> simplify(RE1-RE2);
```

$$[0, 0, 0] \quad (6)$$

```
> term3:=binomial(2*n+alpha+beta,n)*((x-1)/2)^n*hyperterm([-n,-n-alpha],[-2*n-alpha-beta],2/(1-x),k);
```

$$term3 := \frac{\binom{2n+\alpha+\beta}{n} \left(-\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(-n-\alpha,k) \left(\frac{2}{1-x}\right)^k}{\mathrm{pochhammer}(-2n-\alpha-\beta,k)\; k!} \quad (7)$$

```
> RE3:=RecurrenceNormalForm2(term3,k,p(n));
```

$$RE3 := \left[2\,(n+2)\,(2+2n+\alpha+\beta)\,(n+2+\alpha+\beta)\,p(n+2) - (2n+3+\alpha+\beta)\,(\alpha^2 x + 2\alpha\beta x + 4\alpha n x + \beta^2 x + 4\beta n x + 4n^2 x + \alpha^2 + 6\alpha x - \beta^2 + 6\beta x + 12 n x + 8x)\,p(n+1) + 2\,(1+n+\beta)\,(\alpha+1+n)\,(\alpha+\beta+2n+4)\,p(n) = 0, 1, \frac{1}{2}\alpha + x + \frac{1}{2}\alpha x - \frac{1}{2}\beta + \frac{1}{2}\beta x\right] \quad (8)$$

```
> simplify(RE1-RE3);
```

$$[0, 0, 0] \quad (9)$$

```
> term4:=binomial(n+alpha,n)*((1+x)/2)^n*hyperterm([-n,-n-beta],[alpha+1],(x-1)/(x+1),k);
```

$$term4 := \frac{\binom{n+\alpha}{n} \left(\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(-n-\beta,k) \left(\frac{-1+x}{1+x}\right)^k}{\mathrm{pochhammer}(\alpha+1,k)\; k!} \quad (10)$$

```
> RE4:=RecurrenceNormalForm2(term4,k,p(n));
```

$$RE4 := \left[2\,(n+2)\,(2+2n+\alpha+\beta)\,(n+2+\alpha+\beta)\,p(n+2) - (2n+3+\alpha+\beta)\,(\alpha^2 x + 2\alpha\beta x + 4\alpha n x + \beta^2 x + 4\beta n x + 4n^2 x + \alpha^2 + 6\alpha x - \beta^2 + 6\beta x + 12 n x + 8x)\,p(n+1) + 2\,(1+n+\beta)\,(\alpha+1+n)\,(\alpha+\beta+2n+4)\,p(n) = 0, 1, \frac{1}{2}\alpha + x + \frac{1}{2}\alpha x - \frac{1}{2}\beta + \frac{1}{2}\beta x\right] \quad (11)$$

```
> simplify(RE1-RE4);
```

$$[0, 0, 0] \quad (12)$$

```
> term5:=binomial(n+beta,n)*((x-1)/2)^n*hyperterm([-n,-n-alpha],[beta+1],(x+1)/(x-1),k);
```

$$term5 := \frac{\binom{n+\beta}{n} \left(-\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(-n-\alpha,k) \left(\frac{1+x}{-1+x}\right)^k}{\mathrm{pochhammer}(\beta+1,k)\; k!} \quad (13)$$

```
> RE5:=RecurrenceNormalForm2(term5,k,p(n));
```

$$RE5 := \left[2\,(n+2)\,(2+2n+\alpha+\beta)\,(n+2+\alpha+\beta)\,p(n+2) - (2n+3+\alpha+\beta)\,(\alpha^2 x \right. \quad (14)$$

$$+ 2\alpha\beta x + 4\alpha n x + \beta^2 x + 4\beta n x + 4 n^2 x + \alpha^2 + 6\alpha x - \beta^2 + 6\beta x + 12 n x + 8 x\big) p(n+1) + 2(1+n+\beta)(\alpha+1+n)(\alpha+\beta+2n+4) p(n) = 0, 1, \frac{1}{2}\alpha + x + \frac{1}{2}\alpha x - \frac{1}{2}\beta + \frac{1}{2}\beta x\Big]$$

```
> simplify(RE1-RE5);
```

$$[0, 0, 0] \tag{15}$$

Next, we reverse the order of summation of series 1.

```
> convert(Sumtohyper(subs(k=n-k,term1),k),binomial);
```

$$(-1)^n \left(\frac{1}{2} - \frac{x}{2}\right)^n \mathrm{Hypergeom}\left([-n-\alpha, -n], [-2n-\alpha-\beta], \frac{2}{1-x}\right) \binom{2n+\alpha+\beta}{n} \tag{16}$$

```
> term3a:=termtohyper(subs(k=n-k,term1),k);
```

$$term3a := \frac{1}{n!^2 \,\mathrm{pochhammer}(-2n-\alpha-\beta, k)\, k!} \Big(\mathrm{pochhammer}(-n, n)\,\mathrm{pochhammer}(n+\alpha+\beta+1, n) \left(\frac{1}{2} - \frac{x}{2}\right)^n \mathrm{pochhammer}(-n, k)\,\mathrm{pochhammer}(-n-\alpha, k) \left(\frac{2}{1-x}\right)^k\Big) \tag{17}$$

```
> simplify(term3/term3a) assuming n::integer;
```

$$1 \tag{18}$$

This shows that the reverse of series 1 is series 3.

Let's reverse series 2:

```
> convert(Sumtohyper(subs(k=n-k,term2),k),binomial);
```

$$((-1)^n)^2 \left(\frac{1}{2} + \frac{x}{2}\right)^n \mathrm{Hypergeom}\left([-n, -n-\beta], [-2n-\alpha-\beta], -\frac{2}{-1-x}\right) \binom{2n+\alpha+\beta}{n} \tag{19}$$

This is a new identity, not yet in our list.

```
> term6:=binomial(2*n+alpha+beta,n)*((1+x)/2)^n*hyperterm([-n,-n-
  beta],[-2*n-alpha-beta],2/(1+x),k);
```

$$term6 := \frac{\binom{2n+\alpha+\beta}{n} \left(\frac{1}{2} + \frac{x}{2}\right)^n \mathrm{pochhammer}(-n, k)\,\mathrm{pochhammer}(-n-\beta, k) \left(\frac{2}{1+x}\right)^k}{\mathrm{pochhammer}(-2n-\alpha-\beta, k)\, k!} \tag{20}$$

```
> RE6:=RecurrenceNormalForm2(term6,k,p(n));
```

$$RE6 := \Big[2(n+2)(2+2n+\alpha+\beta)(n+2+\alpha+\beta) p(n+2) - (2n+3+\alpha+\beta)\big(\alpha^2 x + 2\alpha\beta x + 4\alpha n x + \beta^2 x + 4\beta n x + 4 n^2 x + \alpha^2 + 6\alpha x - \beta^2 + 6\beta x + 12 n x + 8 x\big) p(n+1) + 2(1+n+\beta)(\alpha+1+n)(\alpha+\beta+2n+4) p(n) = 0, 1, \frac{1}{2}\alpha + x + \frac{1}{2}\alpha x \tag{21}$$

$$-\frac{1}{2}\beta+\frac{1}{2}\beta x]$$

```
> simplify(RE1-RE6);
```

$$[0, 0, 0] \quad \textbf{(22)}$$

Let's reverse series 4:

```
> convert(Sumtohyper(subs(k=n-k,term4),k),binomial);
```

$$\frac{\left(\frac{1}{2}+\frac{x}{2}\right)^n (-1)^n (-1+x)^n \operatorname{Hypergeom}\left([-n-\alpha, -n], [\beta+1], \frac{1+x}{-1+x}\right) \binom{-\beta-1}{n}}{(1+x)^n} \quad \textbf{(23)}$$

```
> term5a:=termtohyper(subs(k=n-k,term4),k);
```

$$term5a := \left(\binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^n \operatorname{pochhammer}(-n, n) \operatorname{pochhammer}(-n-\beta, n) \left(\frac{-1+x}{1+x}\right)^n \operatorname{pochhammer}(-n-\alpha, k) \operatorname{pochhammer}(-n, k) \left(\frac{1+x}{-1+x}\right)^k\right) \Big/ \left(\operatorname{pochhammer}(\alpha+1, n)\, n!\, \operatorname{pochhammer}(\beta+1, k)\, k!\right) \quad \textbf{(24)}$$

```
> simplify(term5/term5a) assuming n::integer;
```

$$1 \quad \textbf{(25)}$$

This shows that the reverse of series 4 is series 5.

Relation (15.8.1) in Olver, Lozier, Boisvert and Clark, 2010

```
> TERM1:= (1-z)^(-a)*Hypergeom([a, c-b],[c],z/(z-1));
```

$$TERM1 := (1-z)^{-a} \operatorname{Hypergeom}\left([a, c-b], [c], \frac{z}{z-1}\right) \quad \textbf{(26)}$$

```
> TERM2:= (1-z)^(-b)*Hypergeom([c-a, b],[c],z/(z-1));
```

$$TERM2 := (1-z)^{-b} \operatorname{Hypergeom}\left([c-a, b], [c], \frac{z}{z-1}\right) \quad \textbf{(27)}$$

```
> TERM3:=(1-z)^(c-a-b)*Hypergeom([c-a, c-b],[c],z);
```

$$TERM3 := (1-z)^{c-a-b} \operatorname{Hypergeom}([c-a, c-b], [c], z) \quad \textbf{(28)}$$

Relation (15.8.6) in Olver, Lozier, Boisvert and Clark, 2010

```
> TERM4:=pochhammer(b,n)/pochhammer(c,n)*(-z)^n*Hypergeom([-n,1-c-
  n],[1-b-n],1/z);
```

$$TERM4 := \frac{\operatorname{pochhammer}(b, n)\, (-z)^n \operatorname{Hypergeom}\left([-n, 1-c-n], [1-b-n], \frac{1}{z}\right)}{\operatorname{pochhammer}(c, n)} \quad \textbf{(29)}$$

```
> TERM5:=pochhammer(b,n)/pochhammer(c,n)*(1-z)^n*Hypergeom([-n,c-
  b],[1-b-n],1/(1-z));
```

$$TERM5 := \frac{\operatorname{pochhammer}(b, n)\, (1-z)^n \operatorname{Hypergeom}\left([-n, c-b], [1-b-n], \frac{1}{1-z}\right)}{\operatorname{pochhammer}(c, n)} \quad \textbf{(30)}$$

Relation (15.8.7) in Olver, Lozier, Boisvert and Clark, 2010

```
> TERM6:=pochhammer(c-b,n)/pochhammer(c,n)*Hypergeom([-n,b],[b-c-
  n+1],1-z);
```

(31)

$$TERM6 := \frac{\text{pochhammer}(c-b, n)\, \text{Hypergeom}([-n, b], [b-c-n+1], 1-z)}{\text{pochhammer}(c, n)} \quad (31)$$

```
> TERM7:=pochhammer(c-b,n)/pochhammer(c,n)*(z)^n*Hypergeom([-n,1-c-n],[b-c-n+1],1-1/z);
```

$$TERM7 := \frac{\text{pochhammer}(c-b, n)\, z^n\, \text{Hypergeom}\left([-n, 1-c-n], [b-c-n+1], 1-\frac{1}{z}\right)}{\text{pochhammer}(c, n)} \quad (32)$$

Substitutions done in order to get series 1 and 2

```
> chang1:= {a=-n, b=n+alpha+beta+1,c=alpha+1, z=(1-x)/2};
```

$$chang1 := \left\{a=-n, b=n+\alpha+\beta+1, c=\alpha+1, z=\frac{1}{2}-\frac{x}{2}\right\} \quad (33)$$

```
> chang2:= {a=-n, b=n+alpha+beta+1,c=beta+1, z=(1+x)/2};
```

$$chang2 := \left\{a=-n, b=n+\alpha+\beta+1, c=\beta+1, z=\frac{1}{2}+\frac{x}{2}\right\} \quad (34)$$

We substitute chang1 in (15.8.6) and (15.8.7)

```
> ser31:=simplify(subs(chang1, binomial(n+alpha, n)*TERM4));
```

$$ser31 := \left(-\frac{1}{2}+\frac{x}{2}\right)^n \text{Hypergeom}\left([-n, -n-\alpha], [-2n-\alpha-\beta], -\frac{2}{-1+x}\right) \binom{2n+\alpha+\beta}{n} \quad (35)$$

```
> term31:=binomial(n+alpha, n)*pochhammer(n+alpha+beta+1, n)*(-1/2+(1/2)*x)^n*hyperterm([-n, -n-alpha], [-2*n-alpha-beta], -2/(-1+x),k)/pochhammer(alpha+1, n);
```

$$term31 := \left(\binom{n+\alpha}{n} \text{pochhammer}(n+\alpha+\beta+1, n) \left(-\frac{1}{2}+\frac{x}{2}\right)^n \text{pochhammer}(-n-\alpha, k)\, \text{pochhammer}(-n, k) \left(-\frac{2}{-1+x}\right)^k\right) \Big/ \left(\text{pochhammer}(-2n-\alpha-\beta, k)\, k!\, \text{pochhammer}(\alpha+1, n)\right) \quad (36)$$

```
> simplify(term3/term31);
```

$$1 \quad (37)$$

We recover here series 3

```
> ser61:=simplify(subs(chang1, binomial(n+alpha, n)*TERM5));
```

$$ser61 := \left(\frac{1}{2}+\frac{x}{2}\right)^n \text{Hypergeom}\left([-n, -n-\beta], [-2n-\alpha-\beta], \frac{2}{1+x}\right) \binom{2n+\alpha+\beta}{n} \quad (38)$$

```
> term61:=binomial(n+alpha, n)*pochhammer(n+alpha+beta+1, n)*(1/2+(1/2)*x)^n*hyperterm([-n, -n-beta], [-2*n-alpha-beta], 2/(1+x),k)/pochhammer(alpha+1, n);
```

$$term61 := \left(\binom{n+\alpha}{n} \text{pochhammer}(n+\alpha+\beta+1, n) \left(\frac{1}{2}+\frac{x}{2}\right)^n \text{pochhammer}(-n, k)\, \text{pochhammer}(-n-\beta, k) \left(\frac{2}{1+x}\right)^k\right) \Big/ \left(\text{pochhammer}(-2n-\alpha-\beta, k)\, k! \right. \quad (39)$$

$\mathrm{pochhammer}(\alpha+1, n))$

```
> simplify(term6/term61);
```

$$1 \qquad \textbf{(40)}$$

We recover here series 6

```
> ser21:=simplify(subs(chang1, binomial(n+alpha, n)*TERM6));
```

$$ser21 := \mathrm{Hypergeom}\left([-n, n+\alpha+\beta+1], [\beta+1], \frac{1}{2}+\frac{x}{2}\right) \binom{-\beta-1}{n} \qquad \textbf{(41)}$$

```
> term21:=GAMMA(-beta)*hyperterm([-n, n+alpha+beta+1], [1+beta],
  1/2+(1/2)*x,k)/(GAMMA(n+1)*GAMMA(-n-beta));
```

$$term21 := \frac{\Gamma(-\beta)\ \mathrm{pochhammer}(-n, k)\ \mathrm{pochhammer}(n+\alpha+\beta+1, k) \left(\frac{1}{2}+\frac{x}{2}\right)^k}{\mathrm{pochhammer}(\beta+1, k)\ k!\ \Gamma(n+1)\ \Gamma(-n-\beta)} \qquad \textbf{(42)}$$

```
> simplify(term2/term21) assuming n::integer;
```

$$1 \qquad \textbf{(43)}$$

We recover here series 2

```
> ser51:=simplify(subs(chang1, binomial(n+alpha, n)*TERM7));
```

$$ser51 := \left(\frac{1}{2}-\frac{x}{2}\right)^n \mathrm{Hypergeom}\left([-n, -n-\alpha], [\beta+1], \frac{1+x}{-1+x}\right) \binom{-\beta-1}{n} \qquad \textbf{(44)}$$

```
> term51:=GAMMA(-beta)*(1/2-(1/2)*x)^n*hyperterm([-n, -n-alpha],
  [1+beta], (1+x)/(-1+x),k)/(GAMMA(n+1)*GAMMA(-n-beta));
```

$$term51 := \frac{\Gamma(-\beta) \left(\frac{1}{2}-\frac{x}{2}\right)^n \mathrm{pochhammer}(-n, k)\ \mathrm{pochhammer}(-n-\alpha, k) \left(\frac{1+x}{-1+x}\right)^k}{\mathrm{pochhammer}(\beta+1, k)\ k!\ \Gamma(n+1)\ \Gamma(-n-\beta)} \qquad \textbf{(45)}$$

```
> simplify(term5/term51) assuming n::integer;
```

$$1 \qquad \textbf{(46)}$$

We recover here series 5

We now substitute chang2 in (15.8.6) and (15.8.7)

```
> simpcomb(subs(chang2, [(-1)^n*binomial(n+beta, n)*TERM4, (-1)^n*
  binomial(n+beta, n)*TERM5, (-1)^n*binomial(n+beta, n)*TERM6, (-1)
  ^n*binomial(n+beta, n)*TERM7]));
```

$$\left[\frac{1}{\Gamma(n+1)\ \Gamma(n+\alpha+\beta+1)} \left((-1)^n\ \Gamma(2n+\alpha+\beta+1) \left(-\frac{1}{2}-\frac{x}{2}\right)^n \mathrm{Hypergeom}\left([\qquad \textbf{(47)}$$

$$-n, -n-\beta], [-2n-\alpha-\beta], \frac{2}{1+x}\right)\right),$$

$$\frac{1}{\Gamma(n+1)\ \Gamma(n+\alpha+\beta+1)} \left((-1)^n\ \Gamma(2n+\alpha+\beta+1) \left(\frac{1}{2}-\frac{x}{2}\right)^n \mathrm{Hypergeom}\left([\right.\right.$$

$$-n, -n-\alpha], [-2\,n-\alpha-\beta], -\frac{2}{-1+x}\Big)\Big),$$

$$\frac{(-1)^n\,\Gamma(-\alpha)\ \mathrm{Hypergeom}\left([-n, n+\alpha+\beta+1], [\alpha+1], \frac{1}{2}-\frac{x}{2}\right)}{\Gamma(n+1)\,\Gamma(-n-\alpha)},$$

$$\frac{(-1)^n\,\Gamma(-\alpha)\left(\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{Hypergeom}\left([-n, -n-\beta], [\alpha+1], \frac{-1+x}{1+x}\right)}{\Gamma(n+1)\,\Gamma(-n-\alpha)}\Bigg]$$

and recover series 6, series 3, series 1, and series 4.

The substitution of chang1 in (15.8.1) yields

```
> sol1:=simplify(subs(chang1, [binomial(n+alpha, n)*TERM1, binomial
  (n+alpha, n)*TERM2, binomial(n+alpha, n)*TERM3]));
```

$$sol1 := \left[\binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{Hypergeom}\left([-n, -n-\beta], [\alpha+1], \frac{-1+x}{1+x}\right), \binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^{-n-\alpha-\beta-1} \mathrm{Hypergeom}\left([\alpha+1+n, n+\alpha+\beta+1], [\alpha+1], \frac{-1+x}{1+x}\right), \binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^{-\beta} \mathrm{Hypergeom}\left([\alpha+1+n, -n-\beta], [\alpha+1], \frac{1}{2}-\frac{x}{2}\right)\right] \quad \textbf{(48)}$$

We recover here series 4 and two series representations of the Jacobi polynomials

```
> series1:=sol1[2];
```

$$series1 := \binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^{-n-\alpha-\beta-1} \mathrm{Hypergeom}\left([\alpha+1+n, n+\alpha+\beta+1], [\alpha+1], \frac{-1+x}{1+x}\right) \quad \textbf{(49)}$$

```
> series2:= sol1[3];
```

$$series2 := \binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^{-\beta} \mathrm{Hypergeom}\left([\alpha+1+n, -n-\beta], [\alpha+1], \frac{1}{2}-\frac{x}{2}\right) \quad \textbf{(50)}$$

```
> termseries1:=binomial(n+alpha, n)*(1/2+(1/2)*x)^(-n-alpha-beta-1)
  *hyperterm([alpha+1+n, n+alpha+beta+1], [alpha+1], (-1+x)/(1+x),
  k);
```

$$termseries1 := \frac{1}{\mathrm{pochhammer}(\alpha+1, k)\ k!}\left(\binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^{-n-\alpha-\beta-1} \mathrm{pochhammer}(\alpha+1+n, k)\ \mathrm{pochhammer}(n+\alpha+\beta+1, k)\left(\frac{-1+x}{1+x}\right)^k\right) \quad \textbf{(51)}$$

```
> termseries2:=binomial(n+alpha, n)*(1/2+(1/2)*x)^(-beta)*hyperterm
  ([alpha+1+n, -n-beta], [alpha+1], 1/2-(1/2)*x, k);
```

$termseries2 :=$ **(52)**

$$\frac{1}{\text{pochhammer}(\alpha+1,k)\,k!}\left(\binom{n+\alpha}{n}\left(\frac{1}{2}+\frac{x}{2}\right)^{-\beta}\text{pochhammer}(\alpha+1+n,k)\,\text{pochhammer}(-n-\beta,k)\left(\frac{1}{2}-\frac{x}{2}\right)^{k}\right)$$

```
> op(1, sumrecursion(termseries1,k,p(n)))-op([1,1],RE1);
  op(1, sumrecursion(termseries2,k,p(n)))-op([1,1],RE1);
```

$$0$$

$$0 \quad \mathbf{(53)}$$

This means the latter two series representations satisfy the same recurrence relation with the same initial conditions given below.

```
> _EnvFormal := true:
> initial1:=simplify([sum(subs(n=0,termseries1), k=0..infinity),
  sum(subs(n=1,termseries1), k=0..infinity)]) assuming(x>0,x<1);
```

$$initial1 := \left[1, \frac{(2+\alpha+\beta)\,x}{2}-\frac{\beta}{2}+\frac{\alpha}{2}\right] \quad \mathbf{(54)}$$

```
> initial2:=simplify([sum(subs(n=0,termseries2), k=0..infinity),
  sum(subs(n=1,termseries2), k=0..infinity)]) assuming(x>0,x<1);
```

$$initial2 := \left[1, \frac{(2+\alpha+\beta)\,x}{2}-\frac{\beta}{2}+\frac{\alpha}{2}\right] \quad \mathbf{(55)}$$

```
> [op(2, RE1),op(3, RE1)];
```

$$\left[1, \frac{1}{2}\alpha+x+\frac{1}{2}\alpha x-\frac{1}{2}\beta+\frac{1}{2}\beta x\right] \quad \mathbf{(56)}$$

```
> simplify(initial2- [op(2, RE1),op(3, RE1)]);
```

$$[0, 0] \quad \mathbf{(57)}$$

The substitution of chang2 in (15.8.1) yields

```
> sol2:=simplify(subs(chang2, [(-1)^n*binomial(n+beta, n)*TERM1,
  (-1)^n*binomial(n+beta, n)*TERM2, (-1)^n*binomial(n+beta, n)*
  TERM3]));
```

$$sol2 := \left[(-1)^n\left(\frac{1}{2}-\frac{x}{2}\right)^n \text{Hypergeom}\left([-n, -n-\alpha], [\beta+1], \frac{1+x}{-1+x}\right)\binom{n+\beta}{\beta}, (-1)^n\left(\frac{1}{2}-\frac{x}{2}\right)^{-n-\alpha-\beta-1}\text{Hypergeom}\left([1+n+\beta, n+\alpha+\beta+1], [\beta+1], \frac{1+x}{-1+x}\right)\binom{n+\beta}{\beta}, (-1)^n\left(\frac{1}{2}-\frac{x}{2}\right)^{-\alpha}\text{Hypergeom}\left([1+n+\beta, -n-\alpha], [\beta+1], \frac{1}{2}+\frac{x}{2}\right)\binom{n+\beta}{\beta}\right] \quad \mathbf{(58)}$$

We recover here series 5 and two series representations of the Jacobi polynomials

```
> series3:=sol2[2];
```

$$series3 := (-1)^n\left(\frac{1}{2}-\frac{x}{2}\right)^{-n-\alpha-\beta-1}\text{Hypergeom}\Big([1+n+\beta, n+\alpha+\beta+1], [\beta+1], \quad \mathbf{(59)}$$

$$\frac{1+x}{-1+x}\Big)\binom{n+\beta}{\beta}$$

```
> series4:= sol2[3];
```

$$series4 := (-1)^n\left(\frac{1}{2}-\frac{x}{2}\right)^{-\alpha}\mathrm{Hypergeom}\left([1+n+\beta, -n-\alpha], [\beta+1], \frac{1}{2}+\frac{x}{2}\right)\binom{n+\beta}{\beta} \quad (60)$$

```
> termseries3:=-(-1)^(n+1)*binomial(n+beta, n)*(1/2-(1/2)*x)^(-n-alpha-beta-1)*hyperterm([n+beta+1, n+alpha+beta+1], [1+beta], (1+x)/(-1+x), k);
```

$$termseries3 := -\frac{1}{\mathrm{pochhammer}(\beta+1,k)\,k!}\left((-1)^{n+1}\binom{n+\beta}{n}\left(\frac{1}{2}-\frac{x}{2}\right)^{-n-\alpha-\beta-1}\mathrm{pochhammer}(1+n+\beta,k)\,\mathrm{pochhammer}(n+\alpha+\beta+1,k)\left(\frac{1+x}{-1+x}\right)^k\right) \quad (61)$$

```
> termseries4:=(-1)^n*binomial(n+beta, n)*(1/2-(1/2)*x)^(-alpha)*hyperterm([n+beta+1, -n-alpha], [1+beta], 1/2+(1/2)*x, k);
```

$$termseries4 := \frac{1}{\mathrm{pochhammer}(\beta+1,k)\,k!}\left((-1)^{n}\binom{n+\beta}{n}\left(\frac{1}{2}-\frac{x}{2}\right)^{-\alpha}\mathrm{pochhammer}(1+n+\beta,k)\,\mathrm{pochhammer}(-n-\alpha,k)\left(\frac{1}{2}+\frac{x}{2}\right)^k\right) \quad (62)$$

```
> op(1, sumrecursion(termseries3,k,p(n)))-op([1,1],RE1);
  op(1, sumrecursion(termseries4,k,p(n)))-op([1,1],RE1);
```

$$0$$

$$0 \quad (63)$$

This means the latter two series representations satisfy the same recurrence relation with the same initial conditions given below.

```
> initial3:=simplify([sum(subs(n=0,termseries3), k=0..infinity), sum(subs(n=1,termseries3), k=0..infinity)]) assuming(x>0,x<1);
```

$$initial3 := \left[1, \frac{(2+\alpha+\beta)\,x}{2}-\frac{\beta}{2}+\frac{\alpha}{2}\right] \quad (64)$$

```
> initial4:=simplify([sum(subs(n=0,termseries4), k=0..infinity), sum(subs(n=1,termseries4), k=0..infinity)]) assuming(x>0,x<1);
```

$$initial4 := \left[1, \frac{(2+\alpha+\beta)\,x}{2}-\frac{\beta}{2}+\frac{\alpha}{2}\right] \quad (65)$$

```
> [op(2, RE1),op(3, RE1)];
```

$$\left[1, \frac{1}{2}\alpha+x+\frac{1}{2}\alpha x-\frac{1}{2}\beta+\frac{1}{2}\beta x\right] \quad (66)$$

```
> simplify(initial4- [op(2, RE1),op(3, RE1)]);
```

$$[0, 0] \quad (67)$$

Gegenbauer polynomials

```
> convert(Sumtohyper(pochhammer(2*lambda,n)/pochhammer(lambda+1/2,
```

```
n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term1),k),binomial);
```

$$\mathrm{Hypergeom}\left(\left[-n, n+2\lambda\right], \left[\lambda+\frac{1}{2}\right], \frac{1}{2}-\frac{x}{2}\right) \binom{n+2\lambda-1}{n} \quad \textbf{(68)}$$

```
> convert(Sumtohyper(pochhammer(2*lambda,n)/pochhammer(lambda+1/2,
  n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term2),k),binomial);
```

$$(-1)^n \,\mathrm{Hypergeom}\left(\left[-n, n+2\lambda\right], \left[\lambda+\frac{1}{2}\right], \frac{1}{2}+\frac{x}{2}\right) \binom{n+2\lambda-1}{n} \quad \textbf{(69)}$$

```
> convert(simplify(Sumtohyper(pochhammer(2*lambda,n)/pochhammer
  (lambda+1/2,n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term3),k)
  ),binomial);
```

$$\mathrm{Hypergeom}\left(\left[-n, -n-\lambda+\frac{1}{2}\right], \left[-2n-2\lambda+1\right], -\frac{2}{-1+x}\right) (2x-2)^n \binom{n+\lambda-1}{n} \quad \textbf{(70)}$$

```
> convert(Sumtohyper(pochhammer(2*lambda,n)/pochhammer(lambda+1/2,
  n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term4),k),binomial);
```

$$\left(\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{Hypergeom}\left(\left[-n, -n-\lambda+\frac{1}{2}\right], \left[\lambda+\frac{1}{2}\right], \frac{2x-2}{2+2x}\right) \binom{n+2\lambda-1}{n} \quad \textbf{(71)}$$

```
> convert(Sumtohyper(pochhammer(2*lambda,n)/pochhammer(lambda+1/2,
  n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term5),k),binomial);
```

$$\left(-\frac{1}{2}+\frac{x}{2}\right)^n \mathrm{Hypergeom}\left(\left[-n, -n-\lambda+\frac{1}{2}\right], \left[\lambda+\frac{1}{2}\right], \frac{2+2x}{2x-2}\right) \binom{n+2\lambda-1}{n} \quad \textbf{(72)}$$

```
> convert(simplify(Sumtohyper(pochhammer(2*lambda,n)/pochhammer
  (lambda+1/2,n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term6),k)
  ),binomial);
```

$$\mathrm{Hypergeom}\left(\left[-n, -n-\lambda+\frac{1}{2}\right], \left[-2n-2\lambda+1\right], \frac{2}{1+x}\right) (2+2x)^n \binom{n+\lambda-1}{n} \quad \textbf{(73)}$$

```
> RE1:=RecurrenceNormalForm2(pochhammer(2*lambda,n)/pochhammer
  (lambda+1/2,n)*subs({alpha=lambda-1/2,beta=lambda-1/2},term1),k,p
  (n));
```

$$RE1 := \left[(n+2)\, p(n+2) - 2x\,(n+\lambda+1)\, p(n+1) + (n+2\lambda)\, p(n) = 0, 1, 2\lambda x\right] \quad \textbf{(74)}$$

```
> term7:=(2*x)^n*pochhammer(lambda,n)/n!*hyperterm([-n/2,-(n-1)/2],
  [1-lambda-n],1/x^2,k);
```

$$term7 := \quad \textbf{(75)}$$

$$\frac{(2x)^n \,\mathrm{pochhammer}(\lambda, n)\, \mathrm{pochhammer}\left(-\frac{n}{2}, k\right) \mathrm{pochhammer}\left(-\frac{n}{2}+\frac{1}{2}, k\right) \left(\frac{1}{x^2}\right)^k}{n!\, \mathrm{pochhammer}(-n-\lambda+1, k)\, k!}$$

```
> RE7:=RecurrenceNormalForm2(term7,k,p(n));
```

$$RE7 := \left[(n+2)\, p(n+2) - 2x\,(n+\lambda+1)\, p(n+1) + (n+2\lambda)\, p(n) = 0, 1, 2\lambda x\right] \quad \textbf{(76)}$$

Reversion yields

```
> res1:=convert(Sumtohyper(subs(k=n-k,subs(n=2*n,term7)),k),
  binomial);
```

$$res1 := \quad \textbf{(77)}$$

$$\frac{\sqrt{\pi}\,\Gamma(-2n-\lambda+1)\,(2^n)^2\,(-1)^n\,\mathrm{Hypergeom}\left([-n,n+\lambda],\left[\frac{1}{2}\right],x^2\right)\binom{2n+\lambda-1}{2n}}{\Gamma\left(-n+\frac{1}{2}\right)\Gamma(-n-\lambda+1)}$$

```
> convert(termtohyper(eval(subs(Hypergeom=1,res1)),n),binomial);
```

$$\binom{n+\lambda-1}{\lambda-1}(-1)^n \qquad \textbf{(78)}$$

Reversion yields

```
> res2:=convert(Sumtohyper(subs(k=n-k,subs(n=2*n+1,term7)),k),
  binomial);
```

$res2 :=$ **(79)**

$$-\frac{1}{\Gamma\left(-n-\frac{1}{2}\right)\Gamma(-n-\lambda)}\left(4\sqrt{\pi}\,\Gamma(-2n-\lambda)\,(2^n)^2\,x\,(-1)^n\,\mathrm{Hypergeom}\left([n+\lambda+1,-n],\left[\frac{3}{2}\right],x^2\right)\binom{2n+\lambda}{\lambda-1}\right)$$

```
> convert(termtohyper(eval(subs(Hypergeom=1,res2)),n),binomial);
```

$$2\lambda x\binom{n+\lambda}{\lambda}(-1)^n \qquad \textbf{(80)}$$

Check (19)=(21)

```
> term8:=subs(n=2*n,term7);
```

$term8 :=$ **(81)**

$$\frac{(2x)^{2n}\,\mathrm{pochhammer}(\lambda,2n)\,\mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}\left(-n+\frac{1}{2},k\right)\left(\frac{1}{x^2}\right)^k}{(2n)!\,\mathrm{pochhammer}(-2n-\lambda+1,k)\,k!}$$

```
> RE8:=RecurrenceNormalForm2(term8,k,p(n));
```

$$RE8 := \Big[(n+2)\,(2n+3)\,(2n+1+\lambda)\,p(n+2)-(2n+2+\lambda)\,(2\lambda^2x^2+8\lambda n x^2+8n^2x^2+8\lambda x^2+16n x^2-4n\lambda-4n^2+6x^2-5\lambda-8n-3)\,p(n+1)+(n+\lambda)\,(2n+2\lambda+1)\,(2n+3+\lambda)\,p(n)=0,1,\frac{\mathrm{pochhammer}(\lambda,2)\,(2\lambda x^2+2x^2-1)}{\lambda+1}\Big] \qquad \textbf{(82)}$$

```
> term9:=(-1)^n*binomial(n+lambda-1,n)*hyperterm([-n,n+lambda],
  [1/2],x^2,k);
```

$$term9 := \frac{(-1)^n\binom{n+\lambda-1}{n}\mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(n+\lambda,k)\,4^k\,(x^2)^k}{(2k)!} \qquad \textbf{(83)}$$

```
> RE9:=RecurrenceNormalForm2(term9,k,p(n));
```

$$RE9 := \left[(n+2)\,(2n+3)\,(2n+1+\lambda)\,p(n+2) - (2n+2+\lambda)\,\left(2\lambda^2 x^2 + 8\lambda n x^2 + 8n^2x^2 + 8\lambda x^2 + 16 n x^2 - 4n\lambda - 4n^2 + 6x^2 - 5\lambda - 8n - 3\right) p(n+1) + (n+\lambda)\,(2n+2\lambda+1)\,(2n+3+\lambda)\,p(n) = 0, 1, 2\lambda^2x^2 + 2\lambda x^2 - \lambda \right] \quad (84)$$

```
> normal(expand(RE8-RE9));
```

$$[0, 0, 0] \quad (85)$$

Check (20)=(22)

```
> term10:=subs(n=2*n+1,term7);
```

$$term10 := \frac{1}{(2n+1)!\,\mathrm{pochhammer}(-2n-\lambda, k)\,k!} \left((2x)^{2n+1}\,\mathrm{pochhammer}(\lambda, 2n+1)\,\mathrm{pochhammer}\left(-n-\frac{1}{2}, k\right)\mathrm{pochhammer}(-n, k)\left(\frac{1}{x^2}\right)^k \right) \quad (86)$$

```
> RE10:=RecurrenceNormalForm2(term10,k,p(n));
```

$$RE10 := \left[(5+2n)\,(n+2)\,(2n+2+\lambda)\,p(n+2) - (2n+3+\lambda)\,\left(2\lambda^2x^2 + 8\lambda n x^2 + 8n^2x^2 + 12\lambda x^2 + 24 n x^2 - 4n\lambda - 4n^2 + 16x^2 - 7\lambda - 12n - 8\right) p(n+1) + (2n+2\lambda+1)\,(n+\lambda+1)\,(2n+4+\lambda)\,p(n) = 0, 2\lambda x, \frac{2x\,\mathrm{pochhammer}(\lambda, 3)\,(2\lambda x^2 + 4x^2 - 3)}{3(\lambda+2)} \right] \quad (87)$$

```
> term11:=2*lambda*x*(-1)^n*binomial(n+lambda,n)*hyperterm([-n,n+
  lambda+1],[3/2],x^2,k);
```

$$term11 := \frac{2\lambda x\,(-1)^n \binom{n+\lambda}{n}\,\mathrm{pochhammer}(-n, k)\,\mathrm{pochhammer}(n+\lambda+1, k)\,(x^2)^k}{\mathrm{pochhammer}\left(\frac{3}{2}, k\right) k!} \quad (88)$$

```
> RE11:=RecurrenceNormalForm2(term11,k,p(n));
```

$$RE11 := \left[(5+2n)\,(n+2)\,(2n+2+\lambda)\,p(n+2) - (2n+3+\lambda)\,\left(2\lambda^2x^2 + 8\lambda n x^2 + 8n^2x^2 + 12\lambda x^2 + 24 n x^2 - 4n\lambda - 4n^2 + 16x^2 - 7\lambda - 12n - 8\right) p(n+1) + (2n+2\lambda+1)\,(n+\lambda+1)\,(2n+4+\lambda)\,p(n) = 0, 2\lambda x, -2\lambda^2 x - 2\lambda x + \frac{4}{3}\lambda^3x^3 + 4\lambda^2x^3 + \frac{8}{3}\lambda x^3 \right] \quad (89)$$

```
> normal(expand(RE10-RE11));
```

$$[0, 0, 0] \quad (90)$$

Legendre polynomials

```
> convert(Sumtohyper(subs({alpha=0,beta=0},term1),k),binomial);
```

$$\mathrm{Hypergeom}\left([n+1,-n],[1],\frac{1}{2}-\frac{x}{2}\right) \tag{91}$$

```
> convert(Sumtohyper(subs({alpha=0,beta=0},term2),k),binomial);
```

$$(-1)^n\,\mathrm{Hypergeom}\left([n+1,-n],[1],\frac{1}{2}+\frac{x}{2}\right) \tag{92}$$

```
> convert(Sumtohyper(subs({alpha=0,beta=0},term3),k),binomial);
```

$$\left(-\frac{1}{2}+\frac{x}{2}\right)^n\mathrm{Hypergeom}\left([-n,-n],[-2\,n],-\frac{2}{-1+x}\right)\binom{2\,n}{n} \tag{93}$$

```
> convert(Sumtohyper(subs({alpha=0,beta=0},term4),k),binomial);
```

$$\left(\frac{1}{2}+\frac{x}{2}\right)^n\mathrm{Hypergeom}\left([-n,-n],[1],\frac{-1+x}{1+x}\right) \tag{94}$$

```
> convert(Sumtohyper(subs({alpha=0,beta=0},term5),k),binomial);
```

$$\left(-\frac{1}{2}+\frac{x}{2}\right)^n\mathrm{Hypergeom}\left([-n,-n],[1],\frac{1+x}{-1+x}\right) \tag{95}$$

```
> convert(Sumtohyper(subs({alpha=0,beta=0},term6),k),binomial);
```

$$\left(\frac{1}{2}+\frac{x}{2}\right)^n\mathrm{Hypergeom}\left([-n,-n],[-2\,n],\frac{2}{1+x}\right)\binom{2\,n}{n} \tag{96}$$

```
> convert(Sumtohyper(subs({lambda=1/2},term7),k),binomial);
```

$$\frac{2^n\,x^n\,\mathrm{Hypergeom}\left(\left[-\frac{n}{2}+\frac{1}{2},-\frac{n}{2}\right],\left[-n+\frac{1}{2}\right],\frac{1}{x^2}\right)\binom{2\,n}{n}}{4^n} \tag{97}$$

```
> simplify(convert(Sumtohyper(subs({lambda=1/2},term8),k),
  binomial),power);
```

$$4^{-n}\,x^{2\,n}\,\mathrm{Hypergeom}\left(\left[-n,-n+\frac{1}{2}\right],\left[-2\,n+\frac{1}{2}\right],\frac{1}{x^2}\right)\binom{4\,n}{2\,n} \tag{98}$$

```
> convert(Sumtohyper(subs({lambda=1/2},term9),k),binomial);
```

$$(-1)^n\,\mathrm{Hypergeom}\left(\left[\frac{1}{2}+n,-n\right],\left[\frac{1}{2}\right],x^2\right)\binom{n-\frac{1}{2}}{-\frac{1}{2}} \tag{99}$$

```
> convert(termtohyper((-1)^n*binomial(-1/2 + n, -1/2),n),binomial);
```

$$\frac{(-1)^n\binom{2\,n}{n}}{4^n} \tag{100}$$

```
> simplify(convert(Sumtohyper(subs({lambda=1/2},term10),k),
  binomial),power);
```

$$\frac{x^{2\,n+1}\,4^{-n}\binom{4\,n+2}{2\,n+1}\mathrm{Hypergeom}\left(\left[-n,-n-\frac{1}{2}\right],\left[-2\,n-\frac{1}{2}\right],\frac{1}{x^2}\right)}{2} \tag{101}$$

```
> convert(Sumtohyper(subs({lambda=1/2},term11),k),binomial);
```

$$2\,x\,(-1)^n\,\mathrm{Hypergeom}\left(\left[-n, \frac{3}{2}+n\right], \left[\frac{3}{2}\right], x^2\right)(n+1)\binom{\frac{1}{2}+n}{-\frac{1}{2}} \tag{102}$$

```
> termtohyper(2*x*(-1)^n*(n + 1)*binomial(n + 1/2, -1/2),n);
```

$$\frac{x\,\mathrm{pochhammer}\left(\frac{3}{2}, n\right)(-1)^n}{n!} \tag{103}$$

```
> term12:=x^n*hyperterm([-n/2,-(n-1)/2],[1],1-1/x^2,k);
```

$$term12 := \frac{x^n\,\mathrm{pochhammer}\left(-\frac{n}{2}, k\right)\mathrm{pochhammer}\left(-\frac{n}{2}+\frac{1}{2}, k\right)\left(1-\frac{1}{x^2}\right)^k}{k!^2} \tag{104}$$

```
> RE1:=RecurrenceNormalForm2(subs({alpha=0,beta=0},term1),k,p(n));
```

$$RE1 := [(n+2)\,p(n+2) - x\,(2\,n+3)\,p(n+1) + (n+1)\,p(n) = 0, 1, x] \tag{105}$$

```
> RE12:=RecurrenceNormalForm2(term12,k,p(n));
```

$$RE12 := [(n+2)\,p(n+2) - x\,(2\,n+3)\,p(n+1) + (n+1)\,p(n) = 0, 1, x] \tag{106}$$

```
> term13:=subs(n=2*n,term12);
```

$$term13 := \frac{x^{2\,n}\,\mathrm{pochhammer}(-n, k)\,\mathrm{pochhammer}\left(-n+\frac{1}{2}, k\right)\left(1-\frac{1}{x^2}\right)^k}{k!^2} \tag{107}$$

```
> RE13:=RecurrenceNormalForm2(term13,k,p(n));
```

$$RE13 := \Big[2\,(4\,n+3)\,(2\,n+3)\,(n+2)\,p(n+2) - (4\,n+5)\,\big(16\,n^2\,x^2 + 40\,n\,x^2 - 8\,n^2 + 21\,x^2 - 20\,n - 11\big)\,p(n+1) + 2\,(n+1)\,(2\,n+1)\,(4\,n+7)\,p(n) = 0, 1, \frac{3\,x^2}{2} - \frac{1}{2}\Big] \tag{108}$$

Reversion yields

```
> simplify(convert(Sumtohyper(subs(k=n-k,subs(n=2*n,term12)),k),
  binomial),power) assuming n::integer;
```

$$(-1)^n\,(x^2-1)^n\binom{-\frac{1}{2}}{n}\mathrm{Hypergeom}\left([-n, -n], \left[\frac{1}{2}\right], \frac{x^2}{(-1+x)\,(1+x)}\right) \tag{109}$$

```
> term14:=binomial(-1/2,n)*(-1)^n*(x^2-1)^n*hyperterm([-n, -n],
  [1/2], x^2/((-1 + x)*(1 + x)),k);
```

$$term14 := \frac{\binom{-\frac{1}{2}}{n}(-1)^n\,(x^2-1)^n\,\mathrm{pochhammer}(-n, k)^2\,4^k\left(\frac{x^2}{(-1+x)\,(1+x)}\right)^k}{(2\,k)!} \tag{110}$$

```
> RE14:=RecurrenceNormalForm2(term14,k,p(n));
```

(111)

$$RE14 := \Big[2\,(4n+3)\,(2n+3)\,(n+2)\,p(n+2) - (4n+5)\,(16n^2x^2+40nx^2-8n^2 + 21x^2-20n-11)\,p(n+1) + 2\,(n+1)\,(2n+1)\,(4n+7)\,p(n)=0, 1, \frac{3x^2}{2} - \frac{1}{2}\Big] \quad \textbf{(111)}$$

```
> RE13-RE14;
```

$$[0, 0, 0] \quad \textbf{(112)}$$

```
> term15:=subs(n=2*n+1,term12);
```

$$term15 := \frac{x^{2n+1}\,\mathrm{pochhammer}\left(-n-\frac{1}{2}, k\right)\mathrm{pochhammer}(-n, k)\left(1-\frac{1}{x^2}\right)^k}{k!^2} \quad \textbf{(113)}$$

```
> RE15:=RecurrenceNormalForm2(term15,k,p(n));
```

$$RE15 := \Big[2\,(5+2n)\,(4n+5)\,(n+2)\,p(n+2) - (4n+7)\,(16n^2x^2+56nx^2-8n^2 + 45x^2-28n-23)\,p(n+1) + 2\,(2n+3)\,(n+1)\,(4n+9)\,p(n)=0, x, \frac{x\,(5x^2-3)}{2}\Big] \quad \textbf{(114)}$$

Reversion yields

```
> simplify(convert(Sumtohyper(subs(k=n-k,subs(n=2*n+1,term12)),k),
  binomial),power) assuming n::integer;
```

$$(-1)^n\,x\,(2n+1)\,(x^2-1)^n\binom{-\frac{1}{2}}{n}\mathrm{Hypergeom}\left([-n, -n], \left[\frac{3}{2}\right], \frac{x^2}{(-1+x)\,(1+x)}\right) \quad \textbf{(115)}$$

```
> term16:=binomial(-1/2,n)*(2*n+1)*(-1)^n*x*(x^2 - 1)^n*hyperterm(
  [-n, -n], [3/2], x^2/((-1 + x)*(1 + x)),k);
```

$$term16 := \frac{\binom{-\frac{1}{2}}{n}(2n+1)\,(-1)^n\,x\,(x^2-1)^n\,\mathrm{pochhammer}(-n, k)^2\left(\frac{x^2}{(-1+x)\,(1+x)}\right)^k}{\mathrm{pochhammer}\left(\frac{3}{2}, k\right)k!} \quad \textbf{(116)}$$

```
> RE16:=RecurrenceNormalForm2(term16,k,p(n));
```

$$RE16 := \Big[2\,(5+2n)\,(4n+5)\,(n+2)\,p(n+2) - (4n+7)\,(16n^2x^2+56nx^2-8n^2 + 45x^2-28n-23)\,p(n+1) + 2\,(2n+3)\,(n+1)\,(4n+9)\,p(n)=0, x, \frac{x\,(5x^2-3)}{2}\Big] \quad \textbf{(117)}$$

```
> RE15-RE16;
```

$$[0, 0, 0] \quad (118)$$

Chebyshev polynomials T_n

```
> convert(Sumtohyper(subs({alpha=-1/2,beta=-1/2},term1)/binomial(2*
  n,n)*4^n,k),binomial);
```

$$\mathrm{Hypergeom}\left([-n, n], \left[\frac{1}{2}\right], \frac{1}{2} - \frac{x}{2}\right) \quad (119)$$

```
> simplify(Sumtohyper(subs({alpha=-1/2,beta=-1/2},term2)/binomial
  (2*n,n)*4^n,k));
```

$$(-1)^n \mathrm{Hypergeom}\left([-n, n], \left[\frac{1}{2}\right], \frac{1}{2} + \frac{x}{2}\right) \quad (120)$$

```
> simplify(Sumtohyper(subs({alpha=-1/2,beta=-1/2},term3)/binomial
  (2*n,n)*4^n,k));
```

$$\frac{\mathrm{Hypergeom}\left(\left[-n, -n + \frac{1}{2}\right], [1 - 2n], -\frac{2}{-1 + x}\right) (2x - 2)^n}{2} \quad (121)$$

```
> simplify(Sumtohyper(subs({alpha=-1/2,beta=-1/2},term4)/binomial
  (2*n,n)*4^n,k));
```

$$\left(\frac{1}{2} + \frac{x}{2}\right)^n \mathrm{Hypergeom}\left(\left[-n, -n + \frac{1}{2}\right], \left[\frac{1}{2}\right], \frac{-1 + x}{1 + x}\right) \quad (122)$$

```
> simplify(Sumtohyper(subs({alpha=-1/2,beta=-1/2},term5)/binomial
  (2*n,n)*4^n,k));
```

$$\left(-\frac{1}{2} + \frac{x}{2}\right)^n \mathrm{Hypergeom}\left(\left[-n, -n + \frac{1}{2}\right], \left[\frac{1}{2}\right], \frac{1 + x}{-1 + x}\right) \quad (123)$$

```
> simplify(Sumtohyper(subs({alpha=-1/2,beta=-1/2},term6)/binomial
  (2*n,n)*4^n,k));
```

$$\frac{\mathrm{Hypergeom}\left(\left[-n, -n + \frac{1}{2}\right], [1 - 2n], \frac{2}{1 + x}\right) (2 + 2x)^n}{2} \quad (124)$$

Chebyshev polynomials U_n

```
> Sumtohyper(subs({alpha=1/2,beta=1/2},term1)*(n+1)/binomial(n+1/2,
  n),k);
```

$$(n + 1) \mathrm{Hypergeom}\left([n + 2, -n], \left[\frac{3}{2}\right], \frac{1}{2} - \frac{x}{2}\right) \quad (125)$$

```
> Sumtohyper(subs({alpha=1/2,beta=1/2},term2)*(n+1)/binomial(n+1/2,
  n),k);
```

$$(-1)^n (n + 1) \mathrm{Hypergeom}\left([n + 2, -n], \left[\frac{3}{2}\right], \frac{1}{2} + \frac{x}{2}\right) \quad (126)$$

```
> simplify(Sumtohyper(subs({alpha=1/2,beta=1/2},term3)*(n+1)
  /binomial(n+1/2,n),k));
```

$$\mathrm{Hypergeom}\left(\left[-n, -n - \frac{1}{2}\right], [-2n - 1], -\frac{2}{-1 + x}\right) (2x - 2)^n \quad (127)$$

```
> Sumtohyper(subs({alpha=1/2,beta=1/2},term4)*(n+1)/binomial(n+1/2,
  n),k);
```

(128)

$$\left(\frac{1}{2}+\frac{x}{2}\right)^n (n+1)\,\mathrm{Hypergeom}\left(\left[-n,-n-\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{2x-2}{2+2x}\right) \quad (128)$$

```
> Sumtohyper(subs({alpha=1/2,beta=1/2},term5)*(n+1)/binomial(n+1/2,
  n),k);
```

$$\left(-\frac{1}{2}+\frac{x}{2}\right)^n (n+1)\,\mathrm{Hypergeom}\left(\left[-n,-n-\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{2+2x}{2x-2}\right) \quad (129)$$

```
> simplify(Sumtohyper(subs({alpha=1/2,beta=1/2},term6)*(n+1)
  /binomial(n+1/2,n),k));
```

$$\mathrm{Hypergeom}\left(\left[-n,-n-\frac{1}{2}\right],[-2n-1],\frac{2}{1+x}\right)(2+2x)^n \quad (130)$$

```
> simplify(pochhammer(3/2,n)/binomial(n+1/2,n)/n!);
```

$$1 \quad (131)$$

```
> convert(Sumtohyper(subs({lambda=1},term7),k),binomial);
```

$$2^n x^n\,\mathrm{Hypergeom}\left(\left[-\frac{n}{2}+\frac{1}{2},-\frac{n}{2}\right],[-n],\frac{1}{x^2}\right) \quad (132)$$

```
> simplify(convert(Sumtohyper(subs({lambda=1},term8),k),binomial),
  power);
```

$$4^n x^{2n}\,\mathrm{Hypergeom}\left(\left[-n,-n+\frac{1}{2}\right],[-2n],\frac{1}{x^2}\right) \quad (133)$$

```
> convert(Sumtohyper(subs({lambda=1},term9),k),binomial);
```

$$(-1)^n\,\mathrm{Hypergeom}\left([n+1,-n],\left[\frac{1}{2}\right],x^2\right) \quad (134)$$

```
> simplify(convert(Sumtohyper(subs({lambda=1},term10),k),binomial),
  power);
```

$$2\,x^{2n+1} 4^n\,\mathrm{Hypergeom}\left(\left[-n,-n-\frac{1}{2}\right],[-2n-1],\frac{1}{x^2}\right) \quad (135)$$

```
> convert(Sumtohyper(subs({lambda=1},term11),k),binomial);
```

$$2x(-1)^n (n+1)\,\mathrm{Hypergeom}\left([n+2,-n],\left[\frac{3}{2}\right],x^2\right) \quad (136)$$

Laguerre polynomials

```
> term17:=pochhammer(alpha+1,n)/n!*hyperterm([-n],[alpha+1],x,k);
```

$$term17 := \frac{\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(-n,k)\,x^k}{n!\,\mathrm{pochhammer}(\alpha+1,k)\,k!} \quad (137)$$

```
> RE17:=RecurrenceNormalForm2(term17,k,p(n));
```

$$RE17 := [(n+2)\,p(n+2)-(\alpha+2n-x+3)\,p(n+1)+(\alpha+1+n)\,p(n)=0,1,\alpha+1-x] \quad (138)$$

```
> convert(Sumtohyper(term17,k),binomial);
```

$$\mathrm{Hypergeom}([-n],[\alpha+1],x)\binom{n+\alpha}{\alpha} \quad (139)$$

Let's reverse this series:

```
> convert(Sumtohyper(subs(k=n-k,term17),k),factorial);
```

$$\frac{(-1)^n x^n \operatorname{Hypergeom}\left([-n-\alpha, -n], [\,], -\frac{1}{x}\right)}{n!} \tag{140}$$

This is a new identity.

```
> term18:=(-1)^n*x^n*hyperterm([-n,-n-alpha],[],-1/x,k)/n!;
```

$$term18 := \frac{(-1)^n x^n \operatorname{pochhammer}(-n, k) \operatorname{pochhammer}(-n-\alpha, k) \left(-\frac{1}{x}\right)^k}{k!\, n!} \tag{141}$$

```
> RE18:=RecurrenceNormalForm2(term18,k,p(n));
```

$$RE18 := [(n+2)\, p(n+2) - (\alpha + 2n - x + 3)\, p(n+1) + (\alpha + 1 + n)\, p(n) = 0, 1, \alpha + 1 - x] \tag{142}$$

```
> RE17-RE18;
```

$$[0, 0, 0] \tag{143}$$

Bessel polynomials

```
> term19:=hyperterm([-n,n+alpha+1],[],-x/2,k);
```

$$term19 := \frac{\operatorname{pochhammer}(-n, k) \operatorname{pochhammer}(\alpha + 1 + n, k) \left(-\frac{x}{2}\right)^k}{k!} \tag{144}$$

```
> RE19:=RecurrenceNormalForm2(term19,k,p(n));
```

$$RE19 := \Big[-2\,(2+\alpha+2n)\,(2+\alpha+n)\,p(n+2) + (\alpha+2n+3)\,\big(\alpha^2 x + 4\,\alpha\, n\, x + 4\, n^2 x + 6\,\alpha\, x + 12\, n\, x + 2\,\alpha + 8\, x\big)\, p(n+1) + 2\,(4+\alpha+2n)\,(n+1)\, p(n) = 0, 1, 1 + \frac{1}{2}\,\alpha\, x + x\Big] \tag{145}$$

Let's reverse this series:

```
> convert(Sumtohyper(subs(k=n-k,term19),k),binomial);
```

$$\frac{(-1)^n\, \Gamma(\alpha + 1 + 2n) \left(-\frac{1}{2}\right)^n x^n \operatorname{Hypergeom}\left([-n], [-\alpha - 2n], \frac{2}{x}\right)}{\Gamma(\alpha + 1 + n)} \tag{146}$$

This is the second hypergeometric representation.

```
> term20:=pochhammer(n+alpha+1,n)*(x/2)^n*hyperterm([-n],[-2*n-
  alpha],2/x,k);
```

$$term20 := \frac{\operatorname{pochhammer}(\alpha + 1 + n, n) \left(\frac{x}{2}\right)^n \operatorname{pochhammer}(-n, k) \left(\frac{2}{x}\right)^k}{\operatorname{pochhammer}(-\alpha - 2n, k)\, k!} \tag{147}$$

```
> RE20:=RecurrenceNormalForm2(term20,k,p(n));
```

$$RE20 := \Big[-2\,(2+\alpha+2n)\,(2+\alpha+n)\,p(n+2) + (\alpha+2n+3)\,\big(\alpha^2 x + 4\,\alpha\, n\, x + 4\, n^2 x + 6\,\alpha\, x + 12\, n\, x + 2\,\alpha + 8\, x\big)\, p(n+1) + 2\,(4+\alpha+2n)\,(n+1)\, p(n) = 0, 1, \tag{148}$$

$$1+\frac{1}{2}\alpha x+x]$$

```
> RE19-RE20;
```

$$[0, 0, 0] \quad (149)$$

Hermite polynomials

```
> term21:=(2*x)^n*hyperterm([-n/2,-(n-1)/2],[],-1/x^2,k);
```

$$term21 := \frac{(2\,x)^n \operatorname{pochhammer}\left(-\frac{n}{2}, k\right) \operatorname{pochhammer}\left(-\frac{n}{2}+\frac{1}{2}, k\right) \left(-\frac{1}{x^2}\right)^k}{k!} \quad (150)$$

```
> RE21:=RecurrenceNormalForm2(term21,k,p(n));
```

$$RE21 := [p(n+2) - 2\,x\,p(n+1) + 2\,(n+1)\,p(n) = 0, 1, 2\,x] \quad (151)$$

Even and odd indices:

```
> term22:=subs(n=2*n,term21);
```

$$term22 := \frac{(2\,x)^{2\,n} \operatorname{pochhammer}(-n, k) \operatorname{pochhammer}\left(-n+\frac{1}{2}, k\right) \left(-\frac{1}{x^2}\right)^k}{k!} \quad (152)$$

```
> RE22:=RecurrenceNormalForm2(term22,k,p(n));
```

$$RE22 := \left[p(n+2) + 2\,\left(-2\,x^2+4\,n+5\right)\,p(n+1) + 8\,(n+1)\,(2\,n+1)\,p(n) = 0, 1, 4\,x^2 - 2\right] \quad (153)$$

```
> term24:=subs(n=2*n+1,term21);
```

$$term24 := \frac{(2\,x)^{2\,n+1} \operatorname{pochhammer}\left(-n-\frac{1}{2}, k\right) \operatorname{pochhammer}(-n, k) \left(-\frac{1}{x^2}\right)^k}{k!} \quad (154)$$

```
> RE24:=RecurrenceNormalForm2(term24,k,p(n));
```

$$RE24 := \left[p(n+2) + 2\,\left(-2\,x^2+4\,n+7\right)\,p(n+1) + 8\,(n+1)\,(2\,n+3)\,p(n) = 0, 2\,x, 8\,x^3 - 12\,x\right] \quad (155)$$

Let's reverse the series for even n:

```
> res23:=simplify(convert(Sumtohyper(subs(k=n-k,term22),k),
  binomial));
```

$$res23 := \frac{4^n\,(-1)^{2\,n}\,\sqrt{\pi}\,\operatorname{Hypergeom}\left([-n], \left[\frac{1}{2}\right], x^2\right)}{\Gamma\left(-n+\frac{1}{2}\right)} \quad (156)$$

```
> simplify(convert(termtohyper(eval(subs(Hypergeom=1,res23)),n),
  binomial)) assuming n::integer;;
```

$$n!\,(-1)^n \binom{2\,n}{n} \quad (157)$$

This is a second hypergeometric representation for even n:

```
> term23:=n!*(-1)^n*binomial(2*n, n)*hyperterm([-n],[1/2],x^2,k);
```

$$term23 := \frac{n!\,(-1)^n \binom{2\,n}{n} \operatorname{pochhammer}(-n, k)\,4^k\,\left(x^2\right)^k}{(2\,k)!} \quad (158)$$

```
> RE23:=RecurrenceNormalForm2(term23,k,p(n));
```

$$RE23 := [p(n+2) + 2(-2x^2 + 4n + 5)\,p(n+1) + 8(n+1)(2n+1)\,p(n) = 0, 1, 4x^2 - 2] \quad (159)$$

```
> RE22-RE23;
```

$$[0, 0, 0] \quad (160)$$

Let's reverse the series for odd n:

```
> res25:=simplify(convert(Sumtohyper(subs(k=n-k,term24),k),
  binomial));
```

$$res25 := -\frac{(-1)^{2n}\, 2^{2n+2}\, x\sqrt{\pi}\ \mathrm{Hypergeom}\left([-n], \left[\frac{3}{2}\right], x^2\right)}{\Gamma\left(-n - \frac{1}{2}\right)} \quad (161)$$

```
> termtohyper(eval(subs(Hypergeom=1,res25)),n);
```

$$2\, x\, \mathrm{pochhammer}\left(\frac{3}{2}, n\right) (-4)^n \quad (162)$$

This is a second hypergeometric representation for odd n:

```
> term25:=2*x*pochhammer(3/2, n)*(-4)^n*hyperterm([-n],[3/2],x^2,k)
  ;
```

$$term25 := \frac{2\, x\, \mathrm{pochhammer}\left(\frac{3}{2}, n\right) (-4)^n\, \mathrm{pochhammer}(-n, k)\, (x^2)^k}{\mathrm{pochhammer}\left(\frac{3}{2}, k\right) k!} \quad (163)$$

```
> RE25:=RecurrenceNormalForm2(term25,k,p(n));
```

$$RE25 := [p(n+2) + 2(-2x^2 + 4n + 7)\,p(n+1) + 8(n+1)(2n+3)\,p(n) = 0, 2x, 8x^3 - 12x] \quad (164)$$

```
> RE24-RE25;
```

$$[0, 0, 0] \quad (165)$$

Masjed-Jamei families

```
> chang:={alpha=-p+I*q/2, beta=-p-I*q/2, x=I*((a^2+c^2)*x+a*b+c*d)/
  (a*d-b*c)};
```

$$chang := \left\{\alpha = -p + \frac{\mathrm{I}\,q}{2}, \beta = -p - \frac{\mathrm{I}\,q}{2}, x = \frac{\mathrm{I}\left((a^2 + c^2)\,x + a\,b + c\,d\right)}{a\,d - b\,c}\right\} \quad (166)$$

```
> an:=2^n*(a*d-b*c)^n*n!/(-I)^n;
```

$$an := \frac{2^n (a\,d - b\,c)^n\, n!}{(-\mathrm{I})^n} \quad (167)$$

```
> convert(Sumtohyper(an*subs(chang, term1),k),binomial);
```

$$\frac{1}{(-1)^n\, \mathrm{I}^n\, \Gamma\left(-p + \frac{\mathrm{I}\,q}{2} + 1\right)} \left(2^n (a\,d - b\,c)^n\, \Gamma\left(n - p + \frac{\mathrm{I}\,q}{2} + 1\right) \mathrm{Hypergeom}\left([n - 2p + 1, -n], \left[-p + \frac{\mathrm{I}\,q}{2} + 1\right], \frac{-\mathrm{I}\,a^2 x - \mathrm{I}\,c^2 x - \mathrm{I}\,a\,b - \mathrm{I}\,c\,d + a\,d - b\,c}{2\,a\,d - 2\,b\,c}\right)\right) \quad (168)$$

```
> convert(Sumtohyper(an*subs(chang, term2),k),binomial);
```

$$\frac{1}{I^n \Gamma\left(-p-\frac{Iq}{2}+1\right)} \left(2^n (a d - b c)^n \Gamma\left(n-p-\frac{Iq}{2}+1\right) \mathrm{Hypergeom}\left([n-2p+1, -n], \left[-p-\frac{Iq}{2}+1\right], \frac{-I a^2 x - I c^2 x - I a b - I c d - a d + b c}{-2 a d + 2 b c}\right)\right) \quad (169)$$

```
> convert(Sumtohyper(an*subs(chang, term3),k),binomial);
```

$$\frac{1}{(-1)^n I^n \Gamma(n-2p+1)} \left(2^n (a d - b c)^n \Gamma(2n-2p+1) \left(\frac{I a^2 x + I c^2 x + I a b + I c d - a d + b c}{2 (a d - b c)}\right)^n \mathrm{Hypergeom}\left(\left[-n, -n+p-\frac{Iq}{2}\right], [-2n+2p], \frac{-2 a d + 2 b c}{I a^2 x + I c^2 x + I a b + I c d - a d + b c}\right)\right) \quad (170)$$

```
> convert(Sumtohyper(an*subs(chang, term4),k),binomial);
```

$$\frac{1}{(-1)^n I^n \Gamma\left(-p+\frac{Iq}{2}+1\right)} \left(2^n (a d - b c)^n \Gamma\left(n-p+\frac{Iq}{2}+1\right) \left(\frac{I a^2 x + I c^2 x + I a b + I c d + a d - b c}{2 (a d - b c)}\right)^n \mathrm{Hypergeom}\left(\left[-n+p+\frac{Iq}{2}, -n\right], \left[-p+\frac{Iq}{2}+1\right], \frac{2 I a^2 x + 2 I c^2 x + 2 I a b + 2 I c d - 2 a d + 2 b c}{2 I a^2 x + 2 I c^2 x + 2 I a b + 2 I c d + 2 a d - 2 b c}\right)\right) \quad (171)$$

```
> convert(Sumtohyper(an*subs(chang, term5),k),binomial);
```

$$\frac{1}{(-1)^n I^n \Gamma\left(-p-\frac{Iq}{2}+1\right)} \left(2^n (a d - b c)^n \Gamma\left(n-p-\frac{Iq}{2}+1\right) \left(\frac{I a^2 x + I c^2 x + I a b + I c d - a d + b c}{2 (a d - b c)}\right)^n \mathrm{Hypergeom}\left(\left[-n, -n+p-\frac{Iq}{2}\right], \left[-p-\frac{Iq}{2}+1\right], \frac{-2 I a^2 x - 2 I c^2 x - 2 I a b - 2 I c d - 2 a d + 2 b c}{-2 I a^2 x - 2 I c^2 x - 2 I a b - 2 I c d + 2 a d - 2 b c}\right)\right) \quad (172)$$

```
> convert(Sumtohyper(an*subs(chang, term6),k),binomial);
```

$$\frac{1}{(-1)^n I^n \Gamma(n-2p+1)} \left(2^n (a d - b c)^n \Gamma(2n-2p+1) \left(\frac{I a^2 x + I c^2 x + I a b + I c d + a d - b c}{2 (a d - b c)}\right)^n \mathrm{Hypergeom}\left(\left[-n+p+\frac{Iq}{2}, -n\right], [-2n+2p], \frac{2 a d - 2 b c}{I a^2 x + I c^2 x + I a b + I c d + a d - b c}\right)\right) \quad (173)$$

```
> TermMJ1:=2^n*(a*d-b*c)^n*GAMMA(-p+(1/2*I)*q+1+n)*hyperterm([n-2*
  p+1, -n], [-p+(1/2*I)*q+1], (-I*a^2*x-I*c^2*x-I*a*b-I*c*d+a*d-b*
```

```
c)/(2*a*d-2*b*c), k)/((-1)^n*I^n*GAMMA(-p+(1/2*I)*q+1));
```

$$TermMJ1 := \left(2^n\,(a\,d-b\,c)^n\,\Gamma\left(n-p+\frac{I\,q}{2}+1\right)\,\mathrm{pochhammer}(n-2\,p+1,\,k)\,\mathrm{pochhammer}(-n,k)\left(\frac{-I\,a^2\,x-I\,c^2\,x-I\,a\,b-I\,c\,d+a\,d-b\,c}{2\,a\,d-2\,b\,c}\right)^k\right)\Big/\left(\mathrm{pochhammer}\left(-p+\frac{I\,q}{2}+1,k\right)k!\,(-1)^n\,I^n\,\Gamma\left(-p+\frac{I\,q}{2}+1\right)\right) \quad \textbf{(174)}$$

```
> REMJ1:=simplify(RecurrenceNormalForm2(TermMJ1,k,S(n)));
```

$$REMJ1 := \left[4\left(\left(a^2\,x+a\,b+c\,(c\,x+d)\right)n^2-2\left(p-\frac{3}{2}\right)\left(a^2\,x+a\,b+c\,(c\,x+d)\right)n+x\,(-1+p)\,(p-2)\,a^2+\left(b\,p^2+\left(-3\,b-\frac{d\,q}{2}\right)p+2\,b\right)a+\frac{\left(2\,x\,(-1+p)\,(p-2)\,c+2\,d\,p^2+(b\,q-6\,d)\,p+4\,d\right)c}{2}\right)\left(n-p+\frac{3}{2}\right)S(n+1)+(n-p+1)\,(n-2\,p+2)\,S(n+2)-4\left(n^2+(-2\,p+2)\,n+p^2+\frac{q^2}{4}-2\,p+1\right)(a\,d-b\,c)^2\,(-p+2+n)\,S(n)\,(n+1)=0,\,1,\,2\,x\,(-1+p)\,a^2+(2\,b\,p-d\,q-2\,b)\,a+c\,(2\,x\,(-1+p)\,c+b\,q+2\,d\,p-2\,d)\right] \quad \textbf{(175)}$$

```
> TermMJ2:=2^n*(a*d-b*c)^n*GAMMA(n-p-(1/2*I)*q+1)*hyperterm([n-2*
  p+1, -n], [-p-(1/2*I)*q+1], (-I*a^2*x-I*c^2*x-I*a*b-I*c*d-a*d+b*
  c)/(-2*a*d+2*b*c), k)/(I^n*GAMMA(-p-(1/2*I)*q+1));
```

$$TermMJ2 := \left(2^n\,(a\,d-b\,c)^n\,\Gamma\left(n-p-\frac{I\,q}{2}+1\right)\,\mathrm{pochhammer}(n-2\,p+1,\,k)\,\mathrm{pochhammer}(-n,k)\left(\frac{-I\,a^2\,x-I\,c^2\,x-I\,a\,b-I\,c\,d-a\,d+b\,c}{-2\,a\,d+2\,b\,c}\right)^k\right)\Big/\left(\mathrm{pochhammer}\left(-p-\frac{I\,q}{2}+1,k\right)k!\,I^n\,\Gamma\left(-p-\frac{I\,q}{2}+1\right)\right) \quad \textbf{(176)}$$

```
> REMJ2:=simplify(RecurrenceNormalForm2(TermMJ2,k,S(n)));
```

$$REMJ2 := \left[4\left(\left(a^2\,x+a\,b+c\,(c\,x+d)\right)n^2-2\left(p-\frac{3}{2}\right)\left(a^2\,x+a\,b+c\,(c\,x+d)\right)n+x\,(-1+p)\,(p-2)\,a^2+\left(b\,p^2+\left(-3\,b-\frac{d\,q}{2}\right)p+2\,b\right)a+\frac{\left(2\,x\,(-1+p)\,(p-2)\,c+2\,d\,p^2+(b\,q-6\,d)\,p+4\,d\right)c}{2}\right)\left(n-p+\frac{3}{2}\right)S(n+1)+(n-p+1)\,(n-2\,p+2)\,S(n+2)-4\left(n^2+(-2\,p+2)\,n+p^2+\frac{q^2}{4}-2\,p\right. \quad \textbf{(177)}$$

$+ 1\Big) (a d - b c)^2 (-p + 2 + n) S(n) (n + 1) = 0, 1, 2 x (-1 + p) a^2 + (2 b p - d q$

$- 2 b) a + c (2 x (-1 + p) c + b q + 2 d p - 2 d)\Big]$

```
> REMJ1-REMJ2;
```

$$[0, 0, 0] \tag{178}$$

```
> TermMJ3:=2^n*(a*d-b*c)^n*GAMMA(2*n-2*p+1)*((1/2)*(I*a^2*x+I*c^2*
  x+I*a*b+I*c*d-a*d+b*c)/(a*d-b*c))^n*hyperterm([-n+p-(1/2*I)*q, -
  n], [-2*n+2*p], (-2*a*d+2*b*c)/(I*a^2*x+I*c^2*x+I*a*b+I*c*d-a*d+
  b*c), k)/((-1)^n*I^n*GAMMA(n-2*p+1));
```

$$TermMJ3 := \Bigg(2^n (a d - b c)^n \Gamma(2 n - 2 p + 1) \left(\frac{I a^2 x + I c^2 x + I a b + I c d - a d + b c}{2 (a d - b c)}\right)^n \text{pochhammer}\left(-n + p - \frac{I q}{2}, k\right) \text{pochhammer}(-n, k) \left(\frac{-2 a d + 2 b c}{I a^2 x + I c^2 x + I a b + I c d - a d + b c}\right)^k\Bigg) \Big/ \left(\text{pochhammer}(-2 n + 2 p, k) k! (-1)^n I^n \Gamma(n - 2 p + 1)\right) \tag{179}$$

```
> REMJ3:=RecurrenceNormalForm2(TermMJ3,k,S(n));
```

$$REMJ3 := \Big[(144 a b c d n^2 p - 240 a b c d n p^2 + 12 a b c d n q^2 - 20 a b c d p q^2 \tag{180}$$

$+ 64 a b c d n p + 16 a^2 b d q - 64 a^2 c d x - 16 a b^2 c q - 64 a b c^2 x + 16 a c d^2 q$

$- 16 b c^2 d q + 116 a^4 n p x^2 - 24 a^3 b n^3 x + 32 a^3 b p^3 x - 104 a^2 c^2 n^2 x^2$

$- 128 a^2 c^2 p^2 x^2 + 116 c^4 n p x^2 - 24 c^3 d n^3 x + 32 c^3 d p^3 x - 104 a^3 b n^2 x$

$- 128 a^3 b p^2 x + 40 a^2 b^2 n^2 p - 44 a^2 b^2 n p^2 - 144 a^2 c^2 n x^2 + 160 a^2 c^2 p x^2$

$- 104 c^3 d n^2 x - 128 c^3 d p^2 x + 40 c^2 d^2 n^2 p - 44 c^2 d^2 n p^2 - 144 a^3 b n x + 160 a^3 b p x$

$+ 16 a^3 d q x + 116 a^2 b^2 n p - 16 b c^3 q x - 144 c^3 d n x + 160 c^3 d p x + 116 c^2 d^2 n p$

$- 32 a b c d n^3 + 144 a b c d p^3 - 48 a b c d n^2 - 16 a b c d p^2 - 80 a b c d n + 32 a^2 d^2$

$+ 32 b^2 c^2 + 4 a^2 d^2 n^3 - 56 a^2 d^2 p^3 + 4 b^2 c^2 n^3 - 56 b^2 c^2 p^3 - 28 a^2 d^2 n^2 - 56 a^2 d^2 p^2$

$- 28 b^2 c^2 n^2 - 56 b^2 c^2 p^2 - 32 a^2 d^2 n + 80 a^2 d^2 p - 32 b^2 c^2 n + 80 b^2 c^2 p - 32 a^4 x^2$

$$- 32 c^4 x^2 - 32 a^2 b^2 - 32 c^2 d^2 - 32 a^2 d^2 n^2 p + 76 a^2 d^2 n p^2 - 6 a^2 d^2 n q^2$$

$$+ 10 a^2 d^2 p q^2 - 32 b^2 c^2 n^2 p + 76 b^2 c^2 n p^2 - 6 b^2 c^2 n q^2 + 10 b^2 c^2 p q^2 + 84 a^2 d^2 n p$$

$$+ 84 b^2 c^2 n p - 128 a b c d + 24 a^2 b c n p q x - 24 a c^2 d n p q x - 24 a^3 d n p q x$$

$$- 12 a^2 b c n^2 q x - 8 a^2 b c p^2 q x + 80 a^2 c d n^2 p x - 88 a^2 c d n p^2 x + 80 a b c^2 n^2 p x$$

$$- 88 a b c^2 n p^2 x + 12 a c^2 d n^2 q x + 8 a c^2 d p^2 q x + 24 b c^3 n p q x - 32 a^2 b c n q x$$

$$+ 44 a^2 b c p q x - 24 a^2 b d n p q + 232 a^2 c d n p x + 24 a b^2 c n p q + 232 a b c^2 n p x$$

$$+ 32 a c^2 d n q x - 44 a c^2 d p q x - 24 a c d^2 n p q + 24 b c^2 d n p q + 40 a^4 n^2 p x^2$$

$$- 44 a^4 n p^2 x^2 - 24 a^2 c^2 n^3 x^2 + 32 a^2 c^2 p^3 x^2 + 40 c^4 n^2 p x^2 - 44 c^4 n p^2 x^2$$

$$+ 12 a c d^2 n^2 q + 8 a c d^2 p^2 q - 32 b c^3 n q x + 44 b c^3 p q x - 12 b c^2 d n^2 q$$

$$- 8 b c^2 d p^2 q + 232 c^3 d n p x - 16 a^2 b c q x + 32 a^2 b d n q - 44 a^2 b d p q$$

$$- 144 a^2 c d n x + 160 a^2 c d p x - 32 a b^2 c n q + 44 a b^2 c p q - 144 a b c^2 n x$$

$$+ 160 a b c^2 p x + 16 a c^2 d q x + 32 a c d^2 n q - 44 a c d^2 p q - 32 b c^2 d n q$$

$$+ 44 b c^2 d p q - 8 b c^3 p^2 q x + 80 c^3 d n^2 p x - 88 c^3 d n p^2 x + 232 a^3 b n p x$$

$$+ 32 a^3 d n q x - 44 a^3 d p q x + 12 a^2 b d n^2 q + 8 a^2 b d p^2 q - 104 a^2 c d n^2 x$$

$$- 128 a^2 c d p^2 x - 12 a b^2 c n^2 q - 8 a b^2 c p^2 q - 104 a b c^2 n^2 x - 128 a b c^2 p^2 x$$

$$+ 80 a^3 b n^2 p x - 88 a^3 b n p^2 x + 12 a^3 d n^2 q x + 8 a^3 d p^2 q x + 232 a^2 c^2 n p x^2$$

$$- 24 a^2 c d n^3 x + 32 a^2 c d p^3 x - 24 a b c^2 n^3 x + 32 a b c^2 p^3 x - 12 b c^3 n^2 q x$$

$$+ 80 a^2 c^2 n^2 p x^2 - 88 a^2 c^2 n p^2 x^2 - 12 a^4 n^3 x^2 + 16 a^4 p^3 x^2 - 12 c^4 n^3 x^2 + 16 c^4 p^3 x^2$$

$$- 52 a^4 n^2 x^2 - 64 a^4 p^2 x^2 - 52 c^4 n^2 x^2 - 64 c^4 p^2 x^2 - 72 a^4 n x^2 + 80 a^4 p x^2$$

$$- 12 a^2 b^2 n^3 + 16 a^2 b^2 p^3 - 72 c^4 n x^2 + 80 c^4 p x^2 - 12 c^2 d^2 n^3 + 16 c^2 d^2 p^3$$

$$- 52 a^2 b^2 n^2 - 64 a^2 b^2 p^2 - 64 a^2 c^2 x^2 - 52 c^2 d^2 n^2 - 64 c^2 d^2 p^2 - 64 a^3 b x - 72 a^2 b^2 n$$

$$+ 80 a^2 b^2 p - 64 c^3 d x - 72 c^2 d^2 n + 80 c^2 d^2 p + 104 \mathrm{I} a^2 b d n p^2 - 2 \mathrm{I} a^2 b d n q^2$$

$$+ 38 \mathrm{I} a^2 d^2 n p q + 72 \mathrm{I} a b^2 c n^2 p - 104 \mathrm{I} a b^2 c n p^2 + 2 \mathrm{I} a b^2 c n q^2 + 24 \mathrm{I} a c^2 d n^2 x$$

$$- 72 \mathrm{I} a c d^2 n^2 p + 104 \mathrm{I} a c d^2 n p^2 - 2 \mathrm{I} a c d^2 n q^2 + 38 \mathrm{I} b^2 c^2 n p q + 112 \mathrm{I} b c^3 n p x$$

$$+ 72 \mathrm{I} b c^2 d n^2 p - 104 \mathrm{I} b c^2 d n p^2 + 2 \mathrm{I} b c^2 d n q^2 + 12 \mathrm{I} c^3 d n q x - 2 \mathrm{I} c^2 d^2 n p q$$

$$+ 56 \mathrm{I} a^2 b c n x - 112 \mathrm{I} a^2 b d n p + 112 \mathrm{I} a b^2 c n p - 56 \mathrm{I} a c^2 d n x - 112 \mathrm{I} a c d^2 n p$$

$$+ 112 \mathrm{I} b c^2 d n p + 48 \mathrm{I} a^2 b c p^3 x - 8 \mathrm{I} a^2 c^2 p q x^2 - 48 \mathrm{I} a c^2 d p^3 x - 8 \mathrm{I} a^3 b p q x$$

$$- 104 \mathrm{I} a^2 b c p^2 x + 4 \mathrm{I} a^2 b c q^2 x - 2 \mathrm{I} a b c d q^3 + 104 \mathrm{I} a c^2 d p^2 x - 4 \mathrm{I} a c^2 d q^2 x$$

$$- 8 \mathrm{I} c^3 d p q x - 24 \mathrm{I} a^2 b c p x + 8 \mathrm{I} a^2 c d q x + 8 \mathrm{I} a b c^2 q x + 24 \mathrm{I} a c^2 d p x$$

$$- 40 \mathrm{I} a b c d q - 2 \mathrm{I} a^4 n p q x^2 + 4 \mathrm{I} a^3 b n^2 q x - 72 \mathrm{I} a^3 d n^2 p x + 104 \mathrm{I} a^3 d n p^2 x$$

$$- 2 \mathrm{I} a^3 d n q^2 x - 16 \mathrm{I} a^2 b c n^3 x + 12 \mathrm{I} a^2 c^2 n q x^2 + 16 \mathrm{I} a c^2 d n^3 x + 72 \mathrm{I} b c^3 n^2 p x$$

$$- 104 \mathrm{I} b c^3 n p^2 x + 2 \mathrm{I} b c^3 n q^2 x + 4 \mathrm{I} c^3 d n^2 q x + 12 \mathrm{I} a^3 b n q x - 112 \mathrm{I} a^3 d n p x$$

$$- 2 \mathrm{I} a^2 b^2 n p q - 24 \mathrm{I} a^2 b c n^2 x - 72 \mathrm{I} a^2 b d n^2 p + \mathrm{I} a^2 d^2 q^3 + \mathrm{I} b^2 c^2 q^3 + 4 \mathrm{I} a^4 q x^2$$

$$+ 4 \mathrm{I} c^4 q x^2 - 80 \mathrm{I} a^3 d x + 4 \mathrm{I} q b^2 a^2 + 24 \mathrm{I} q d^2 a^2 + 24 \mathrm{I} b^2 c^2 q + 80 \mathrm{I} b c^3 x + 4 \mathrm{I} c^2 d^2 q$$

$$- 80 \mathrm{I} d b a^2 + 80 \mathrm{I} a b^2 c - 80 \mathrm{I} a c d^2 + 80 \mathrm{I} b c^2 d + 6 \mathrm{I} c^2 d^2 n q - 56 \mathrm{I} a^2 b d n$$

$$+ 56 \mathrm{I} a b^2 c n - 56 \mathrm{I} a c d^2 n + 56 \mathrm{I} b c^2 d n + 16 \mathrm{I} a^3 d n^3 x - 16 \mathrm{I} b c^3 n^3 x + 6 \mathrm{I} c^4 n q x^2$$

$$+ 24 I a^3 d n^2 x + 2 I a^2 b^2 n^2 q + 16 I a^2 b d n^3 - 10 I a^2 d^2 n^2 q - 16 I a b^2 c n^3$$

$$+ 16 I a c d^2 n^3 - 10 I b^2 c^2 n^2 q - 24 I b c^3 n^2 x - 16 I b c^2 d n^3 + 2 I c^2 d^2 n^2 q$$

$$- 56 I a^3 d n x + 6 I a^2 b^2 n q + 24 I a^2 b d n^2 + 14 I a^2 d^2 n q - 24 I a b^2 c n^2$$

$$+ 24 I a c d^2 n^2 + 14 I b^2 c^2 n q + 56 I b c^3 n x + 2 I a^4 n^2 q x^2 + 2 I c^4 n^2 q x^2 + 6 I a^4 n q x^2$$

$$- 48 I p^3 d b a^2 + 8 I a^2 c^2 q x^2 - 36 I a^2 d^2 p^2 q + 48 I a b^2 c p^3 - 48 I a c d^2 p^3$$

$$- 36 I b^2 c^2 p^2 q - 104 I b c^3 p^2 x + 4 I b c^3 q^2 x + 48 I b c^2 d p^3 + 8 I a^3 b q x + 24 I a^3 d p x$$

$$- 4 I q p b^2 a^2 + 104 I p^2 d b a^2 - 4 I a^2 b d q^2 - 16 I q p d^2 a^2 - 104 I a b^2 c p^2$$

$$+ 4 I a b^2 c q^2 + 104 I a c d^2 p^2 - 4 I a c d^2 q^2 - 16 I b^2 c^2 p q - 24 I b c^3 p x$$

$$- 104 I b c^2 d p^2 + 4 I b c^2 d q^2 + 8 I c^3 d q x - 4 I c^2 d^2 p q + 80 I a^2 b c x + 24 I p d b a^2$$

$$- 24 I a b^2 c p - 80 I a c^2 d x + 24 I a c d^2 p - 24 I b c^2 d p - 4 I a^4 p q x^2 - 48 I a^3 d p^3 x$$

$$+ 48 I b c^3 p^3 x - 4 I c^4 p q x^2 + 104 I a^3 d p^2 x - 4 I a^3 d q^2 x - 24 I b c^2 d n^2$$

$$+ 4 I a^2 c^2 n^2 q x^2 - 2 I c^4 n p q x^2 - 4 I a^2 c d n p q x - 4 I a b c^2 n p q x - 80 I a b c d n p q$$

$$- 8 I a^2 c d p q x - 8 I a b c^2 p q x + 72 I a b c d p^2 q + 24 I a b c d p q + 12 I a b c^2 n q x$$

$$+ 24 I a b c d n^2 q - 112 I a c^2 d n p x - 16 I a b c d n q - 4 I a^3 b n p q x$$

$$+ 72 I a^2 b c n^2 p x - 104 I a^2 b c n p^2 x + 2 I a^2 b c n q^2 x + 4 I a^2 c d n^2 q x$$

$$+ 4 I a b c^2 n^2 q x - 72 I a c^2 d n^2 p x + 104 I a c^2 d n p^2 x - 2 I a c^2 d n q^2 x$$

$$- 4 I c^3 d n p q x + 112 I a^2 b c n p x + 12 I a^2 c d n q x - 4 I a^2 c^2 n p q x^2) (I q - 2 n + 2 p$$

$$+ 2) (I q - 2 n + 2 p) (I q + 2 n + 4 - 2 p) (n - p + 1) (n - 2 p + 2) (a x + b$$

$$+ \mathrm{I}\,c\,x + \mathrm{I}\,d)^2\,(\mathrm{I}\,c - a)^2\,(\mathrm{I}\,q - 4\,n + 6\,p - 6)^5\,S(n+2) + \big(144\,a\,b\,c\,d\,n^2\,p$$

$$- 240\,a\,b\,c\,d\,n\,p^2 + 12\,a\,b\,c\,d\,n\,q^2 - 20\,a\,b\,c\,d\,p\,q^2 + 64\,a\,b\,c\,d\,n\,p + 16\,a^2\,b\,d\,q$$

$$- 64\,a^2\,c\,d\,x - 16\,a\,b^2\,c\,q - 64\,a\,b\,c^2\,x + 16\,a\,c\,d^2\,q - 16\,b\,c^2\,d\,q + 116\,a^4\,n\,p\,x^2$$

$$- 24\,a^3\,b\,n^3\,x + 32\,a^3\,b\,p^3\,x - 104\,a^2\,c^2\,n^2\,x^2 - 128\,a^2\,c^2\,p^2\,x^2 + 116\,c^4\,n\,p\,x^2$$

$$- 24\,c^3\,d\,n^3\,x + 32\,c^3\,d\,p^3\,x - 104\,a^3\,b\,n^2\,x - 128\,a^3\,b\,p^2\,x + 40\,a^2\,b^2\,n^2\,p - 44\,a^2\,b^2\,n\,p^2$$

$$- 144\,a^2\,c^2\,n\,x^2 + 160\,a^2\,c^2\,p\,x^2 - 104\,c^3\,d\,n^2\,x - 128\,c^3\,d\,p^2\,x + 40\,c^2\,d^2\,n^2\,p$$

$$- 44\,c^2\,d^2\,n\,p^2 - 144\,a^3\,b\,n\,x + 160\,a^3\,b\,p\,x + 16\,a^3\,d\,q\,x + 116\,a^2\,b^2\,n\,p - 16\,b\,c^3\,q\,x$$

$$- 144\,c^3\,d\,n\,x + 160\,c^3\,d\,p\,x + 116\,c^2\,d^2\,n\,p - 32\,a\,b\,c\,d\,n^3 + 144\,a\,b\,c\,d\,p^3$$

$$- 48\,a\,b\,c\,d\,n^2 - 16\,a\,b\,c\,d\,p^2 - 80\,a\,b\,c\,d\,n + 32\,a^2\,d^2 + 32\,b^2\,c^2 + 4\,a^2\,d^2\,n^3$$

$$- 56\,a^2\,d^2\,p^3 + 4\,b^2\,c^2\,n^3 - 56\,b^2\,c^2\,p^3 - 28\,a^2\,d^2\,n^2 - 56\,a^2\,d^2\,p^2 - 28\,b^2\,c^2\,n^2$$

$$- 56\,b^2\,c^2\,p^2 - 32\,a^2\,d^2\,n + 80\,a^2\,d^2\,p - 32\,b^2\,c^2\,n + 80\,b^2\,c^2\,p - 32\,a^4\,x^2 - 32\,c^4\,x^2$$

$$- 32\,a^2\,b^2 - 32\,c^2\,d^2 - 32\,a^2\,d^2\,n^2\,p + 76\,a^2\,d^2\,n\,p^2 - 6\,a^2\,d^2\,n\,q^2 + 10\,a^2\,d^2\,p\,q^2$$

$$- 32\,b^2\,c^2\,n^2\,p + 76\,b^2\,c^2\,n\,p^2 - 6\,b^2\,c^2\,n\,q^2 + 10\,b^2\,c^2\,p\,q^2 + 84\,a^2\,d^2\,n\,p + 84\,b^2\,c^2\,n\,p$$

$$- 128\,a\,b\,c\,d + 24\,a^2\,b\,c\,n\,p\,q\,x - 24\,a\,c^2\,d\,n\,p\,q\,x - 24\,a^3\,d\,n\,p\,q\,x - 12\,a^2\,b\,c\,n^2\,q\,x$$

$$- 8\,a^2\,b\,c\,p^2\,q\,x + 80\,a^2\,c\,d\,n^2\,p\,x - 88\,a^2\,c\,d\,n\,p^2\,x + 80\,a\,b\,c^2\,n^2\,p\,x - 88\,a\,b\,c^2\,n\,p^2\,x$$

$$+ 12\,a\,c^2\,d\,n^2\,q\,x + 8\,a\,c^2\,d\,p^2\,q\,x + 24\,b\,c^3\,n\,p\,q\,x - 32\,a^2\,b\,c\,n\,q\,x + 44\,a^2\,b\,c\,p\,q\,x$$

$$- 24\,a^2\,b\,d\,n\,p\,q + 232\,a^2\,c\,d\,n\,p\,x + 24\,a\,b^2\,c\,n\,p\,q + 232\,a\,b\,c^2\,n\,p\,x + 32\,a\,c^2\,d\,n\,q\,x$$

$$- 44\,a\,c^2\,d\,p\,q\,x - 24\,a\,c\,d^2\,n\,p\,q + 24\,b\,c^2\,d\,n\,p\,q + 40\,a^4\,n^2\,p\,x^2 - 44\,a^4\,n\,p^2\,x^2$$

$$- 24\, a^2\, c^2\, n^3\, x^2 + 32\, a^2\, c^2\, p^3\, x^2 + 40\, c^4\, n^2\, p\, x^2 - 44\, c^4\, n\, p^2\, x^2 + 12\, a\, c\, d^2\, n^2\, q$$

$$+ 8\, a\, c\, d^2\, p^2\, q - 32\, b\, c^3\, n\, q\, x + 44\, b\, c^3\, p\, q\, x - 12\, b\, c^2\, d\, n^2\, q - 8\, b\, c^2\, d\, p^2\, q$$

$$+ 232\, c^3\, d\, n\, p\, x - 16\, a^2\, b\, c\, q\, x + 32\, a^2\, b\, d\, n\, q - 44\, a^2\, b\, d\, p\, q - 144\, a^2\, c\, d\, n\, x$$

$$+ 160\, a^2\, c\, d\, p\, x - 32\, a\, b^2\, c\, n\, q + 44\, a\, b^2\, c\, p\, q - 144\, a\, b\, c^2\, n\, x + 160\, a\, b\, c^2\, p\, x$$

$$+ 16\, a\, c^2\, d\, q\, x + 32\, a\, c\, d^2\, n\, q - 44\, a\, c\, d^2\, p\, q - 32\, b\, c^2\, d\, n\, q + 44\, b\, c^2\, d\, p\, q - 8\, b\, c^3\, p^2\, q\, x$$

$$+ 80\, c^3\, d\, n^2\, p\, x - 88\, c^3\, d\, n\, p^2\, x + 232\, a^3\, b\, n\, p\, x + 32\, a^3\, d\, n\, q\, x - 44\, a^3\, d\, p\, q\, x$$

$$+ 12\, a^2\, b\, d\, n^2\, q + 8\, a^2\, b\, d\, p^2\, q - 104\, a^2\, c\, d\, n^2\, x - 128\, a^2\, c\, d\, p^2\, x - 12\, a\, b^2\, c\, n^2\, q$$

$$- 8\, a\, b^2\, c\, p^2\, q - 104\, a\, b\, c^2\, n^2\, x - 128\, a\, b\, c^2\, p^2\, x + 80\, a^3\, b\, n^2\, p\, x - 88\, a^3\, b\, n\, p^2\, x$$

$$+ 12\, a^3\, d\, n^2\, q\, x + 8\, a^3\, d\, p^2\, q\, x + 232\, a^2\, c^2\, n\, p\, x^2 - 24\, a^2\, c\, d\, n^3\, x + 32\, a^2\, c\, d\, p^3\, x$$

$$- 24\, a\, b\, c^2\, n^3\, x + 32\, a\, b\, c^2\, p^3\, x - 12\, b\, c^3\, n^2\, q\, x + 80\, a^2\, c^2\, n^2\, p\, x^2 - 88\, a^2\, c^2\, n\, p^2\, x^2$$

$$- 12\, a^4\, n^3\, x^2 + 16\, a^4\, p^3\, x^2 - 12\, c^4\, n^3\, x^2 + 16\, c^4\, p^3\, x^2 - 52\, a^4\, n^2\, x^2 - 64\, a^4\, p^2\, x^2$$

$$- 52\, c^4\, n^2\, x^2 - 64\, c^4\, p^2\, x^2 - 72\, a^4\, n\, x^2 + 80\, a^4\, p\, x^2 - 12\, a^2\, b^2\, n^3 + 16\, a^2\, b^2\, p^3 - 72\, c^4\, n\, x^2$$

$$+ 80\, c^4\, p\, x^2 - 12\, c^2\, d^2\, n^3 + 16\, c^2\, d^2\, p^3 - 52\, a^2\, b^2\, n^2 - 64\, a^2\, b^2\, p^2 - 64\, a^2\, c^2\, x^2$$

$$- 52\, c^2\, d^2\, n^2 - 64\, c^2\, d^2\, p^2 - 64\, a^3\, b\, x - 72\, a^2\, b^2\, n + 80\, a^2\, b^2\, p - 64\, c^3\, d\, x - 72\, c^2\, d^2\, n$$

$$+ 80\, c^2\, d^2\, p + 104\, \mathrm{I}\, a^2\, b\, d\, n\, p^2 - 2\, \mathrm{I}\, a^2\, b\, d\, n\, q^2 + 38\, \mathrm{I}\, a^2\, d^2\, n\, p\, q + 72\, \mathrm{I}\, a\, b^2\, c\, n^2\, p$$

$$- 104\, \mathrm{I}\, a\, b^2\, c\, n\, p^2 + 2\, \mathrm{I}\, a\, b^2\, c\, n\, q^2 + 24\, \mathrm{I}\, a\, c^2\, d\, n^2\, x - 72\, \mathrm{I}\, a\, c\, d^2\, n^2\, p + 104\, \mathrm{I}\, a\, c\, d^2\, n\, p^2$$

$$- 2\, \mathrm{I}\, a\, c\, d^2\, n\, q^2 + 38\, \mathrm{I}\, b^2\, c^2\, n\, p\, q + 112\, \mathrm{I}\, b\, c^3\, n\, p\, x + 72\, \mathrm{I}\, b\, c^2\, d\, n^2\, p - 104\, \mathrm{I}\, b\, c^2\, d\, n\, p^2$$

$$+ 2\, \mathrm{I}\, b\, c^2\, d\, n\, q^2 + 12\, \mathrm{I}\, c^3\, d\, n\, q\, x - 2\, \mathrm{I}\, c^2\, d^2\, n\, p\, q + 56\, \mathrm{I}\, a^2\, b\, c\, n\, x - 112\, \mathrm{I}\, a^2\, b\, d\, n\, p$$

$$+ 112\, I\, a\, b^2\, c\, n\, p - 56\, I\, a\, c^2\, d\, n\, x - 112\, I\, a\, c\, d^2\, n\, p + 112\, I\, b\, c^2\, d\, n\, p + 48\, I\, a^2\, b\, c\, p^3\, x$$

$$- 8\, I\, a^2\, c^2\, p\, q\, x^2 - 48\, I\, a\, c^2\, d\, p^3\, x - 8\, I\, a^3\, b\, p\, q\, x - 104\, I\, a^2\, b\, c\, p^2\, x + 4\, I\, a^2\, b\, c\, q^2\, x$$

$$- 2\, I\, a\, b\, c\, d\, q^3 + 104\, I\, a\, c^2\, d\, p^2\, x - 4\, I\, a\, c^2\, d\, q^2\, x - 8\, I\, c^3\, d\, p\, q\, x - 24\, I\, a^2\, b\, c\, p\, x$$

$$+ 8\, I\, a^2\, c\, d\, q\, x + 8\, I\, a\, b\, c^2\, q\, x + 24\, I\, a\, c^2\, d\, p\, x - 40\, I\, a\, b\, c\, d\, q - 2\, I\, a^4\, n\, p\, q\, x^2$$

$$+ 4\, I\, a^3\, b\, n^2\, q\, x - 72\, I\, a^3\, d\, n^2\, p\, x + 104\, I\, a^3\, d\, n\, p^2\, x - 2\, I\, a^3\, d\, n\, q^2\, x - 16\, I\, a^2\, b\, c\, n^3\, x$$

$$+ 12\, I\, a^2\, c^2\, n\, q\, x^2 + 16\, I\, a\, c^2\, d\, n^3\, x + 72\, I\, b\, c^3\, n^2\, p\, x - 104\, I\, b\, c^3\, n\, p^2\, x + 2\, I\, b\, c^3\, n\, q^2\, x$$

$$+ 4\, I\, c^3\, d\, n^2\, q\, x + 12\, I\, a^3\, b\, n\, q\, x - 112\, I\, a^3\, d\, n\, p\, x - 2\, I\, a^2\, b^2\, n\, p\, q - 24\, I\, a^2\, b\, c\, n^2\, x$$

$$- 72\, I\, a^2\, b\, d\, n^2\, p + I\, a^2\, d^2\, q^3 + I\, b^2\, c^2\, q^3 + 4\, I\, a^4\, q\, x^2 + 4\, I\, c^4\, q\, x^2 - 80\, I\, a^3\, d\, x + 4\, I\, q\, b^2\, a^2$$

$$+ 24\, I\, q\, d^2\, a^2 + 24\, I\, b^2\, c^2\, q + 80\, I\, b\, c^3\, x + 4\, I\, c^2\, d^2\, q - 80\, I\, d\, b\, a^2 + 80\, I\, a\, b^2\, c$$

$$- 80\, I\, a\, c\, d^2 + 80\, I\, b\, c^2\, d + 6\, I\, c^2\, d^2\, n\, q - 56\, I\, a^2\, b\, d\, n + 56\, I\, a\, b^2\, c\, n - 56\, I\, a\, c\, d^2\, n$$

$$+ 56\, I\, b\, c^2\, d\, n + 16\, I\, a^3\, d\, n^3\, x - 16\, I\, b\, c^3\, n^3\, x + 6\, I\, c^4\, n\, q\, x^2 + 24\, I\, a^3\, d\, n^2\, x + 2\, I\, a^2\, b^2\, n^2\, q$$

$$+ 16\, I\, a^2\, b\, d\, n^3 - 10\, I\, a^2\, d^2\, n^2\, q - 16\, I\, a\, b^2\, c\, n^3 + 16\, I\, a\, c\, d^2\, n^3 - 10\, I\, b^2\, c^2\, n^2\, q$$

$$- 24\, I\, b\, c^3\, n^2\, x - 16\, I\, b\, c^2\, d\, n^3 + 2\, I\, c^2\, d^2\, n^2\, q - 56\, I\, a^3\, d\, n\, x + 6\, I\, a^2\, b^2\, n\, q + 24\, I\, a^2\, b\, d\, n^2$$

$$+ 14\, I\, a^2\, d^2\, n\, q - 24\, I\, a\, b^2\, c\, n^2 + 24\, I\, a\, c\, d^2\, n^2 + 14\, I\, b^2\, c^2\, n\, q + 56\, I\, b\, c^3\, n\, x$$

$$+ 2\, I\, a^4\, n^2\, q\, x^2 + 2\, I\, c^4\, n^2\, q\, x^2 + 6\, I\, a^4\, n\, q\, x^2 - 48\, I\, p^3\, d\, b\, a^2 + 8\, I\, a^2\, c^2\, q\, x^2$$

$$- 36\, I\, a^2\, d^2\, p^2\, q + 48\, I\, a\, b^2\, c\, p^3 - 48\, I\, a\, c\, d^2\, p^3 - 36\, I\, b^2\, c^2\, p^2\, q - 104\, I\, b\, c^3\, p^2\, x$$

$$+ 4\, I\, b\, c^3\, q^2\, x + 48\, I\, b\, c^2\, d\, p^3 + 8\, I\, a^3\, b\, q\, x + 24\, I\, a^3\, d\, p\, x - 4\, I\, q\, p\, b^2\, a^2 + 104\, I\, p^2\, d\, b\, a^2$$

$$- 4\, I\, a^2\, b\, d\, q^2 - 16\, I\, q\, p\, d^2\, a^2 - 104\, I\, a\, b^2\, c\, p^2 + 4\, I\, a\, b^2\, c\, q^2 + 104\, I\, a\, c\, d^2\, p^2$$

$$- 4 I a c d^2 q^2 - 16 I b^2 c^2 p q - 24 I b c^3 p x - 104 I b c^2 d p^2 + 4 I b c^2 d q^2 + 8 I c^3 d q x$$

$$- 4 I c^2 d^2 p q + 80 I a^2 b c x + 24 I p d b a^2 - 24 I a b^2 c p - 80 I a c^2 d x + 24 I a c d^2 p$$

$$- 24 I b c^2 d p - 4 I a^4 p q x^2 - 48 I a^3 d p^3 x + 48 I b c^3 p^3 x - 4 I c^4 p q x^2 + 104 I a^3 d p^2 x$$

$$- 4 I a^3 d q^2 x - 24 I b c^2 d n^2 + 4 I a^2 c^2 n^2 q x^2 - 2 I c^4 n p q x^2 - 4 I a^2 c d n p q x$$

$$- 4 I a b c^2 n p q x - 80 I a b c d n p q - 8 I a^2 c d p q x - 8 I a b c^2 p q x + 72 I a b c d p^2 q$$

$$+ 24 I a b c d p q + 12 I a b c^2 n q x + 24 I a b c d n^2 q - 112 I a c^2 d n p x - 16 I a b c d n q$$

$$- 4 I a^3 b n p q x + 72 I a^2 b c n^2 p x - 104 I a^2 b c n p^2 x + 2 I a^2 b c n q^2 x$$

$$+ 4 I a^2 c d n^2 q x + 4 I a b c^2 n^2 q x - 72 I a c^2 d n^2 p x + 104 I a c^2 d n p^2 x$$

$$- 2 I a c^2 d n q^2 x - 4 I c^3 d n p q x + 112 I a^2 b c n p x + 12 I a^2 c d n q x$$

$$- 4 I a^2 c^2 n p q x^2 \big) \big(2 a^2 x n^2 - 4 a^2 x p n + 2 a^2 p^2 x + 2 c^2 x n^2 - 4 c^2 x p n + 2 c^2 p^2 x$$

$$+ 6 a^2 x n - 6 a^2 x p + 2 a b n^2 - 4 a b p n + 2 a b p^2 - a d p q + b c p q + 6 c^2 x n$$

$$- 6 c^2 x p + 2 c d n^2 - 4 c d p n + 2 c d p^2 + 4 a^2 x + 6 a b n - 6 a b p + 4 c^2 x + 6 c d n$$

$$- 6 c d p + 4 a b + 4 c d\big) (I q + 2 n + 4 - 2 p) (I q - 2 n + 2 p) (I q - 2 n + 2 p$$

$$+ 2) (2 n - 2 p + 3) (a x + b + I c x + I d)^2 (I c - a)^2 (I q - 4 n + 6 p - 6)^5 S(n + 1)$$

$$+ \big(144 a b c d n^2 p - 240 a b c d n p^2 + 12 a b c d n q^2 - 20 a b c d p q^2 + 64 a b c d n p$$

$$+ 16 a^2 b d q - 64 a^2 c d x - 16 a b^2 c q - 64 a b c^2 x + 16 a c d^2 q - 16 b c^2 d q$$

$$+ 116 a^4 n p x^2 - 24 a^3 b n^3 x + 32 a^3 b p^3 x - 104 a^2 c^2 n^2 x^2 - 128 a^2 c^2 p^2 x^2$$

$$+ 116 c^4 n p x^2 - 24 c^3 d n^3 x + 32 c^3 d p^3 x - 104 a^3 b n^2 x - 128 a^3 b p^2 x + 40 a^2 b^2 n^2 p$$

$$
\begin{aligned}
&- 44 a^2 b^2 n p^2 - 144 a^2 c^2 n x^2 + 160 a^2 c^2 p x^2 - 104 c^3 d n^2 x - 128 c^3 d p^2 x \\
&+ 40 c^2 d^2 n^2 p - 44 c^2 d^2 n p^2 - 144 a^3 b n x + 160 a^3 b p x + 16 a^3 d q x + 116 a^2 b^2 n p \\
&- 16 b c^3 q x - 144 c^3 d n x + 160 c^3 d p x + 116 c^2 d^2 n p - 32 a b c d n^3 + 144 a b c d p^3 \\
&- 48 a b c d n^2 - 16 a b c d p^2 - 80 a b c d n + 32 a^2 d^2 + 32 b^2 c^2 + 4 a^2 d^2 n^3 \\
&- 56 a^2 d^2 p^3 + 4 b^2 c^2 n^3 - 56 b^2 c^2 p^3 - 28 a^2 d^2 n^2 - 56 a^2 d^2 p^2 - 28 b^2 c^2 n^2 \\
&- 56 b^2 c^2 p^2 - 32 a^2 d^2 n + 80 a^2 d^2 p - 32 b^2 c^2 n + 80 b^2 c^2 p - 32 a^4 x^2 - 32 c^4 x^2 \\
&- 32 a^2 b^2 - 32 c^2 d^2 - 32 a^2 d^2 n^2 p + 76 a^2 d^2 n p^2 - 6 a^2 d^2 n q^2 + 10 a^2 d^2 p q^2 \\
&- 32 b^2 c^2 n^2 p + 76 b^2 c^2 n p^2 - 6 b^2 c^2 n q^2 + 10 b^2 c^2 p q^2 + 84 a^2 d^2 n p + 84 b^2 c^2 n p \\
&- 128 a b c d + 24 a^2 b c n p q x - 24 a c^2 d n p q x - 24 a^3 d n p q x - 12 a^2 b c n^2 q x \\
&- 8 a^2 b c p^2 q x + 80 a^2 c d n^2 p x - 88 a^2 c d n p^2 x + 80 a b c^2 n^2 p x - 88 a b c^2 n p^2 x \\
&+ 12 a c^2 d n^2 q x + 8 a c^2 d p^2 q x + 24 b c^3 n p q x - 32 a^2 b c n q x + 44 a^2 b c p q x \\
&- 24 a^2 b d n p q + 232 a^2 c d n p x + 24 a b^2 c n p q + 232 a b c^2 n p x + 32 a c^2 d n q x \\
&- 44 a c^2 d p q x - 24 a c d^2 n p q + 24 b c^2 d n p q + 40 a^4 n^2 p x^2 - 44 a^4 n p^2 x^2 \\
&- 24 a^2 c^2 n^3 x^2 + 32 a^2 c^2 p^3 x^2 + 40 c^4 n^2 p x^2 - 44 c^4 n p^2 x^2 + 12 a c d^2 n^2 q \\
&+ 8 a c d^2 p^2 q - 32 b c^3 n q x + 44 b c^3 p q x - 12 b c^2 d n^2 q - 8 b c^2 d p^2 q \\
&+ 232 c^3 d n p x - 16 a^2 b c q x + 32 a^2 b d n q - 44 a^2 b d p q - 144 a^2 c d n x \\
&+ 160 a^2 c d p x - 32 a b^2 c n q + 44 a b^2 c p q - 144 a b c^2 n x + 160 a b c^2 p x \\
&+ 16 a c^2 d q x + 32 a c d^2 n q - 44 a c d^2 p q - 32 b c^2 d n q + 44 b c^2 d p q - 8 b c^3 p^2 q x
\end{aligned}
$$

$$+ 80 c^3 d n^2 p x - 88 c^3 d n p^2 x + 232 a^3 b n p x + 32 a^3 d n q x - 44 a^3 d p q x$$

$$+ 12 a^2 b d n^2 q + 8 a^2 b d p^2 q - 104 a^2 c d n^2 x - 128 a^2 c d p^2 x - 12 a b^2 c n^2 q$$

$$- 8 a b^2 c p^2 q - 104 a b c^2 n^2 x - 128 a b c^2 p^2 x + 80 a^3 b n^2 p x - 88 a^3 b n p^2 x$$

$$+ 12 a^3 d n^2 q x + 8 a^3 d p^2 q x + 232 a^2 c^2 n p x^2 - 24 a^2 c d n^3 x + 32 a^2 c d p^3 x$$

$$- 24 a b c^2 n^3 x + 32 a b c^2 p^3 x - 12 b c^3 n^2 q x + 80 a^2 c^2 n^2 p x^2 - 88 a^2 c^2 n p^2 x^2$$

$$- 12 a^4 n^3 x^2 + 16 a^4 p^3 x^2 - 12 c^4 n^3 x^2 + 16 c^4 p^3 x^2 - 52 a^4 n^2 x^2 - 64 a^4 p^2 x^2$$

$$- 52 c^4 n^2 x^2 - 64 c^4 p^2 x^2 - 72 a^4 n x^2 + 80 a^4 p x^2 - 12 a^2 b^2 n^3 + 16 a^2 b^2 p^3 - 72 c^4 n x^2$$

$$+ 80 c^4 p x^2 - 12 c^2 d^2 n^3 + 16 c^2 d^2 p^3 - 52 a^2 b^2 n^2 - 64 a^2 b^2 p^2 - 64 a^2 c^2 x^2$$

$$- 52 c^2 d^2 n^2 - 64 c^2 d^2 p^2 - 64 a^3 b x - 72 a^2 b^2 n + 80 a^2 b^2 p - 64 c^3 d x - 72 c^2 d^2 n$$

$$+ 80 c^2 d^2 p + 104 \mathrm{I} a^2 b d n p^2 - 2 \mathrm{I} a^2 b d n q^2 + 38 \mathrm{I} a^2 d^2 n p q + 72 \mathrm{I} a b^2 c n^2 p$$

$$- 104 \mathrm{I} a b^2 c n p^2 + 2 \mathrm{I} a b^2 c n q^2 + 24 \mathrm{I} a c^2 d n^2 x - 72 \mathrm{I} a c d^2 n^2 p + 104 \mathrm{I} a c d^2 n p^2$$

$$- 2 \mathrm{I} a c d^2 n q^2 + 38 \mathrm{I} b^2 c^2 n p q + 112 \mathrm{I} b c^3 n p x + 72 \mathrm{I} b c^2 d n^2 p - 104 \mathrm{I} b c^2 d n p^2$$

$$+ 2 \mathrm{I} b c^2 d n q^2 + 12 \mathrm{I} c^3 d n q x - 2 \mathrm{I} c^2 d^2 n p q + 56 \mathrm{I} a^2 b c n x - 112 \mathrm{I} a^2 b d n p$$

$$+ 112 \mathrm{I} a b^2 c n p - 56 \mathrm{I} a c^2 d n x - 112 \mathrm{I} a c d^2 n p + 112 \mathrm{I} b c^2 d n p + 48 \mathrm{I} a^2 b c p^3 x$$

$$- 8 \mathrm{I} a^2 c^2 p q x^2 - 48 \mathrm{I} a c^2 d p^3 x - 8 \mathrm{I} a^3 b p q x - 104 \mathrm{I} a^2 b c p^2 x + 4 \mathrm{I} a^2 b c q^2 x$$

$$- 2 \mathrm{I} a b c d q^3 + 104 \mathrm{I} a c^2 d p^2 x - 4 \mathrm{I} a c^2 d q^2 x - 8 \mathrm{I} c^3 d p q x - 24 \mathrm{I} a^2 b c p x$$

$$+ 8 \mathrm{I} a^2 c d q x + 8 \mathrm{I} a b c^2 q x + 24 \mathrm{I} a c^2 d p x - 40 \mathrm{I} a b c d q - 2 \mathrm{I} a^4 n p q x^2$$

$$+ 4 \mathrm{I} a^3 b n^2 q x - 72 \mathrm{I} a^3 d n^2 p x + 104 \mathrm{I} a^3 d n p^2 x - 2 \mathrm{I} a^3 d n q^2 x - 16 \mathrm{I} a^2 b c n^3 x$$

$$+12\,\mathrm{I}\,a^2\,c^2\,n\,q\,x^2+16\,\mathrm{I}\,a\,c^2\,d\,n^3\,x+72\,\mathrm{I}\,b\,c^3\,n^2\,p\,x-104\,\mathrm{I}\,b\,c^3\,n\,p^2\,x+2\,\mathrm{I}\,b\,c^3\,n\,q^2\,x$$

$$+4\,\mathrm{I}\,c^3\,d\,n^2\,q\,x+12\,\mathrm{I}\,a^3\,b\,n\,q\,x-112\,\mathrm{I}\,a^3\,d\,n\,p\,x-2\,\mathrm{I}\,a^2\,b^2\,n\,p\,q-24\,\mathrm{I}\,a^2\,b\,c\,n^2\,x$$

$$-72\,\mathrm{I}\,a^2\,b\,d\,n^2\,p+\mathrm{I}\,a^2\,d^2\,q^3+\mathrm{I}\,b^2\,c^2\,q^3+4\,\mathrm{I}\,a^4\,q\,x^2+4\,\mathrm{I}\,c^4\,q\,x^2-80\,\mathrm{I}\,a^3\,d\,x+4\,\mathrm{I}\,q\,b^2\,a^2$$

$$+24\,\mathrm{I}\,q\,d^2\,a^2+24\,\mathrm{I}\,b^2\,c^2\,q+80\,\mathrm{I}\,b\,c^3\,x+4\,\mathrm{I}\,c^2\,d^2\,q-80\,\mathrm{I}\,d\,b\,a^2+80\,\mathrm{I}\,a\,b^2\,c$$

$$-80\,\mathrm{I}\,a\,c\,d^2+80\,\mathrm{I}\,b\,c^2\,d+6\,\mathrm{I}\,c^2\,d^2\,n\,q-56\,\mathrm{I}\,a^2\,b\,d\,n+56\,\mathrm{I}\,a\,b^2\,c\,n-56\,\mathrm{I}\,a\,c\,d^2\,n$$

$$+56\,\mathrm{I}\,b\,c^2\,d\,n+16\,\mathrm{I}\,a^3\,d\,n^3\,x-16\,\mathrm{I}\,b\,c^3\,n^3\,x+6\,\mathrm{I}\,c^4\,n\,q\,x^2+24\,\mathrm{I}\,a^3\,d\,n^2\,x+2\,\mathrm{I}\,a^2\,b^2\,n^2\,q$$

$$+16\,\mathrm{I}\,a^2\,b\,d\,n^3-10\,\mathrm{I}\,a^2\,d^2\,n^2\,q-16\,\mathrm{I}\,a\,b^2\,c\,n^3+16\,\mathrm{I}\,a\,c\,d^2\,n^3-10\,\mathrm{I}\,b^2\,c^2\,n^2\,q$$

$$-24\,\mathrm{I}\,b\,c^3\,n^2\,x-16\,\mathrm{I}\,b\,c^2\,d\,n^3+2\,\mathrm{I}\,c^2\,d^2\,n^2\,q-56\,\mathrm{I}\,a^3\,d\,n\,x+6\,\mathrm{I}\,a^2\,b^2\,n\,q+24\,\mathrm{I}\,a^2\,b\,d\,n^2$$

$$+14\,\mathrm{I}\,a^2\,d^2\,n\,q-24\,\mathrm{I}\,a\,b^2\,c\,n^2+24\,\mathrm{I}\,a\,c\,d^2\,n^2+14\,\mathrm{I}\,b^2\,c^2\,n\,q+56\,\mathrm{I}\,b\,c^3\,n\,x$$

$$+2\,\mathrm{I}\,a^4\,n^2\,q\,x^2+2\,\mathrm{I}\,c^4\,n^2\,q\,x^2+6\,\mathrm{I}\,a^4\,n\,q\,x^2-48\,\mathrm{I}\,p^3\,d\,b\,a^2+8\,\mathrm{I}\,a^2\,c^2\,q\,x^2$$

$$-36\,\mathrm{I}\,a^2\,d^2\,p^2\,q+48\,\mathrm{I}\,a\,b^2\,c\,p^3-48\,\mathrm{I}\,a\,c\,d^2\,p^3-36\,\mathrm{I}\,b^2\,c^2\,p^2\,q-104\,\mathrm{I}\,b\,c^3\,p^2\,x$$

$$+4\,\mathrm{I}\,b\,c^3\,q^2\,x+48\,\mathrm{I}\,b\,c^2\,d\,p^3+8\,\mathrm{I}\,a^3\,b\,q\,x+24\,\mathrm{I}\,a^3\,d\,p\,x-4\,\mathrm{I}\,q\,p\,b^2\,a^2+104\,\mathrm{I}\,p^2\,d\,b\,a^2$$

$$-4\,\mathrm{I}\,a^2\,b\,d\,q^2-16\,\mathrm{I}\,q\,p\,d^2\,a^2-104\,\mathrm{I}\,a\,b^2\,c\,p^2+4\,\mathrm{I}\,a\,b^2\,c\,q^2+104\,\mathrm{I}\,a\,c\,d^2\,p^2$$

$$-4\,\mathrm{I}\,a\,c\,d^2\,q^2-16\,\mathrm{I}\,b^2\,c^2\,p\,q-24\,\mathrm{I}\,b\,c^3\,p\,x-104\,\mathrm{I}\,b\,c^2\,d\,p^2+4\,\mathrm{I}\,b\,c^2\,d\,q^2+8\,\mathrm{I}\,c^3\,d\,q\,x$$

$$-4\,\mathrm{I}\,c^2\,d^2\,p\,q+80\,\mathrm{I}\,a^2\,b\,c\,x+24\,\mathrm{I}\,p\,d\,b\,a^2-24\,\mathrm{I}\,a\,b^2\,c\,p-80\,\mathrm{I}\,a\,c^2\,d\,x+24\,\mathrm{I}\,a\,c\,d^2\,p$$

$$-24\,\mathrm{I}\,b\,c^2\,d\,p-4\,\mathrm{I}\,a^4\,p\,q\,x^2-48\,\mathrm{I}\,a^3\,d\,p^3\,x+48\,\mathrm{I}\,b\,c^3\,p^3\,x-4\,\mathrm{I}\,c^4\,p\,q\,x^2+104\,\mathrm{I}\,a^3\,d\,p^2\,x$$

$$-4\,\mathrm{I}\,a^3\,d\,q^2\,x-24\,\mathrm{I}\,b\,c^2\,d\,n^2+4\,\mathrm{I}\,a^2\,c^2\,n^2\,q\,x^2-2\,\mathrm{I}\,c^4\,n\,p\,q\,x^2-4\,\mathrm{I}\,a^2\,c\,d\,n\,p\,q\,x$$

$$-4\,\mathrm{I}\,a\,b\,c^2\,n\,p\,q\,x-80\,\mathrm{I}\,a\,b\,c\,d\,n\,p\,q-8\,\mathrm{I}\,a^2\,c\,d\,p\,q\,x-8\,\mathrm{I}\,a\,b\,c^2\,p\,q\,x+72\,\mathrm{I}\,a\,b\,c\,d\,p^2\,q$$

$$+ 24 \mathrm{I} a b c d p q + 12 \mathrm{I} a b c^2 n q x + 24 \mathrm{I} a b c d n^2 q - 112 \mathrm{I} a c^2 d n p x - 16 \mathrm{I} a b c d n q$$

$$- 4 \mathrm{I} a^3 b n p q x + 72 \mathrm{I} a^2 b c n^2 p x - 104 \mathrm{I} a^2 b c n p^2 x + 2 \mathrm{I} a^2 b c n q^2 x$$

$$+ 4 \mathrm{I} a^2 c d n^2 q x + 4 \mathrm{I} a b c^2 n^2 q x - 72 \mathrm{I} a c^2 d n^2 p x + 104 \mathrm{I} a c^2 d n p^2 x$$

$$- 2 \mathrm{I} a c^2 d n q^2 x - 4 \mathrm{I} c^3 d n p q x + 112 \mathrm{I} a^2 b c n p x + 12 \mathrm{I} a^2 c d n q x$$

$$- 4 \mathrm{I} a^2 c^2 n p q x^2) (\mathrm{I} q - 2 n + 2 p + 2) (\mathrm{I} q - 2 n + 2 p) (\mathrm{I} q + 2 n + 4 - 2 p) (\mathrm{I} q$$

$$- 2 n + 2 p - 2) (\mathrm{I} q + 2 n - 2 p + 2) (-p + 2 + n) (n + 1) (a x + b + \mathrm{I} c x$$

$$+ \mathrm{I} d)^2 (a d - b c)^2 (\mathrm{I} c - a)^2 (\mathrm{I} q - 4 n + 6 p - 6)^5 S(n) = 0, 1,$$

$$- \frac{1}{2 (-1 + p) \Gamma(-2 p + 2)} (\Gamma(3 - 2 p) (2 a^2 x p + 2 c^2 x p - 2 a^2 x + 2 a b p - q a d$$

$$+ q b c - 2 c^2 x + 2 c d p - 2 a b - 2 c d)) \Big]$$

```
> solJ3:=solve(REMJ3[1], S(n+2)):
> solJ1:=solve(REMJ1[1], S(n+2)):
> normal(solJ1-solJ3);
```

$$0 \qquad \textbf{(181)}$$

```
> simplify(REMJ1[2]-REMJ3[2]);
```

$$0 \qquad \textbf{(182)}$$

```
> simplify(REMJ1[3]-REMJ3[3]);
```

$$0 \qquad \textbf{(183)}$$

```
> TermMJ4:=2^n*(a*d-b*c)^n*GAMMA(-p+(1/2*I)*q+1+n)*((1/2)*(I*a^2*x+
  I*c^2*x+I*a*b+I*c*d+a*d-b*c)/(a*d-b*c))^n*hyperterm([-n+p+(1/2*I)
  *q, -n], [-p+(1/2*I)*q+1], ((2*I)*a^2*x+(2*I)*c^2*x+(2*I)*a*b+(2*
  I)*c*d-2*a*d+2*b*c)/((2*I)*a^2*x+(2*I)*c^2*x+(2*I)*a*b+(2*I)*c*
  d+2*a*d-2*b*c), k)/((-1)^n*I^n*GAMMA(-p+(1/2*I)*q+1));
```

$$TermMJ4 := \left(2^n (a d - b c)^n \Gamma\left(n - p + \frac{\mathrm{I} q}{2} + 1\right) \left(\frac{\mathrm{I} a^2 x + \mathrm{I} c^2 x + \mathrm{I} a b + \mathrm{I} c d + a d - b c}{2 (a d - b c)}\right)^n \mathrm{pochhammer}\left(-n + p + \frac{\mathrm{I} q}{2}, k\right) \mathrm{pochhammer}(-n, k) \left(\frac{2 \mathrm{I} a^2 x + 2 \mathrm{I} c^2 x + 2 \mathrm{I} a b + 2 \mathrm{I} c d - 2 a d + 2 b c}{2 \mathrm{I} a^2 x + 2 \mathrm{I} c^2 x + 2 \mathrm{I} a b + 2 \mathrm{I} c d + 2 a d - 2 b c}\right)^k\right) \Big/ \left(\mathrm{pochhammer}\left(-p + \frac{\mathrm{I} q}{2} + 1, k\right) k! \, (-1)^n \mathrm{I}^n \Gamma\left(-p + \frac{\mathrm{I} q}{2} + 1\right)\right) \qquad \textbf{(184)}$$

```
> REMJ4:=RecurrenceNormalForm2(TermMJ4,k,S(n));
```

$$REMJ4 := \Big[(2\,\mathrm{I}\,n-2\,\mathrm{I}\,p+4\,\mathrm{I}-q)\,(2\,\mathrm{I}\,n-2\,\mathrm{I}\,p+q+4\,\mathrm{I})\,(n-2\,p+2)\,(n-p+1)\,(\mathrm{I}\,c\,x+\mathrm{I}\,d-a\,x-b)^{3}\,(a+\mathrm{I}\,c)^{3}\,S(n+2)+\big(2\,a^{2}\,x\,n^{2}-4\,a^{2}\,x\,p\,n+2\,a^{2}\,p^{2}\,x+2\,c^{2}\,x\,n^{2}-4\,c^{2}\,x\,p\,n+2\,c^{2}\,p^{2}\,x+6\,a^{2}\,x\,n-6\,a^{2}\,x\,p+2\,a\,b\,n^{2}-4\,a\,b\,p\,n+2\,a\,b\,p^{2}-a\,d\,p\,q+b\,c\,p\,q+6\,c^{2}\,x\,n-6\,c^{2}\,x\,p+2\,c\,d\,n^{2}-4\,c\,d\,p\,n+2\,c\,d\,p^{2}+4\,a^{2}\,x+6\,a\,b\,n-6\,a\,b\,p+4\,c^{2}\,x+6\,c\,d\,n-6\,c\,d\,p+4\,a\,b+4\,c\,d\big)\,(2\,\mathrm{I}\,n-2\,\mathrm{I}\,p+4\,\mathrm{I}-q)\,(2\,\mathrm{I}\,n-2\,\mathrm{I}\,p+q+4\,\mathrm{I})\,(2\,n-2\,p+3)\,(\mathrm{I}\,c\,x+\mathrm{I}\,d-a\,x-b)^{3}\,(a+\mathrm{I}\,c)^{3}\,S(n+1)+(\mathrm{I}\,q-2\,n+2\,p-2)\,(\mathrm{I}\,q+2\,n-2\,p+2)\,(\mathrm{I}\,q+2\,n+4-2\,p)\,(\mathrm{I}\,q-2\,n-4+2\,p)\,(-p+2+n)\,(n+1)\,(a\,d-b\,c)^{2}\,(\mathrm{I}\,c\,x+\mathrm{I}\,d-a\,x-b)^{3}\,(a+\mathrm{I}\,c)^{3}\,S(n)=0,\ 1,\ \frac{1}{\Gamma\left(-p+\frac{\mathrm{I}\,q}{2}+1\right)\,(\mathrm{I}\,q-2\,p+2)}\Big(2\,\big(2\,a^{2}\,x\,p+2\,c^{2}\,x\,p-2\,a^{2}\,x+2\,a\,b\,p-q\,a\,d+q\,b\,c-2\,c^{2}\,x+2\,c\,d\,p-2\,a\,b-2\,c\,d\big)\,\Gamma\left(2-p+\frac{\mathrm{I}\,q}{2}\right)\Big)\Big] \quad \textbf{(185)}$$

```
> solJ4:=solve(REMJ4[1], S(n+2)):
> normal(solJ1-solJ4);
```

$$0 \quad \textbf{(186)}$$

```
> simplify(REMJ1[2]-REMJ4[2]);
```

$$0 \tag{187}$$

```
> simplify(REMJ1[3]-REMJ4[3]);
> TermMJ5:=2^n*(a*d-b*c)^n*GAMMA(n-p-(1/2*I)*q+1)*((1/2)*(I*a^2*x+
  I*c^2*x+I*a*b+I*c*d-a*d+b*c)/(a*d-b*c))^n*hyperterm([-n+p-(1/2*I)
  *q, -n], [-p-(1/2*I)*q+1], (-(2*I)*a^2*x-(2*I)*c^2*x-(2*I)*a*b-
  (2*I)*c*d-2*a*d+2*b*c)/(-(2*I)*a^2*x-(2*I)*c^2*x-(2*I)*a*b-(2*I)*
  c*d+2*a*d-2*b*c), k)/((-1)^n*I^n*GAMMA(-p-(1/2*I)*q+1));
```

$$TermMJ5 := \left(2^n (a d - b c)^n \Gamma\left(n - p - \frac{I q}{2} + 1\right) \left(\frac{I a^2 x + I c^2 x + I a b + I c d - a d + b c}{2 (a d - b c)}\right)^n \mathrm{pochhammer}\left(-n + p - \frac{I q}{2}, k\right) \mathrm{pochhammer}(-n, k) \left(\frac{-2 I a^2 x - 2 I c^2 x - 2 I a b - 2 I c d - 2 a d + 2 b c}{-2 I a^2 x - 2 I c^2 x - 2 I a b - 2 I c d + 2 a d - 2 b c}\right)^k\right) \Big/ \left(\mathrm{pochhammer}\left(-p - \frac{I q}{2} + 1, k\right) k! \, (-1)^n I^n \Gamma\left(-p - \frac{I q}{2} + 1\right)\right) \tag{188}$$

```
> REMJ5:=RecurrenceNormalForm2(TermMJ5,k,S(n));
```

$$REMJ5 := \Big[-(2 I n - 2 I p + 4 I - q) (2 I n - 2 I p + q + 4 I) (n - 2 p + 2) (n - p + 1) (a x + b + I c x + I d)^3 (I c - a)^3 S(n + 2) - \big(2 a^2 x n^2 - 4 a^2 x p n + 2 a^2 p^2 x + 2 c^2 x n^2 - 4 c^2 x p n + 2 c^2 p^2 x + 6 a^2 x n - 6 a^2 x p + 2 a b n^2 - 4 a b p n + 2 a b p^2 - a d p q + b c p q + 6 c^2 x n - 6 c^2 x p + 2 c d n^2 - 4 c d p n + 2 c d p^2 + 4 a^2 x + 6 a b n - 6 a b p + 4 c^2 x + 6 c d n - 6 c d p + 4 a b + 4 c d\big) (2 I n - 2 I p + q + 4 I) (2 I n - 2 I p + 4 I - q) (2 n - 2 p + 3) (a x + b + I c x + I d)^3 (I c - a)^3 S(n + 1) - (I q \tag{189}$$

$+ 2 n + 4 - 2 p) (I q + 2 n - 2 p + 2) (I q - 2 n + 2 p - 2) (I q - 2 n - 4 + 2 p) (-p$

$+ 2 + n) (n + 1) (a d - b c)^2 (a x + b + I c x + I d)^3 (I c - a)^3 S(n) = 0, 1,$

$$- \frac{1}{\Gamma\left(-p - \frac{I q}{2} + 1\right) (I q + 2 p - 2)} \left(2 \left(2 a^2 x p + 2 c^2 x p - 2 a^2 x + 2 a b p - q a d + q b c - 2 c^2 x + 2 c d p - 2 a b - 2 c d\right) \Gamma\left(2 - p - \frac{I q}{2}\right)\right)\Bigg]$$

```
> solJ5:=solve(REMJ5[1], S(n+2)):
> normal(solJ1-solJ5);
```

$$0 \qquad \textbf{(190)}$$

```
> simplify(REMJ1[2]-REMJ5[2]);
```

$$0 \qquad \textbf{(191)}$$

```
> simplify(REMJ1[3]-REMJ5[3]);
```

$$0 \qquad \textbf{(192)}$$

```
> TermMJ6:=2^n*(a*d-b*c)^n*GAMMA(2*n-2*p+1)*((1/2)*(I*a^2*x+I*c^2*
  x+I*a*b+I*c*d+a*d-b*c)/(a*d-b*c))^n*hyperterm([-n+p+(1/2*I)*q, -
  n], [-2*n+2*p], (2*a*d-2*b*c)/(I*a^2*x+I*c^2*x+I*a*b+I*c*d+a*d-b*
  c), k)/((-1)^n*I^n*GAMMA(n-2*p+1));
```

$$TermMJ6 := \left(2^n (a d - b c)^n \Gamma(2 n - 2 p + 1) \left(\frac{I a^2 x + I c^2 x + I a b + I c d + a d - b c}{2 (a d - b c)}\right)^n \mathrm{pochhammer}\left(-n + p + \frac{I q}{2}, k\right) \mathrm{pochhammer}(-n, k) \left(\frac{2 a d - 2 b c}{I a^2 x + I c^2 x + I a b + I c d + a d - b c}\right)^k\right) \Big/ \left(\mathrm{pochhammer}(-2 n + 2 p, k)\, k!\, (-1)^n I^n \Gamma(n - 2 p + 1)\right) \qquad \textbf{(193)}$$

```
> REMJ6:=RecurrenceNormalForm2(TermMJ6,k,S(n));
```

$REMJ6 := \Big[(-144 a b c d n^2 p + 240 a b c d n p^2 - 12 a b c d n q^2 + 20 a b c d p q^2$ **(194)**

$- 64 a b c d n p - 16 a^2 b d q + 64 a^2 c d x + 16 a b^2 c q + 64 a b c^2 x - 16 a c d^2 q$

$+ 16 b c^2 d q - 116 a^4 n p x^2 + 24 a^3 b n^3 x - 32 a^3 b p^3 x + 104 a^2 c^2 n^2 x^2$

$+ 128 a^2 c^2 p^2 x^2 - 116 c^4 n p x^2 + 24 c^3 d n^3 x - 32 c^3 d p^3 x + 104 a^3 b n^2 x$

$$+ 128 a^3 b p^2 x - 40 a^2 b^2 n^2 p + 44 a^2 b^2 n p^2 + 144 a^2 c^2 n x^2 - 160 a^2 c^2 p x^2$$

$$+ 104 c^3 d n^2 x + 128 c^3 d p^2 x - 40 c^2 d^2 n^2 p + 44 c^2 d^2 n p^2 + 144 a^3 b n x - 160 a^3 b p x$$

$$- 16 a^3 d q x - 116 a^2 b^2 n p + 16 b c^3 q x + 144 c^3 d n x - 160 c^3 d p x - 116 c^2 d^2 n p$$

$$+ 32 a b c d n^3 - 144 a b c d p^3 + 48 a b c d n^2 + 16 a b c d p^2 + 80 a b c d n - 32 a^2 d^2$$

$$- 32 b^2 c^2 - 4 a^2 d^2 n^3 + 56 a^2 d^2 p^3 - 4 b^2 c^2 n^3 + 56 b^2 c^2 p^3 + 28 a^2 d^2 n^2 + 56 a^2 d^2 p^2$$

$$+ 28 b^2 c^2 n^2 + 56 b^2 c^2 p^2 + 32 a^2 d^2 n - 80 a^2 d^2 p + 32 b^2 c^2 n - 80 b^2 c^2 p + 32 a^4 x^2$$

$$+ 32 c^4 x^2 + 32 a^2 b^2 + 32 c^2 d^2 + 32 a^2 d^2 n^2 p - 76 a^2 d^2 n p^2 + 6 a^2 d^2 n q^2$$

$$- 10 a^2 d^2 p q^2 + 32 b^2 c^2 n^2 p - 76 b^2 c^2 n p^2 + 6 b^2 c^2 n q^2 - 10 b^2 c^2 p q^2 - 84 a^2 d^2 n p$$

$$- 84 b^2 c^2 n p + 128 a b c d - 24 a^2 b c n p q x + 24 a c^2 d n p q x + 24 a^3 d n p q x$$

$$+ 12 a^2 b c n^2 q x + 8 a^2 b c p^2 q x - 80 a^2 c d n^2 p x + 88 a^2 c d n p^2 x - 80 a b c^2 n^2 p x$$

$$+ 88 a b c^2 n p^2 x - 12 a c^2 d n^2 q x - 8 a c^2 d p^2 q x - 24 b c^3 n p q x + 32 a^2 b c n q x$$

$$- 44 a^2 b c p q x + 24 a^2 b d n p q - 232 a^2 c d n p x - 24 a b^2 c n p q - 232 a b c^2 n p x$$

$$- 32 a c^2 d n q x + 44 a c^2 d p q x + 24 a c d^2 n p q - 24 b c^2 d n p q - 40 a^4 n^2 p x^2$$

$$+ 44 a^4 n p^2 x^2 + 24 a^2 c^2 n^3 x^2 - 32 a^2 c^2 p^3 x^2 - 40 c^4 n^2 p x^2 + 44 c^4 n p^2 x^2$$

$$- 12 a c d^2 n^2 q - 8 a c d^2 p^2 q + 32 b c^3 n q x - 44 b c^3 p q x + 12 b c^2 d n^2 q$$

$$+ 8 b c^2 d p^2 q - 232 c^3 d n p x + 16 a^2 b c q x - 32 a^2 b d n q + 44 a^2 b d p q$$

$$+ 144 a^2 c d n x - 160 a^2 c d p x + 32 a b^2 c n q - 44 a b^2 c p q + 144 a b c^2 n x$$

$$- 160 a b c^2 p x - 16 a c^2 d q x - 32 a c d^2 n q + 44 a c d^2 p q + 32 b c^2 d n q$$

$$- 44\, b\, c^{2}\, d\, p\, q + 8\, b\, c^{3}\, p^{2}\, q\, x - 80\, c^{3}\, d\, n^{2}\, p\, x + 88\, c^{3}\, d\, n\, p^{2}\, x - 232\, a^{3}\, b\, n\, p\, x$$

$$- 32\, a^{3}\, d\, n\, q\, x + 44\, a^{3}\, d\, p\, q\, x - 12\, a^{2}\, b\, d\, n^{2}\, q - 8\, a^{2}\, b\, d\, p^{2}\, q + 104\, a^{2}\, c\, d\, n^{2}\, x$$

$$+ 128\, a^{2}\, c\, d\, p^{2}\, x + 12\, a\, b^{2}\, c\, n^{2}\, q + 8\, a\, b^{2}\, c\, p^{2}\, q + 104\, a\, b\, c^{2}\, n^{2}\, x + 128\, a\, b\, c^{2}\, p^{2}\, x$$

$$- 80\, a^{3}\, b\, n^{2}\, p\, x + 88\, a^{3}\, b\, n\, p^{2}\, x - 12\, a^{3}\, d\, n^{2}\, q\, x - 8\, a^{3}\, d\, p^{2}\, q\, x - 232\, a^{2}\, c^{2}\, n\, p\, x^{2}$$

$$+ 24\, a^{2}\, c\, d\, n^{3}\, x - 32\, a^{2}\, c\, d\, p^{3}\, x + 24\, a\, b\, c^{2}\, n^{3}\, x - 32\, a\, b\, c^{2}\, p^{3}\, x + 12\, b\, c^{3}\, n^{2}\, q\, x$$

$$- 80\, a^{2}\, c^{2}\, n^{2}\, p\, x^{2} + 88\, a^{2}\, c^{2}\, n\, p^{2}\, x^{2} + 12\, a^{4}\, n^{3}\, x^{2} - 16\, a^{4}\, p^{3}\, x^{2} + 12\, c^{4}\, n^{3}\, x^{2} - 16\, c^{4}\, p^{3}\, x^{2}$$

$$+ 52\, a^{4}\, n^{2}\, x^{2} + 64\, a^{4}\, p^{2}\, x^{2} + 52\, c^{4}\, n^{2}\, x^{2} + 64\, c^{4}\, p^{2}\, x^{2} + 72\, a^{4}\, n\, x^{2} - 80\, a^{4}\, p\, x^{2}$$

$$+ 12\, a^{2}\, b^{2}\, n^{3} - 16\, a^{2}\, b^{2}\, p^{3} + 72\, c^{4}\, n\, x^{2} - 80\, c^{4}\, p\, x^{2} + 12\, c^{2}\, d^{2}\, n^{3} - 16\, c^{2}\, d^{2}\, p^{3}$$

$$+ 52\, a^{2}\, b^{2}\, n^{2} + 64\, a^{2}\, b^{2}\, p^{2} + 64\, a^{2}\, c^{2}\, x^{2} + 52\, c^{2}\, d^{2}\, n^{2} + 64\, c^{2}\, d^{2}\, p^{2} + 64\, a^{3}\, b\, x + 72\, a^{2}\, b^{2}\, n$$

$$- 80\, a^{2}\, b^{2}\, p + 64\, c^{3}\, d\, x + 72\, c^{2}\, d^{2}\, n - 80\, c^{2}\, d^{2}\, p + 104\, \mathrm{I}\, a^{2}\, b\, d\, n\, p^{2} - 2\, \mathrm{I}\, a^{2}\, b\, d\, n\, q^{2}$$

$$+ 38\, \mathrm{I}\, a^{2}\, d^{2}\, n\, p\, q + 72\, \mathrm{I}\, a\, b^{2}\, c\, n^{2}\, p - 104\, \mathrm{I}\, a\, b^{2}\, c\, n\, p^{2} + 2\, \mathrm{I}\, a\, b^{2}\, c\, n\, q^{2} + 24\, \mathrm{I}\, a\, c^{2}\, d\, n^{2}\, x$$

$$- 72\, \mathrm{I}\, a\, c\, d^{2}\, n^{2}\, p + 104\, \mathrm{I}\, a\, c\, d^{2}\, n\, p^{2} - 2\, \mathrm{I}\, a\, c\, d^{2}\, n\, q^{2} + 38\, \mathrm{I}\, b^{2}\, c^{2}\, n\, p\, q + 112\, \mathrm{I}\, b\, c^{3}\, n\, p\, x$$

$$+ 72\, \mathrm{I}\, b\, c^{2}\, d\, n^{2}\, p - 104\, \mathrm{I}\, b\, c^{2}\, d\, n\, p^{2} + 2\, \mathrm{I}\, b\, c^{2}\, d\, n\, q^{2} + 12\, \mathrm{I}\, c^{3}\, d\, n\, q\, x - 2\, \mathrm{I}\, c^{2}\, d^{2}\, n\, p\, q$$

$$+ 56\, \mathrm{I}\, a^{2}\, b\, c\, n\, x - 112\, \mathrm{I}\, a^{2}\, b\, d\, n\, p + 112\, \mathrm{I}\, a\, b^{2}\, c\, n\, p - 56\, \mathrm{I}\, a\, c^{2}\, d\, n\, x - 112\, \mathrm{I}\, a\, c\, d^{2}\, n\, p$$

$$+ 112\, \mathrm{I}\, b\, c^{2}\, d\, n\, p + 48\, \mathrm{I}\, a^{2}\, b\, c\, p^{3}\, x - 8\, \mathrm{I}\, a^{2}\, c^{2}\, p\, q\, x^{2} - 48\, \mathrm{I}\, a\, c^{2}\, d\, p^{3}\, x - 8\, \mathrm{I}\, a^{3}\, b\, p\, q\, x$$

$$- 104\, \mathrm{I}\, a^{2}\, b\, c\, p^{2}\, x + 4\, \mathrm{I}\, a^{2}\, b\, c\, q^{2}\, x - 2\, \mathrm{I}\, a\, b\, c\, d\, q^{3} + 104\, \mathrm{I}\, a\, c^{2}\, d\, p^{2}\, x - 4\, \mathrm{I}\, a\, c^{2}\, d\, q^{2}\, x$$

$$- 8\, \mathrm{I}\, c^{3}\, d\, p\, q\, x - 24\, \mathrm{I}\, a^{2}\, b\, c\, p\, x + 8\, \mathrm{I}\, a^{2}\, c\, d\, q\, x + 8\, \mathrm{I}\, a\, b\, c^{2}\, q\, x + 24\, \mathrm{I}\, a\, c^{2}\, d\, p\, x$$

$$- 40\, \mathrm{I}\, a\, b\, c\, d\, q - 2\, \mathrm{I}\, a^{4}\, n\, p\, q\, x^{2} + 4\, \mathrm{I}\, a^{3}\, b\, n^{2}\, q\, x - 72\, \mathrm{I}\, a^{3}\, d\, n^{2}\, p\, x + 104\, \mathrm{I}\, a^{3}\, d\, n\, p^{2}\, x$$

$- 2\,I\,a^3\,d\,n\,q^2\,x - 16\,I\,a^2\,b\,c\,n^3\,x + 12\,I\,a^2\,c^2\,n\,q\,x^2 + 16\,I\,a\,c^2\,d\,n^3\,x + 72\,I\,b\,c^3\,n^2\,p\,x$

$- 104\,I\,b\,c^3\,n\,p^2\,x + 2\,I\,b\,c^3\,n\,q^2\,x + 4\,I\,c^3\,d\,n^2\,q\,x + 12\,I\,a^3\,b\,n\,q\,x - 112\,I\,a^3\,d\,n\,p\,x$

$- 2\,I\,a^2\,b^2\,n\,p\,q - 24\,I\,a^2\,b\,c\,n^2\,x - 72\,I\,a^2\,b\,d\,n^2\,p + I\,a^2\,d^2\,q^3 + I\,b^2\,c^2\,q^3 + 4\,I\,a^4\,q\,x^2$

$+ 4\,I\,c^4\,q\,x^2 - 80\,I\,a^3\,d\,x + 4\,I\,q\,b^2\,a^2 + 24\,I\,q\,d^2\,a^2 + 24\,I\,b^2\,c^2\,q + 80\,I\,b\,c^3\,x + 4\,I\,c^2\,d^2\,q$

$- 80\,I\,d\,b\,a^2 + 80\,I\,a\,b^2\,c - 80\,I\,a\,c\,d^2 + 80\,I\,b\,c^2\,d + 6\,I\,c^2\,d^2\,n\,q - 56\,I\,a^2\,b\,d\,n$

$+ 56\,I\,a\,b^2\,c\,n - 56\,I\,a\,c\,d^2\,n + 56\,I\,b\,c^2\,d\,n + 16\,I\,a^3\,d\,n^3\,x - 16\,I\,b\,c^3\,n^3\,x + 6\,I\,c^4\,n\,q\,x^2$

$+ 24\,I\,a^3\,d\,n^2\,x + 2\,I\,a^2\,b^2\,n^2\,q + 16\,I\,a^2\,b\,d\,n^3 - 10\,I\,a^2\,d^2\,n^2\,q - 16\,I\,a\,b^2\,c\,n^3$

$+ 16\,I\,a\,c\,d^2\,n^3 - 10\,I\,b^2\,c^2\,n^2\,q - 24\,I\,b\,c^3\,n^2\,x - 16\,I\,b\,c^2\,d\,n^3 + 2\,I\,c^2\,d^2\,n^2\,q$

$- 56\,I\,a^3\,d\,n\,x + 6\,I\,a^2\,b^2\,n\,q + 24\,I\,a^2\,b\,d\,n^2 + 14\,I\,a^2\,d^2\,n\,q - 24\,I\,a\,b^2\,c\,n^2$

$+ 24\,I\,a\,c\,d^2\,n^2 + 14\,I\,b^2\,c^2\,n\,q + 56\,I\,b\,c^3\,n\,x + 2\,I\,a^4\,n^2\,q\,x^2 + 2\,I\,c^4\,n^2\,q\,x^2 + 6\,I\,a^4\,n\,q\,x^2$

$- 48\,I\,p^3\,d\,b\,a^2 + 8\,I\,a^2\,c^2\,q\,x^2 - 36\,I\,a^2\,d^2\,p^2\,q + 48\,I\,a\,b^2\,c\,p^3 - 48\,I\,a\,c\,d^2\,p^3$

$- 36\,I\,b^2\,c^2\,p^2\,q - 104\,I\,b\,c^3\,p^2\,x + 4\,I\,b\,c^3\,q^2\,x + 48\,I\,b\,c^2\,d\,p^3 + 8\,I\,a^3\,b\,q\,x + 24\,I\,a^3\,d\,p\,x$

$- 4\,I\,q\,p\,b^2\,a^2 + 104\,I\,p^2\,d\,b\,a^2 - 4\,I\,a^2\,b\,d\,q^2 - 16\,I\,q\,p\,d^2\,a^2 - 104\,I\,a\,b^2\,c\,p^2$

$+ 4\,I\,a\,b^2\,c\,q^2 + 104\,I\,a\,c\,d^2\,p^2 - 4\,I\,a\,c\,d^2\,q^2 - 16\,I\,b^2\,c^2\,p\,q - 24\,I\,b\,c^3\,p\,x$

$- 104\,I\,b\,c^2\,d\,p^2 + 4\,I\,b\,c^2\,d\,q^2 + 8\,I\,c^3\,d\,q\,x - 4\,I\,c^2\,d^2\,p\,q + 80\,I\,a^2\,b\,c\,x + 24\,I\,p\,d\,b\,a^2$

$- 24\,I\,a\,b^2\,c\,p - 80\,I\,a\,c^2\,d\,x + 24\,I\,a\,c\,d^2\,p - 24\,I\,b\,c^2\,d\,p - 4\,I\,a^4\,p\,q\,x^2 - 48\,I\,a^3\,d\,p^3\,x$

$+ 48\,I\,b\,c^3\,p^3\,x - 4\,I\,c^4\,p\,q\,x^2 + 104\,I\,a^3\,d\,p^2\,x - 4\,I\,a^3\,d\,q^2\,x - 24\,I\,b\,c^2\,d\,n^2$

$+ 4\,I\,a^2\,c^2\,n^2\,q\,x^2 - 2\,I\,c^4\,n\,p\,q\,x^2 - 4\,I\,a^2\,c\,d\,n\,p\,q\,x - 4\,I\,a\,b\,c^2\,n\,p\,q\,x - 80\,I\,a\,b\,c\,d\,n\,p\,q$

$$- 8 I a^2 c d p q x - 8 I a b c^2 p q x + 72 I a b c d p^2 q + 24 I a b c d p q + 12 I a b c^2 n q x$$

$$+ 24 I a b c d n^2 q - 112 I a c^2 d n p x - 16 I a b c d n q - 4 I a^3 b n p q x$$

$$+ 72 I a^2 b c n^2 p x - 104 I a^2 b c n p^2 x + 2 I a^2 b c n q^2 x + 4 I a^2 c d n^2 q x$$

$$+ 4 I a b c^2 n^2 q x - 72 I a c^2 d n^2 p x + 104 I a c^2 d n p^2 x - 2 I a c^2 d n q^2 x$$

$$- 4 I c^3 d n p q x + 112 I a^2 b c n p x + 12 I a^2 c d n q x - 4 I a^2 c^2 n p q x^2) (I q + 2 n - 2 p$$

$$- 2) (I q - 2 n - 4 + 2 p) (I q + 2 n - 2 p) (n - 2 p + 2) (n - p + 1) (I c x + I d$$

$$- a x - b)^2 (a + I c)^2 (I q + 4 n - 6 p + 6)^5 S(n + 2) + \big(2 a^2 x n^2 - 4 a^2 x p n$$

$$+ 2 a^2 p^2 x + 2 c^2 x n^2 - 4 c^2 x p n + 2 c^2 p^2 x + 6 a^2 x n - 6 a^2 x p + 2 a b n^2 - 4 a b p n$$

$$+ 2 a b p^2 - a d p q + b c p q + 6 c^2 x n - 6 c^2 x p + 2 c d n^2 - 4 c d p n + 2 c d p^2$$

$$+ 4 a^2 x + 6 a b n - 6 a b p + 4 c^2 x + 6 c d n - 6 c d p + 4 a b + 4 c d\big) \big($$

$$-144 a b c d n^2 p + 240 a b c d n p^2 - 12 a b c d n q^2 + 20 a b c d p q^2 - 64 a b c d n p$$

$$- 16 a^2 b d q + 64 a^2 c d x + 16 a b^2 c q + 64 a b c^2 x - 16 a c d^2 q + 16 b c^2 d q$$

$$- 116 a^4 n p x^2 + 24 a^3 b n^3 x - 32 a^3 b p^3 x + 104 a^2 c^2 n^2 x^2 + 128 a^2 c^2 p^2 x^2$$

$$- 116 c^4 n p x^2 + 24 c^3 d n^3 x - 32 c^3 d p^3 x + 104 a^3 b n^2 x + 128 a^3 b p^2 x - 40 a^2 b^2 n^2 p$$

$$+ 44 a^2 b^2 n p^2 + 144 a^2 c^2 n x^2 - 160 a^2 c^2 p x^2 + 104 c^3 d n^2 x + 128 c^3 d p^2 x$$

$$- 40 c^2 d^2 n^2 p + 44 c^2 d^2 n p^2 + 144 a^3 b n x - 160 a^3 b p x - 16 a^3 d q x - 116 a^2 b^2 n p$$

$$+ 16 b c^3 q x + 144 c^3 d n x - 160 c^3 d p x - 116 c^2 d^2 n p + 32 a b c d n^3 - 144 a b c d p^3$$

$$+ 48 a b c d n^2 + 16 a b c d p^2 + 80 a b c d n - 32 a^2 d^2 - 32 b^2 c^2 - 4 a^2 d^2 n^3$$

$$+ 56 a^2 d^2 p^3 - 4 b^2 c^2 n^3 + 56 b^2 c^2 p^3 + 28 a^2 d^2 n^2 + 56 a^2 d^2 p^2 + 28 b^2 c^2 n^2$$

$$+ 56 b^2 c^2 p^2 + 32 a^2 d^2 n - 80 a^2 d^2 p + 32 b^2 c^2 n - 80 b^2 c^2 p + 32 a^4 x^2 + 32 c^4 x^2$$

$$+ 32 a^2 b^2 + 32 c^2 d^2 + 32 a^2 d^2 n^2 p - 76 a^2 d^2 n p^2 + 6 a^2 d^2 n q^2 - 10 a^2 d^2 p q^2$$

$$+ 32 b^2 c^2 n^2 p - 76 b^2 c^2 n p^2 + 6 b^2 c^2 n q^2 - 10 b^2 c^2 p q^2 - 84 a^2 d^2 n p - 84 b^2 c^2 n p$$

$$+ 128 a b c d - 24 a^2 b c n p q x + 24 a c^2 d n p q x + 24 a^3 d n p q x + 12 a^2 b c n^2 q x$$

$$+ 8 a^2 b c p^2 q x - 80 a^2 c d n^2 p x + 88 a^2 c d n p^2 x - 80 a b c^2 n^2 p x + 88 a b c^2 n p^2 x$$

$$- 12 a c^2 d n^2 q x - 8 a c^2 d p^2 q x - 24 b c^3 n p q x + 32 a^2 b c n q x - 44 a^2 b c p q x$$

$$+ 24 a^2 b d n p q - 232 a^2 c d n p x - 24 a b^2 c n p q - 232 a b c^2 n p x - 32 a c^2 d n q x$$

$$+ 44 a c^2 d p q x + 24 a c d^2 n p q - 24 b c^2 d n p q - 40 a^4 n^2 p x^2 + 44 a^4 n p^2 x^2$$

$$+ 24 a^2 c^2 n^3 x^2 - 32 a^2 c^2 p^3 x^2 - 40 c^4 n^2 p x^2 + 44 c^4 n p^2 x^2 - 12 a c d^2 n^2 q$$

$$- 8 a c d^2 p^2 q + 32 b c^3 n q x - 44 b c^3 p q x + 12 b c^2 d n^2 q + 8 b c^2 d p^2 q$$

$$- 232 c^3 d n p x + 16 a^2 b c q x - 32 a^2 b d n q + 44 a^2 b d p q + 144 a^2 c d n x$$

$$- 160 a^2 c d p x + 32 a b^2 c n q - 44 a b^2 c p q + 144 a b c^2 n x - 160 a b c^2 p x$$

$$- 16 a c^2 d q x - 32 a c d^2 n q + 44 a c d^2 p q + 32 b c^2 d n q - 44 b c^2 d p q + 8 b c^3 p^2 q x$$

$$- 80 c^3 d n^2 p x + 88 c^3 d n p^2 x - 232 a^3 b n p x - 32 a^3 d n q x + 44 a^3 d p q x$$

$$- 12 a^2 b d n^2 q - 8 a^2 b d p^2 q + 104 a^2 c d n^2 x + 128 a^2 c d p^2 x + 12 a b^2 c n^2 q$$

$$+ 8 a b^2 c p^2 q + 104 a b c^2 n^2 x + 128 a b c^2 p^2 x - 80 a^3 b n^2 p x + 88 a^3 b n p^2 x$$

$$- 12 a^3 d n^2 q x - 8 a^3 d p^2 q x - 232 a^2 c^2 n p x^2 + 24 a^2 c d n^3 x - 32 a^2 c d p^3 x$$

$$+ 24\, a\, b\, c^2\, n^3\, x - 32\, a\, b\, c^2\, p^3\, x + 12\, b\, c^3\, n^2\, q\, x - 80\, a^2\, c^2\, n^2\, p\, x^2 + 88\, a^2\, c^2\, n\, p^2\, x^2$$

$$+ 12\, a^4\, n^3\, x^2 - 16\, a^4\, p^3\, x^2 + 12\, c^4\, n^3\, x^2 - 16\, c^4\, p^3\, x^2 + 52\, a^4\, n^2\, x^2 + 64\, a^4\, p^2\, x^2$$

$$+ 52\, c^4\, n^2\, x^2 + 64\, c^4\, p^2\, x^2 + 72\, a^4\, n\, x^2 - 80\, a^4\, p\, x^2 + 12\, a^2\, b^2\, n^3 - 16\, a^2\, b^2\, p^3 + 72\, c^4\, n\, x^2$$

$$- 80\, c^4\, p\, x^2 + 12\, c^2\, d^2\, n^3 - 16\, c^2\, d^2\, p^3 + 52\, a^2\, b^2\, n^2 + 64\, a^2\, b^2\, p^2 + 64\, a^2\, c^2\, x^2$$

$$+ 52\, c^2\, d^2\, n^2 + 64\, c^2\, d^2\, p^2 + 64\, a^3\, b\, x + 72\, a^2\, b^2\, n - 80\, a^2\, b^2\, p + 64\, c^3\, d\, x + 72\, c^2\, d^2\, n$$

$$- 80\, c^2\, d^2\, p + 104\, \mathrm{I}\, a^2\, b\, d\, n\, p^2 - 2\, \mathrm{I}\, a^2\, b\, d\, n\, q^2 + 38\, \mathrm{I}\, a^2\, d^2\, n\, p\, q + 72\, \mathrm{I}\, a\, b^2\, c\, n^2\, p$$

$$- 104\, \mathrm{I}\, a\, b^2\, c\, n\, p^2 + 2\, \mathrm{I}\, a\, b^2\, c\, n\, q^2 + 24\, \mathrm{I}\, a\, c^2\, d\, n^2\, x - 72\, \mathrm{I}\, a\, c\, d^2\, n^2\, p + 104\, \mathrm{I}\, a\, c\, d^2\, n\, p^2$$

$$- 2\, \mathrm{I}\, a\, c\, d^2\, n\, q^2 + 38\, \mathrm{I}\, b^2\, c^2\, n\, p\, q + 112\, \mathrm{I}\, b\, c^3\, n\, p\, x + 72\, \mathrm{I}\, b\, c^2\, d\, n^2\, p - 104\, \mathrm{I}\, b\, c^2\, d\, n\, p^2$$

$$+ 2\, \mathrm{I}\, b\, c^2\, d\, n\, q^2 + 12\, \mathrm{I}\, c^3\, d\, n\, q\, x - 2\, \mathrm{I}\, c^2\, d^2\, n\, p\, q + 56\, \mathrm{I}\, a^2\, b\, c\, n\, x - 112\, \mathrm{I}\, a^2\, b\, d\, n\, p$$

$$+ 112\, \mathrm{I}\, a\, b^2\, c\, n\, p - 56\, \mathrm{I}\, a\, c^2\, d\, n\, x - 112\, \mathrm{I}\, a\, c\, d^2\, n\, p + 112\, \mathrm{I}\, b\, c^2\, d\, n\, p + 48\, \mathrm{I}\, a^2\, b\, c\, p^3\, x$$

$$- 8\, \mathrm{I}\, a^2\, c^2\, p\, q\, x^2 - 48\, \mathrm{I}\, a\, c^2\, d\, p^3\, x - 8\, \mathrm{I}\, a^3\, b\, p\, q\, x - 104\, \mathrm{I}\, a^2\, b\, c\, p^2\, x + 4\, \mathrm{I}\, a^2\, b\, c\, q^2\, x$$

$$- 2\, \mathrm{I}\, a\, b\, c\, d\, q^3 + 104\, \mathrm{I}\, a\, c^2\, d\, p^2\, x - 4\, \mathrm{I}\, a\, c^2\, d\, q^2\, x - 8\, \mathrm{I}\, c^3\, d\, p\, q\, x - 24\, \mathrm{I}\, a^2\, b\, c\, p\, x$$

$$+ 8\, \mathrm{I}\, a^2\, c\, d\, q\, x + 8\, \mathrm{I}\, a\, b\, c^2\, q\, x + 24\, \mathrm{I}\, a\, c^2\, d\, p\, x - 40\, \mathrm{I}\, a\, b\, c\, d\, q - 2\, \mathrm{I}\, a^4\, n\, p\, q\, x^2$$

$$+ 4\, \mathrm{I}\, a^3\, b\, n^2\, q\, x - 72\, \mathrm{I}\, a^3\, d\, n^2\, p\, x + 104\, \mathrm{I}\, a^3\, d\, n\, p^2\, x - 2\, \mathrm{I}\, a^3\, d\, n\, q^2\, x - 16\, \mathrm{I}\, a^2\, b\, c\, n^3\, x$$

$$+ 12\, \mathrm{I}\, a^2\, c^2\, n\, q\, x^2 + 16\, \mathrm{I}\, a\, c^2\, d\, n^3\, x + 72\, \mathrm{I}\, b\, c^3\, n^2\, p\, x - 104\, \mathrm{I}\, b\, c^3\, n\, p^2\, x + 2\, \mathrm{I}\, b\, c^3\, n\, q^2\, x$$

$$+ 4\, \mathrm{I}\, c^3\, d\, n^2\, q\, x + 12\, \mathrm{I}\, a^3\, b\, n\, q\, x - 112\, \mathrm{I}\, a^3\, d\, n\, p\, x - 2\, \mathrm{I}\, a^2\, b^2\, n\, p\, q - 24\, \mathrm{I}\, a^2\, b\, c\, n^2\, x$$

$$- 72\, \mathrm{I}\, a^2\, b\, d\, n^2\, p + \mathrm{I}\, a^2\, d^2\, q^3 + \mathrm{I}\, b^2\, c^2\, q^3 + 4\, \mathrm{I}\, a^4\, q\, x^2 + 4\, \mathrm{I}\, c^4\, q\, x^2 - 80\, \mathrm{I}\, a^3\, d\, x + 4\, \mathrm{I}\, q\, b^2\, a^2$$

$$+ 24\, \mathrm{I}\, q\, d^2\, a^2 + 24\, \mathrm{I}\, b^2\, c^2\, q + 80\, \mathrm{I}\, b\, c^3\, x + 4\, \mathrm{I}\, c^2\, d^2\, q - 80\, \mathrm{I}\, d\, b\, a^2 + 80\, \mathrm{I}\, a\, b^2\, c$$

$$- 80\, I\, a\, c\, d^2 + 80\, I\, b\, c^2\, d + 6\, I\, c^2\, d^2\, n\, q - 56\, I\, a^2\, b\, d\, n + 56\, I\, a\, b^2\, c\, n - 56\, I\, a\, c\, d^2\, n$$

$$+ 56\, I\, b\, c^2\, d\, n + 16\, I\, a^3\, d\, n^3\, x - 16\, I\, b\, c^3\, n^3\, x + 6\, I\, c^4\, n\, q\, x^2 + 24\, I\, a^3\, d\, n^2\, x + 2\, I\, a^2\, b^2\, n^2\, q$$

$$+ 16\, I\, a^2\, b\, d\, n^3 - 10\, I\, a^2\, d^2\, n^2\, q - 16\, I\, a\, b^2\, c\, n^3 + 16\, I\, a\, c\, d^2\, n^3 - 10\, I\, b^2\, c^2\, n^2\, q$$

$$- 24\, I\, b\, c^3\, n^2\, x - 16\, I\, b\, c^2\, d\, n^3 + 2\, I\, c^2\, d^2\, n^2\, q - 56\, I\, a^3\, d\, n\, x + 6\, I\, a^2\, b^2\, n\, q + 24\, I\, a^2\, b\, d\, n^2$$

$$+ 14\, I\, a^2\, d^2\, n\, q - 24\, I\, a\, b^2\, c\, n^2 + 24\, I\, a\, c\, d^2\, n^2 + 14\, I\, b^2\, c^2\, n\, q + 56\, I\, b\, c^3\, n\, x$$

$$+ 2\, I\, a^4\, n^2\, q\, x^2 + 2\, I\, c^4\, n^2\, q\, x^2 + 6\, I\, a^4\, n\, q\, x^2 - 48\, I\, p^3\, d\, b\, a^2 + 8\, I\, a^2\, c^2\, q\, x^2$$

$$- 36\, I\, a^2\, d^2\, p^2\, q + 48\, I\, a\, b^2\, c\, p^3 - 48\, I\, a\, c\, d^2\, p^3 - 36\, I\, b^2\, c^2\, p^2\, q - 104\, I\, b\, c^3\, p^2\, x$$

$$+ 4\, I\, b\, c^3\, q^2\, x + 48\, I\, b\, c^2\, d\, p^3 + 8\, I\, a^3\, b\, q\, x + 24\, I\, a^3\, d\, p\, x - 4\, I\, q\, p\, b^2\, a^2 + 104\, I\, p^2\, d\, b\, a^2$$

$$- 4\, I\, a^2\, b\, d\, q^2 - 16\, I\, q\, p\, d^2\, a^2 - 104\, I\, a\, b^2\, c\, p^2 + 4\, I\, a\, b^2\, c\, q^2 + 104\, I\, a\, c\, d^2\, p^2$$

$$- 4\, I\, a\, c\, d^2\, q^2 - 16\, I\, b^2\, c^2\, p\, q - 24\, I\, b\, c^3\, p\, x - 104\, I\, b\, c^2\, d\, p^2 + 4\, I\, b\, c^2\, d\, q^2 + 8\, I\, c^3\, d\, q\, x$$

$$- 4\, I\, c^2\, d^2\, p\, q + 80\, I\, a^2\, b\, c\, x + 24\, I\, p\, d\, b\, a^2 - 24\, I\, a\, b^2\, c\, p - 80\, I\, a\, c^2\, d\, x + 24\, I\, a\, c\, d^2\, p$$

$$- 24\, I\, b\, c^2\, d\, p - 4\, I\, a^4\, p\, q\, x^2 - 48\, I\, a^3\, d\, p^3\, x + 48\, I\, b\, c^3\, p^3\, x - 4\, I\, c^4\, p\, q\, x^2 + 104\, I\, a^3\, d\, p^2\, x$$

$$- 4\, I\, a^3\, d\, q^2\, x - 24\, I\, b\, c^2\, d\, n^2 + 4\, I\, a^2\, c^2\, n^2\, q\, x^2 - 2\, I\, c^4\, n\, p\, q\, x^2 - 4\, I\, a^2\, c\, d\, n\, p\, q\, x$$

$$- 4\, I\, a\, b\, c^2\, n\, p\, q\, x - 80\, I\, a\, b\, c\, d\, n\, p\, q - 8\, I\, a^2\, c\, d\, p\, q\, x - 8\, I\, a\, b\, c^2\, p\, q\, x + 72\, I\, a\, b\, c\, d\, p^2\, q$$

$$+ 24\, I\, a\, b\, c\, d\, p\, q + 12\, I\, a\, b\, c^2\, n\, q\, x + 24\, I\, a\, b\, c\, d\, n^2\, q - 112\, I\, a\, c^2\, d\, n\, p\, x - 16\, I\, a\, b\, c\, d\, n\, q$$

$$- 4\, I\, a^3\, b\, n\, p\, q\, x + 72\, I\, a^2\, b\, c\, n^2\, p\, x - 104\, I\, a^2\, b\, c\, n\, p^2\, x + 2\, I\, a^2\, b\, c\, n\, q^2\, x$$

$$+ 4\, I\, a^2\, c\, d\, n^2\, q\, x + 4\, I\, a\, b\, c^2\, n^2\, q\, x - 72\, I\, a\, c^2\, d\, n^2\, p\, x + 104\, I\, a\, c^2\, d\, n\, p^2\, x$$

$$- 2\, I\, a\, c^2\, d\, n\, q^2\, x - 4\, I\, c^3\, d\, n\, p\, q\, x + 112\, I\, a^2\, b\, c\, n\, p\, x + 12\, I\, a^2\, c\, d\, n\, q\, x$$

$$- 4 \mathrm{I} a^2 c^2 n p q x^2) (\mathrm{I} q - 2 n - 4 + 2 p) (\mathrm{I} q + 2 n - 2 p) (\mathrm{I} q + 2 n - 2 p - 2) (2 n$$

$$- 2 p + 3) (\mathrm{I} c x + \mathrm{I} d - a x - b)^2 (a + \mathrm{I} c)^2 (\mathrm{I} q + 4 n - 6 p + 6)^5 S(n + 1) + ($$

$$-144 a b c d n^2 p + 240 a b c d n p^2 - 12 a b c d n q^2 + 20 a b c d p q^2 - 64 a b c d n p$$

$$- 16 a^2 b d q + 64 a^2 c d x + 16 a b^2 c q + 64 a b c^2 x - 16 a c d^2 q + 16 b c^2 d q$$

$$- 116 a^4 n p x^2 + 24 a^3 b n^3 x - 32 a^3 b p^3 x + 104 a^2 c^2 n^2 x^2 + 128 a^2 c^2 p^2 x^2$$

$$- 116 c^4 n p x^2 + 24 c^3 d n^3 x - 32 c^3 d p^3 x + 104 a^3 b n^2 x + 128 a^3 b p^2 x - 40 a^2 b^2 n^2 p$$

$$+ 44 a^2 b^2 n p^2 + 144 a^2 c^2 n x^2 - 160 a^2 c^2 p x^2 + 104 c^3 d n^2 x + 128 c^3 d p^2 x$$

$$- 40 c^2 d^2 n^2 p + 44 c^2 d^2 n p^2 + 144 a^3 b n x - 160 a^3 b p x - 16 a^3 d q x - 116 a^2 b^2 n p$$

$$+ 16 b c^3 q x + 144 c^3 d n x - 160 c^3 d p x - 116 c^2 d^2 n p + 32 a b c d n^3 - 144 a b c d p^3$$

$$+ 48 a b c d n^2 + 16 a b c d p^2 + 80 a b c d n - 32 a^2 d^2 - 32 b^2 c^2 - 4 a^2 d^2 n^3$$

$$+ 56 a^2 d^2 p^3 - 4 b^2 c^2 n^3 + 56 b^2 c^2 p^3 + 28 a^2 d^2 n^2 + 56 a^2 d^2 p^2 + 28 b^2 c^2 n^2$$

$$+ 56 b^2 c^2 p^2 + 32 a^2 d^2 n - 80 a^2 d^2 p + 32 b^2 c^2 n - 80 b^2 c^2 p + 32 a^4 x^2 + 32 c^4 x^2$$

$$+ 32 a^2 b^2 + 32 c^2 d^2 + 32 a^2 d^2 n^2 p - 76 a^2 d^2 n p^2 + 6 a^2 d^2 n q^2 - 10 a^2 d^2 p q^2$$

$$+ 32 b^2 c^2 n^2 p - 76 b^2 c^2 n p^2 + 6 b^2 c^2 n q^2 - 10 b^2 c^2 p q^2 - 84 a^2 d^2 n p - 84 b^2 c^2 n p$$

$$+ 128 a b c d - 24 a^2 b c n p q x + 24 a c^2 d n p q x + 24 a^3 d n p q x + 12 a^2 b c n^2 q x$$

$$+ 8 a^2 b c p^2 q x - 80 a^2 c d n^2 p x + 88 a^2 c d n p^2 x - 80 a b c^2 n^2 p x + 88 a b c^2 n p^2 x$$

$$- 12 a c^2 d n^2 q x - 8 a c^2 d p^2 q x - 24 b c^3 n p q x + 32 a^2 b c n q x - 44 a^2 b c p q x$$

$$+ 24 a^2 b d n p q - 232 a^2 c d n p x - 24 a b^2 c n p q - 232 a b c^2 n p x - 32 a c^2 d n q x$$

$+ 44\, a\, c^2\, d\, p\, q\, x + 24\, a\, c\, d^2\, n\, p\, q - 24\, b\, c^2\, d\, n\, p\, q - 40\, a^4\, n^2\, p\, x^2 + 44\, a^4\, n\, p^2\, x^2$

$+ 24\, a^2\, c^2\, n^3\, x^2 - 32\, a^2\, c^2\, p^3\, x^2 - 40\, c^4\, n^2\, p\, x^2 + 44\, c^4\, n\, p^2\, x^2 - 12\, a\, c\, d^2\, n^2\, q$

$- 8\, a\, c\, d^2\, p^2\, q + 32\, b\, c^3\, n\, q\, x - 44\, b\, c^3\, p\, q\, x + 12\, b\, c^2\, d\, n^2\, q + 8\, b\, c^2\, d\, p^2\, q$

$- 232\, c^3\, d\, n\, p\, x + 16\, a^2\, b\, c\, q\, x - 32\, a^2\, b\, d\, n\, q + 44\, a^2\, b\, d\, p\, q + 144\, a^2\, c\, d\, n\, x$

$- 160\, a^2\, c\, d\, p\, x + 32\, a\, b^2\, c\, n\, q - 44\, a\, b^2\, c\, p\, q + 144\, a\, b\, c^2\, n\, x - 160\, a\, b\, c^2\, p\, x$

$- 16\, a\, c^2\, d\, q\, x - 32\, a\, c\, d^2\, n\, q + 44\, a\, c\, d^2\, p\, q + 32\, b\, c^2\, d\, n\, q - 44\, b\, c^2\, d\, p\, q + 8\, b\, c^3\, p^2\, q\, x$

$- 80\, c^3\, d\, n^2\, p\, x + 88\, c^3\, d\, n\, p^2\, x - 232\, a^3\, b\, n\, p\, x - 32\, a^3\, d\, n\, q\, x + 44\, a^3\, d\, p\, q\, x$

$- 12\, a^2\, b\, d\, n^2\, q - 8\, a^2\, b\, d\, p^2\, q + 104\, a^2\, c\, d\, n^2\, x + 128\, a^2\, c\, d\, p^2\, x + 12\, a\, b^2\, c\, n^2\, q$

$+ 8\, a\, b^2\, c\, p^2\, q + 104\, a\, b\, c^2\, n^2\, x + 128\, a\, b\, c^2\, p^2\, x - 80\, a^3\, b\, n^2\, p\, x + 88\, a^3\, b\, n\, p^2\, x$

$- 12\, a^3\, d\, n^2\, q\, x - 8\, a^3\, d\, p^2\, q\, x - 232\, a^2\, c^2\, n\, p\, x^2 + 24\, a^2\, c\, d\, n^3\, x - 32\, a^2\, c\, d\, p^3\, x$

$+ 24\, a\, b\, c^2\, n^3\, x - 32\, a\, b\, c^2\, p^3\, x + 12\, b\, c^3\, n^2\, q\, x - 80\, a^2\, c^2\, n^2\, p\, x^2 + 88\, a^2\, c^2\, n\, p^2\, x^2$

$+ 12\, a^4\, n^3\, x^2 - 16\, a^4\, p^3\, x^2 + 12\, c^4\, n^3\, x^2 - 16\, c^4\, p^3\, x^2 + 52\, a^4\, n^2\, x^2 + 64\, a^4\, p^2\, x^2$

$+ 52\, c^4\, n^2\, x^2 + 64\, c^4\, p^2\, x^2 + 72\, a^4\, n\, x^2 - 80\, a^4\, p\, x^2 + 12\, a^2\, b^2\, n^3 - 16\, a^2\, b^2\, p^3 + 72\, c^4\, n\, x^2$

$- 80\, c^4\, p\, x^2 + 12\, c^2\, d^2\, n^3 - 16\, c^2\, d^2\, p^3 + 52\, a^2\, b^2\, n^2 + 64\, a^2\, b^2\, p^2 + 64\, a^2\, c^2\, x^2$

$+ 52\, c^2\, d^2\, n^2 + 64\, c^2\, d^2\, p^2 + 64\, a^3\, b\, x + 72\, a^2\, b^2\, n - 80\, a^2\, b^2\, p + 64\, c^3\, d\, x + 72\, c^2\, d^2\, n$

$- 80\, c^2\, d^2\, p + 104\, \mathrm{I}\, a^2\, b\, d\, n\, p^2 - 2\, \mathrm{I}\, a^2\, b\, d\, n\, q^2 + 38\, \mathrm{I}\, a^2\, d^2\, n\, p\, q + 72\, \mathrm{I}\, a\, b^2\, c\, n^2\, p$

$- 104\, \mathrm{I}\, a\, b^2\, c\, n\, p^2 + 2\, \mathrm{I}\, a\, b^2\, c\, n\, q^2 + 24\, \mathrm{I}\, a\, c^2\, d\, n^2\, x - 72\, \mathrm{I}\, a\, c\, d^2\, n^2\, p + 104\, \mathrm{I}\, a\, c\, d^2\, n\, p^2$

$- 2\, \mathrm{I}\, a\, c\, d^2\, n\, q^2 + 38\, \mathrm{I}\, b^2\, c^2\, n\, p\, q + 112\, \mathrm{I}\, b\, c^3\, n\, p\, x + 72\, \mathrm{I}\, b\, c^2\, d\, n^2\, p - 104\, \mathrm{I}\, b\, c^2\, d\, n\, p^2$

$$+ 2 I b c^2 d n q^2 + 12 I c^3 d n q x - 2 I c^2 d^2 n p q + 56 I a^2 b c n x - 112 I a^2 b d n p$$

$$+ 112 I a b^2 c n p - 56 I a c^2 d n x - 112 I a c d^2 n p + 112 I b c^2 d n p + 48 I a^2 b c p^3 x$$

$$- 8 I a^2 c^2 p q x^2 - 48 I a c^2 d p^3 x - 8 I a^3 b p q x - 104 I a^2 b c p^2 x + 4 I a^2 b c q^2 x$$

$$- 2 I a b c d q^3 + 104 I a c^2 d p^2 x - 4 I a c^2 d q^2 x - 8 I c^3 d p q x - 24 I a^2 b c p x$$

$$+ 8 I a^2 c d q x + 8 I a b c^2 q x + 24 I a c^2 d p x - 40 I a b c d q - 2 I a^4 n p q x^2$$

$$+ 4 I a^3 b n^2 q x - 72 I a^3 d n^2 p x + 104 I a^3 d n p^2 x - 2 I a^3 d n q^2 x - 16 I a^2 b c n^3 x$$

$$+ 12 I a^2 c^2 n q x^2 + 16 I a c^2 d n^3 x + 72 I b c^3 n^2 p x - 104 I b c^3 n p^2 x + 2 I b c^3 n q^2 x$$

$$+ 4 I c^3 d n^2 q x + 12 I a^3 b n q x - 112 I a^3 d n p x - 2 I a^2 b^2 n p q - 24 I a^2 b c n^2 x$$

$$- 72 I a^2 b d n^2 p + I a^2 d^2 q^3 + I b^2 c^2 q^3 + 4 I a^4 q x^2 + 4 I c^4 q x^2 - 80 I a^3 d x + 4 I q b^2 a^2$$

$$+ 24 I q d^2 a^2 + 24 I b^2 c^2 q + 80 I b c^3 x + 4 I c^2 d^2 q - 80 I d b a^2 + 80 I a b^2 c$$

$$- 80 I a c d^2 + 80 I b c^2 d + 6 I c^2 d^2 n q - 56 I a^2 b d n + 56 I a b^2 c n - 56 I a c d^2 n$$

$$+ 56 I b c^2 d n + 16 I a^3 d n^3 x - 16 I b c^3 n^3 x + 6 I c^4 n q x^2 + 24 I a^3 d n^2 x + 2 I a^2 b^2 n^2 q$$

$$+ 16 I a^2 b d n^3 - 10 I a^2 d^2 n^2 q - 16 I a b^2 c n^3 + 16 I a c d^2 n^3 - 10 I b^2 c^2 n^2 q$$

$$- 24 I b c^3 n^2 x - 16 I b c^2 d n^3 + 2 I c^2 d^2 n^2 q - 56 I a^3 d n x + 6 I a^2 b^2 n q + 24 I a^2 b d n^2$$

$$+ 14 I a^2 d^2 n q - 24 I a b^2 c n^2 + 24 I a c d^2 n^2 + 14 I b^2 c^2 n q + 56 I b c^3 n x$$

$$+ 2 I a^4 n^2 q x^2 + 2 I c^4 n^2 q x^2 + 6 I a^4 n q x^2 - 48 I p^3 d b a^2 + 8 I a^2 c^2 q x^2$$

$$- 36 I a^2 d^2 p^2 q + 48 I a b^2 c p^3 - 48 I a c d^2 p^3 - 36 I b^2 c^2 p^2 q - 104 I b c^3 p^2 x$$

$$+ 4 I b c^3 q^2 x + 48 I b c^2 d p^3 + 8 I a^3 b q x + 24 I a^3 d p x - 4 I q p b^2 a^2 + 104 I p^2 d b a^2$$

$$- 4 I a^2 b d q^2 - 16 I q p d^2 a^2 - 104 I a b^2 c p^2 + 4 I a b^2 c q^2 + 104 I a c d^2 p^2$$

$$- 4 I a c d^2 q^2 - 16 I b^2 c^2 p q - 24 I b c^3 p x - 104 I b c^2 d p^2 + 4 I b c^2 d q^2 + 8 I c^3 d q x$$

$$- 4 I c^2 d^2 p q + 80 I a^2 b c x + 24 I p d b a^2 - 24 I a b^2 c p - 80 I a c^2 d x + 24 I a c d^2 p$$

$$- 24 I b c^2 d p - 4 I a^4 p q x^2 - 48 I a^3 d p^3 x + 48 I b c^3 p^3 x - 4 I c^4 p q x^2 + 104 I a^3 d p^2 x$$

$$- 4 I a^3 d q^2 x - 24 I b c^2 d n^2 + 4 I a^2 c^2 n^2 q x^2 - 2 I c^4 n p q x^2 - 4 I a^2 c d n p q x$$

$$- 4 I a b c^2 n p q x - 80 I a b c d n p q - 8 I a^2 c d p q x - 8 I a b c^2 p q x + 72 I a b c d p^2 q$$

$$+ 24 I a b c d p q + 12 I a b c^2 n q x + 24 I a b c d n^2 q - 112 I a c^2 d n p x - 16 I a b c d n q$$

$$- 4 I a^3 b n p q x + 72 I a^2 b c n^2 p x - 104 I a^2 b c n p^2 x + 2 I a^2 b c n q^2 x$$

$$+ 4 I a^2 c d n^2 q x + 4 I a b c^2 n^2 q x - 72 I a c^2 d n^2 p x + 104 I a c^2 d n p^2 x$$

$$- 2 I a c^2 d n q^2 x - 4 I c^3 d n p q x + 112 I a^2 b c n p x + 12 I a^2 c d n q x$$

$$- 4 I a^2 c^2 n p q x^2) (I q - 2 n - 4 + 2 p) (I q + 2 n - 2 p - 2) (I q + 2 n - 2 p) (I q$$

$$- 2 n + 2 p - 2) (I q + 2 n - 2 p + 2) (-p + 2 + n) (n + 1) (I c x + I d - a x$$

$$- b)^2 (a d - b c)^2 (a + I c)^2 (I q + 4 n - 6 p + 6)^5 S(n) = 0, 1,$$

$$- \frac{1}{2 (-1 + p) \Gamma(-2 p + 2)} (\Gamma(3 - 2 p) (2 a^2 x p + 2 c^2 x p - 2 a^2 x + 2 a b p - q a d$$

$$+ q b c - 2 c^2 x + 2 c d p - 2 a b - 2 c d)) \Big]$$

```
> solJ6:=solve(REMJ6[1], S(n+2)):
> normal(solJ1-solJ6);
```

$$0 \qquad (195)$$

```
> simplify(REMJ1[2]-REMJ6[2]);
```

$$0 \qquad (196)$$

```
> simplify(REMJ1[3]-REMJ6[3]);
```

$$0 \qquad (197)$$