```
> restart;
> read "hsum19.mpl";
```

Package "Hypergeometric Summation", Maple V - Maple 2019

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```
> RecurrenceNormalForm2:=proc(P,k,pn)
  local p,n;
  p:=op(0,pn);
  n:=op(1,pn);
  [sumrecursion(P,k,p(n)),normal(subs[eval]({k=0,n=0},P)),normal
  (subs[eval]({k=0,n=1},P)+subs[eval]({k=1,n=1},P))]
  end proc:
```

Charlier

```
> charlierterm1:=hyperterm([-n,-x],[],-1/a,k);
```

$$charlierterm1 := \frac{\text{pochhammer}(-n,k)\,\text{pochhammer}(-x,k)\left(-\frac{1}{a}\right)^k}{k!} \tag{2}$$

```
> charlierterm2:=(-1)^n/a^n*pochhammer(x-n+1,n)*hyperterm([-n],[x-
  n+1],a,k);
```

$$charlierterm2 := \frac{(-1)^n\,\text{pochhammer}(x-n+1,n)\,\text{pochhammer}(-n,k)\,a^k}{a^n\,\text{pochhammer}(x-n+1,k)\,k!} \tag{3}$$

```
> RE1:=RecurrenceNormalForm2(charlierterm1,k,p(n));
```

$$RE1 := \left[a\,p(2+n) - (a+n-x+1)\,p(n+1) + (n+1)\,p(n) = 0,\ 1,\ \frac{a-x}{a}\right] \tag{4}$$

```
> RE2:=RecurrenceNormalForm2(charlierterm2,k,p(n));
```

$$RE2 := \left[a\,p(2+n) - (a+n-x+1)\,p(n+1) + (n+1)\,p(n) = 0,\ 1,\ \frac{a-x}{a}\right] \tag{5}$$

Reversion from right to left

```
> Sumtohyper(subs(k=n-k,charlierterm2),k)
```

$$\left((-1)^n\right)^2 \text{Hypergeom}\left([-x,-n],[\,],-\frac{1}{a}\right) \tag{6}$$

Meixner (11)

```
> meixnerterm1:=hyperterm([-n,-x],[beta],1-1/c,k);
```

$$meixnerterm1 := \frac{\text{pochhammer}(-n,k)\,\text{pochhammer}(-x,k)\left(1-\frac{1}{c}\right)^k}{\text{pochhammer}(\beta,k)\,k!} \tag{7}$$

```
> RE1:=RecurrenceNormalForm2(meixnerterm1,k,p(n));
```

$$RE1 := \left[c\,(\beta+n+1)\,p(2+n) - (\beta c + c\,n + x\,c + c + n - x + 1)\,p(n+1) + (n+1)\,p(n) = 0,\ 1,\ \frac{\beta c + x\,c - x}{\beta c}\right] \tag{8}$$

Reversion (12)

```
> Sumtohyper(subs(k=n-k,meixnerterm1),k)
```

$$\frac{(-1)^n\,\Gamma(n-x)\,\Gamma(\beta)\left(\frac{c-1}{c}\right)^n \text{Hypergeom}\left([1-n-\beta,\,-n],\,[x-n+1],\,\frac{c}{c-1}\right)}{\Gamma(-x)\,\Gamma(n+\beta)} \tag{9}$$

```
> termtohyper((-1)^n*GAMMA(-x + n)*GAMMA(beta)*((c - 1)/c)^n/(GAMMA
  (-x)*GAMMA(n + beta)),n);
```

$$\frac{\text{pochhammer}(-x,n)\left(\frac{-c+1}{c}\right)^n}{\text{pochhammer}(\beta,n)} \tag{10}$$

```
> meixnerterm2:=pochhammer(-x, n)*((-c + 1)/c)^n/pochhammer(beta,
  n)*hyperterm([1-n-beta,-n],[x-n+1],c/(c-1),k)
```

$$meixnerterm2 := \left(\text{pochhammer}(-x,n)\left(\frac{-c+1}{c}\right)^n \text{pochhammer}(1-n-\beta,\,k)\,\text{pochhammer}(-n,k)\left(\frac{c}{c-1}\right)^k\right) \Big/ \left(\text{pochhammer}(\beta,n)\,\text{pochhammer}(x-n+1,k)\,k!\right) \tag{11}$$

```
> RE2:=RecurrenceNormalForm2(meixnerterm2,k,p(n));
```

$$RE2 := \left[c\,(\beta+n+1)\,p(2+n) - (\beta c + c\,n + x\,c + c + n - x + 1)\,p(n+1) + (n+1)\,p(n) = 0,\,1,\,\frac{\beta c + x\,c - x}{\beta c}\right] \tag{12}$$

```
> RE1-RE2;
```

$$[0,\,0,\,0] \tag{13}$$

(13)

```
> meixnerterm3:=(1/c)^n*hyperterm([-n,x+beta],[beta],1-c,k);
```

$$meixnerterm3 := \frac{\left(\frac{1}{c}\right)^n \text{pochhammer}(-n,k)\,\text{pochhammer}(x+\beta,k)\,(-c+1)^k}{\text{pochhammer}(\beta,k)\,k!} \tag{14}$$

```
> RE3:=RecurrenceNormalForm2(meixnerterm3,k,p(n));
```

$$RE3 := \left[c\,(\beta+n+1)\,p(2+n) - (\beta c + c\,n + x\,c + c + n - x + 1)\,p(n+1) + (n+1)\,p(n) = 0,\,1,\,\frac{\beta c + x\,c - x}{\beta c}\right] \tag{15}$$

```
> RE1-RE3;
```

$$[0,\,0,\,0] \tag{16}$$

Reversion (16)

```
> Sumtohyper(subs(k=n-k,meixnerterm3),k)
```

$$\frac{1}{c^n\,\Gamma(x+\beta)\,\Gamma(n+\beta)}\left((-1)^n\,\Gamma(n+\beta+x)\,\Gamma(\beta)\,(-c+1)^n \text{Hypergeom}\left([1-n-\beta, \right.\right. \tag{17}$$

$$-n], [-n+1-\beta-x], \frac{1}{-c+1}\Bigg)\Bigg)$$

```
> termtohyper((-1)^n*GAMMA(n + x + beta)*GAMMA(beta)*(-c + 1)^n/
  (c^n*GAMMA(x + beta)*GAMMA(n + beta)),n);
```

$$\frac{\text{pochhammer}(x+\beta, n)\left(\frac{c-1}{c}\right)^n}{\text{pochhammer}(\beta, n)} \tag{18}$$

```
> meixnerterm4:=pochhammer(x + beta, n)*((c - 1)/c)^n/pochhammer
  (beta, n)*hyperterm([-n, 1 - n - beta], [-n + 1 - x - beta], 1/(-
  c + 1),k);
```

$$meixnerterm4 := \left(\text{pochhammer}(x+\beta, n)\left(\frac{c-1}{c}\right)^n \text{pochhammer}(1-n-\beta, k)\ \text{pochhammer}(-n, k)\left(\frac{1}{-c+1}\right)^k\right) \Big/ \left(\text{pochhammer}(\beta, n)\ \text{pochhammer}(-n+1-\beta-x, k)\ k!\right) \tag{19}$$

```
> RE4:=RecurrenceNormalForm2(meixnerterm4,k,p(n));
```

$$RE4 := \left[c\,(\beta+n+1)\,p(2+n) - (\beta c + c n + x c + c + n - x + 1)\,p(n+1) + (n+1)\,p(n) = 0,\ 1,\ \frac{\beta c + x c - x}{\beta c}\right] \tag{20}$$

```
> RE1-RE4;
```

$$[0, 0, 0] \tag{21}$$

(14)

```
> meixnerterm5:=(1/c)^(beta+n+x)*hyperterm([beta+n,beta+x],[beta],1
  -1/c,k);
```

$$meixnerterm5 := \frac{\left(\frac{1}{c}\right)^{n+\beta+x}\text{pochhammer}(n+\beta, k)\ \text{pochhammer}(x+\beta, k)\left(1-\frac{1}{c}\right)^k}{\text{pochhammer}(\beta, k)\ k!} \tag{22}$$

```
> RE5:=RecurrenceNormalForm2(meixnerterm5,k,p(n));
```

$$RE5 := \left[c\,(\beta+n+1)\,p(2+n) - (\beta c + c n + x c + c + n - x + 1)\,p(n+1) + (n+1)\,p(n) = 0,\ \left(\frac{1}{c}\right)^{x+\beta},\ \frac{\left(\frac{1}{c}\right)^{1+x+\beta}\left(\beta^2 c + \beta c x - \beta^2 + 2\beta c - \beta x + x c - \beta - x\right)}{\beta c}\right] \tag{23}$$

```
> RE1-RE5;
```

$$\left[0, -\left(\frac{1}{c}\right)^{x+\beta}+1, -\frac{\left(\frac{1}{c}\right)^{1+x+\beta}\left(\beta^2 c + \beta c x - \beta^2 + 2\beta c - \beta x + x c - \beta - x\right)}{\beta c}\right] \tag{24}$$

$$+\frac{\beta c + x c - x}{\beta c}\Bigg]$$

In this case we get the same recurrence. However, the computation of the initial values for this infinite sum is incomplete and will be proven by Mathematica.

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```
> meixnerterm6:=pochhammer(beta+x,n)/pochhammer(beta,n)*hyperterm(
  [-n,-x],[1-n-x-beta],1/c,k);
```

$$meixnerterm6 := \frac{\text{pochhammer}(x+\beta, n)\ \text{pochhammer}(-n, k)\ \text{pochhammer}(-x, k)\left(\frac{1}{c}\right)^k}{\text{pochhammer}(\beta, n)\ \text{pochhammer}(-n+1-\beta-x, k)\ k!} \quad (25)$$

```
> RE6:=RecurrenceNormalForm2(meixnerterm6,k,p(n));
```

$$RE6 := \Bigg[c\,(\beta+n+1)\,p(2+n) - (\beta c + c\,n + x\,c + c + n - x + 1)\,p(n+1) + (n + 1)\,p(n) = 0,\ 1,\ \frac{\beta c + x c - x}{\beta c}\Bigg] \quad (26)$$

```
> RE1-RE6;
```

$$[0, 0, 0] \quad (27)$$

Reversion (17)

```
> Sumtohyper(subs(k=n-k,meixnerterm6),k)
```

$$\frac{1}{\Gamma(x+\beta)\,\Gamma(n+\beta)\,\Gamma(-x)\,\Gamma(1-x-\beta)\,c^n}\left(\Gamma(n+\beta+x)\,\Gamma(\beta)\,(-1)^n\,\Gamma(n-x)\,\Gamma(-n+1-\beta-x)\,\text{Hypergeom}([x+\beta, -n], [x-n+1], c)\right) \quad (28)$$

```
> termtohyper(GAMMA(n + beta + x)*GAMMA(beta)*(-1)^n*GAMMA(-x + n)*
  GAMMA(-n + 1 - beta - x)/(GAMMA(x + beta)*GAMMA(n + beta)*GAMMA(-
  x)*GAMMA(1 - x - beta)*c^n),n);
```

$$\frac{\text{pochhammer}(-x, n)\left(\frac{1}{c}\right)^n}{\text{pochhammer}(\beta, n)} \quad (29)$$

```
> meixnerterm7:=pochhammer(-x, n)*(1/c)^n/pochhammer(beta, n)*
  hyperterm([-n, x + beta], [-n + 1 + x],c,k);
```

$$meixnerterm7 := \frac{\text{pochhammer}(-x, n)\left(\frac{1}{c}\right)^n\ \text{pochhammer}(-n, k)\ \text{pochhammer}(x+\beta, k)\ c^k}{\text{pochhammer}(\beta, n)\ \text{pochhammer}(x-n+1, k)\ k!} \quad (30)$$

```
> RE7:=RecurrenceNormalForm2(meixnerterm7,k,p(n));
```

$$RE7 := \Bigg[c\,(\beta+n+1)\,p(2+n) - (\beta c + c\,n + x\,c + c + n - x + 1)\,p(n+1) + (n + 1)\,p(n) = 0,\ 1,\ \frac{\beta c + x c - x}{\beta c}\Bigg] \quad (31)$$

```
> RE1-RE7;
```

$$[0, 0, 0] \quad (32)$$

Krawtchouk (18)

```
> krawtchoukterm1:=hyperterm([-n,-x],[-N],1/p,k);
```

$$krawtchoukterm1 := \frac{\text{pochhammer}(-n, k)\ \text{pochhammer}(-x, k)\left(\frac{1}{p}\right)^k}{\text{pochhammer}(-N, k)\ k!} \tag{33}$$

```
> RE1:=RecurrenceNormalForm2(krawtchoukterm1,k,p(n));
```

$$RE1 := \left[-p\ (-1-n+N)\ p(2+n) + (Np - 2\,p\,n + n - 2\,p - x + 1)\ p(n+1) + (n + 1)\ (-1+p)\ p(n) = 0,\ 1,\ \frac{Np-x}{Np}\right] \tag{34}$$

Reversion (19)

```
> Sumtohyper(subs(k=n-k,krawtchoukterm1),k)
```

$$\frac{(-1)^n\,\Gamma(n-x)\,\Gamma(-N)\,\text{Hypergeom}([1-n+N, -n], [x-n+1], p)}{\Gamma(-x)\,\Gamma(n-N)\,p^n} \tag{35}$$

```
> termtohyper((-1)^n*GAMMA(-x + n)*GAMMA(-N)/(GAMMA(-x)*GAMMA(n -
  N)*p^n),n);
```

$$\frac{\text{pochhammer}(-x, n)\left(-\frac{1}{p}\right)^n}{\text{pochhammer}(-N, n)} \tag{36}$$

```
> krawtchoukterm2:=(-1/p)^n*pochhammer(-x, n)/pochhammer(-N, n)*
  hyperterm([1-n+N,-n],[x-n+1],p,k)
```

$$krawtchoukterm2 := \frac{\left(-\frac{1}{p}\right)^n \text{pochhammer}(-x, n)\ \text{pochhammer}(1-n+N, k)\ \text{pochhammer}(-n, k)\ p^k}{\text{pochhammer}(-N, n)\ \text{pochhammer}(x-n+1, k)\ k!} \tag{37}$$

```
> RE2:=RecurrenceNormalForm2(krawtchoukterm2,k,p(n));
```

$$RE2 := \left[-p\ (-1-n+N)\ p(2+n) + (Np - 2\,p\,n + n - 2\,p - x + 1)\ p(n+1) + (n + 1)\ (-1+p)\ p(n) = 0,\ 1,\ \frac{Np-x}{Np}\right] \tag{38}$$

```
> RE1-RE2;
```

$$[0, 0, 0] \tag{39}$$

(20)

```
> krawtchoukterm3:=(1-1/p)^n*hyperterm([-N+x,-n],[-N],1/(1-p),k)
```

$$krawtchoukterm3 := \frac{\left(1-\frac{1}{p}\right)^n \text{pochhammer}(x-N, k)\ \text{pochhammer}(-n, k)\left(\frac{1}{1-p}\right)^k}{\text{pochhammer}(-N, k)\ k!} \tag{40}$$

```
> RE3:=RecurrenceNormalForm2(krawtchoukterm3,k,p(n));
```

$$RE3 := \left[-p\ (-1-n+N)\ p(2+n) + (Np - 2\,p\,n + n - 2\,p - x + 1)\ p(n+1) + (n + 1)\ (-1+p)\ p(n) = 0,\ 1,\ \frac{Np-x}{Np}\right] \tag{41}$$

```
> RE1-RE3;
```

$$[0, 0, 0] \tag{42}$$

Reversion (23)

```
> Sumtohyper(subs(k=n-k,krawtchoukterm3),k)
```

$$\frac{1}{\Gamma(x-N)\,\Gamma(n-N)\,(1-p)^n}\left(\left(\frac{-1+p}{p}\right)^n \Gamma(n-N+x)\,(-1)^n\,\Gamma(-N)\,\mathrm{Hypergeom}([1 \right. \tag{43}$$

$$\left. -n+N, -n], [-n+1+N-x], 1-p)\right)$$

```
> termtohyper(((-1 + p)/p)^n*GAMMA(n - N + x)*(-1)^n*GAMMA(-N)/
  (GAMMA(x - N)*GAMMA(n - N)*(1 - p)^n),n);
```

$$\frac{\mathrm{pochhammer}(x-N, n)\left(\frac{1}{p}\right)^n}{\mathrm{pochhammer}(-N, n)} \tag{44}$$

```
> krawtchoukterm4:=pochhammer(x - N, n)/(pochhammer(-N, n)*p^n)*
  hyperterm([-n, 1 - n + N], [-n + 1 - x + N],1-p,k);
```

$$krawtchoukterm4 := \tag{45}$$

$$\frac{\mathrm{pochhammer}(x-N, n)\,\mathrm{pochhammer}(1-n+N, k)\,\mathrm{pochhammer}(-n, k)\,(1-p)^k}{\mathrm{pochhammer}(-N, n)\,p^n\,\mathrm{pochhammer}(-n+1+N-x, k)\,k!}$$

```
> RE4:=RecurrenceNormalForm2(krawtchoukterm4,k,p(n));
```

$$RE4 := \left[-p\,(-1-n+N)\,p(2+n) + (Np - 2pn + n - 2p - x + 1)\,p(n+1) + (n \right. \tag{46}$$

$$\left. +1)\,(-1+p)\,p(n) = 0, 1, \frac{Np-x}{Np}\right]$$

```
> RE1-RE4;
```

$$[0, 0, 0] \tag{47}$$

(21)

```
> krawtchoukterm5:=(1-1/p)^(-N+n+x)*hyperterm([-N+n,-N+x],[-N],1/p,
  k);
```

$$krawtchoukterm5 := \frac{\left(1-\frac{1}{p}\right)^{n-N+x}\mathrm{pochhammer}(n-N, k)\,\mathrm{pochhammer}(x-N, k)\left(\frac{1}{p}\right)^k}{\mathrm{pochhammer}(-N, k)\,k!} \tag{48}$$

```
> RE5:=RecurrenceNormalForm2(krawtchoukterm5,k,p(n));
```

$$RE5 := \left[-p\,(-1-n+N)\,p(2+n) + (Np - 2pn + n - 2p - x + 1)\,p(n+1) + (n \right. \tag{49}$$

$$\left. +1)\,(-1+p)\,p(n) = 0, \left(\frac{-1+p}{p}\right)^{x-N}, \right.$$

$$\left. -\frac{\left(\frac{-1+p}{p}\right)^{1-N+x}\left(N^2 - Np - Nx - N + x\right)}{Np}\right]$$

```
> RE1-RE5;
```

$$\left[0,\ -\left(\frac{-1+p}{p}\right)^{x-N}+1,\ \frac{\left(\frac{-1+p}{p}\right)^{1-N+x}\left(N^2-Np-Nx-N+x\right)}{Np}+\frac{Np-x}{Np}\right] \tag{50}$$

In this case we get the same recurrence. However, the computation of the initial values for this infinite sum is incomplete and will be proven by Mathematica.

(22)

```
> krawtchoukterm6:=pochhammer(x-N,n)/pochhammer(-N,n)*hyperterm([-
  n,-x],[-n-x+N+1],1-1/p,k);
```

$$krawtchoukterm6 := \frac{\text{pochhammer}(x-N,n)\ \text{pochhammer}(-n,k)\ \text{pochhammer}(-x,k)\left(1-\frac{1}{p}\right)^{k}}{\text{pochhammer}(-N,n)\ \text{pochhammer}(-n+1+N-x,k)\ k!} \tag{51}$$

```
> RE6:=RecurrenceNormalForm2(krawtchoukterm6,k,p(n));
```

$$RE6 := \left[-p\ (-1-n+N)\ p(2+n)+(Np-2pn+n-2p-x+1)\ p(n+1)+(n+1)\ (-1+p)\ p(n)=0,\ 1,\ \frac{Np-x}{Np}\right] \tag{52}$$

```
> RE1-RE6;
```

$$[0,\ 0,\ 0] \tag{53}$$

Reversion (24)

```
> Sumtohyper(subs(k=n-k,krawtchoukterm6),k)
```

$$\frac{1}{\Gamma(x-N)\ \Gamma(n-N)\ \Gamma(-x)\ \Gamma(1-x+N)}\left(\Gamma(n-N+x)\ \Gamma(-N)\ (-1)^{n}\ \Gamma(n-x)\ \Gamma(-n+1+N-x)\left(\frac{-1+p}{p}\right)^{n}\text{Hypergeom}\left([x-N,\ -n],\ [x-n+1],\ \frac{p}{-1+p}\right)\right) \tag{54}$$

```
> termtohyper(GAMMA(n - N + x)*GAMMA(-N)*(-1)^n*GAMMA(-x + n)*GAMMA
  (-n + 1 + N - x)*((-1 + p)/p)^n/(GAMMA(x - N)*GAMMA(n - N)*GAMMA
  (-x)*GAMMA(1 + N - x)),n);
```

$$\frac{\text{pochhammer}(-x,n)\left(-\frac{1-p}{p}\right)^{n}}{\text{pochhammer}(-N,n)} \tag{55}$$

```
> krawtchoukterm7:=pochhammer(-x, n)*(-(1 - p)/p)^n/pochhammer(-N,
  n)*hyperterm([-n, x - N], [-n + 1 + x], p/(-1 + p),k);
```

$$krawtchoukterm7 := \left(\text{pochhammer}(-x,n)\left(-\frac{1-p}{p}\right)^{n}\text{pochhammer}(-n,k)\ \text{pochhammer}(x-N,k)\left(\frac{p}{-1+p}\right)^{k}\right)\Big/\left(\text{pochhammer}(-N,n)\ \text{pochhammer}(x-n+1,k)\ k!\right) \tag{56}$$

```
> RE7:=RecurrenceNormalForm2(krawtchoukterm7,k,p(n));
```

$$RE7 := \left[-p\ (-1-n+N)\ p(2+n)+(Np-2pn+n-2p-x+1)\ p(n+1)+(n \right. \tag{57}$$

$$+1)\ (-1+p)\ p(n)=0,\ 1,\ \frac{Np-x}{Np}\ \Bigg]$$

```
> RE1-RE7;
```

$$[0,\ 0,\ 0] \tag{58}$$

Hahn

(25)

```
> hahnterm1:=hyperterm([-n,-x,alpha+beta+n+1],[alpha+1,-N],1,k);
```

$$hahnterm1 := \frac{\text{pochhammer}(-n,k)\ \text{pochhammer}(-x,k)\ \text{pochhammer}(\alpha+\beta+n+1,k)}{\text{pochhammer}(\alpha+1,k)\ \text{pochhammer}(-N,k)\ k!} \tag{59}$$

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```
> hahnterm2:=pochhammer(alpha+x+1,n)*pochhammer(x-N,n)/(pochhammer
  (alpha+1,n)*pochhammer(-N,n))*hyperterm([-n,-x,beta+N+1-x],[N+1-
  x-n,-alpha-x-n],1,k)
```

$$hahnterm2 := (\text{pochhammer}(\alpha+x+1,n)\ \text{pochhammer}(x-N,n)\ \text{pochhammer}(-n,k)\ \text{pochhammer}(-x,k)\ \text{pochhammer}(\beta+N+1-x,k))\ /\ (\text{pochhammer}(\alpha+1,n)\ \text{pochhammer}(-N,n)\ \text{pochhammer}(N+1-x-n,k)\ \text{pochhammer}(-\alpha-x-n,k)\ k!) \tag{60}$$

```
> RE1:=RecurrenceNormalForm2(hahnterm1,k,p(n));
```

$$RE1 := \Big[(\alpha+n+2)\ (\alpha+\beta+2n+2)\ (\alpha+\beta+n+2)\ (-1-n+N)\ p(2+n) - (2n+3+\alpha+\beta)\ (N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x)\ p(n+1)+(n+1)\ (\beta+n+1)\ (4+2n+\alpha+\beta)\ (N+\alpha+\beta+n+2)\ p(n)=0,\ 1,\ \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)\ N}\Big] \tag{61}$$

```
> RE2:=RecurrenceNormalForm2(hahnterm2,k,p(n));
```

$$RE2 := \Big[(\alpha+n+2)\ (\alpha+\beta+2n+2)\ (\alpha+\beta+n+2)\ (-1-n+N)\ p(2+n) - (2n+3+\alpha+\beta)\ (N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n \tag{62}$$

$$-6\,x\,\alpha+\beta^2+3\,\beta\,n-6\,x\,\beta-12\,x\,n+4\,N-2\,\alpha+2\,\beta-8\,x\Big)\,p(n+1)+(n+1)\,(\beta$$

$$+n+1)\,(4+2\,n+\alpha+\beta)\,(N+\alpha+\beta+n+2)\,p(n)=0,\ 1,$$

$$\frac{\alpha\,N-x\,\alpha-x\,\beta+N-2\,x}{(\alpha+1)\,N}\Bigg]$$

```
> RE1-RE2;
```

$$[0,\ 0,\ 0] \tag{63}$$

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```
> hahnterm3:=(-1)^n*pochhammer(beta+1,n)/pochhammer(alpha+1,n)*
  hyperterm([-n,n+alpha+beta+1,x-N],[beta+1,-N],1,k);
```

$$hahnterm3 := \left((-1)^n\,\mathrm{pochhammer}(\beta+1,n)\,\mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(\alpha+\beta+n+1,k)\,\mathrm{pochhammer}(x-N,k)\right)\Big/\left(\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(\beta+1,k)\,\mathrm{pochhammer}(-N,k)\,k!\right) \tag{64}$$

```
> RE3:=RecurrenceNormalForm2(hahnterm3,k,p(n));
```

$$RE3 := \Bigg[(\alpha+n+2)\,(\alpha+\beta+2\,n+2)\,(\alpha+\beta+n+2)\,(-1-n+N)\,p(2+n)-(2\,n+3+\alpha+\beta)\,\left(N\,\alpha^2+N\,\alpha\,\beta+2\,N\,\alpha\,n+2\,N\,\beta\,n+2\,N\,n^2-\alpha^2\,n-\alpha^2\,x-2\,\alpha\,\beta\,x-\alpha\,n^2\right.$$

$$\left.-4\,\alpha\,n\,x+\beta^2\,n-\beta^2\,x+\beta\,n^2-4\,\beta\,n\,x-4\,n^2\,x+3\,\alpha\,N+3\,N\,\beta+6\,N\,n-\alpha^2-3\,\alpha\,n\right.$$

$$\left.-6\,x\,\alpha+\beta^2+3\,\beta\,n-6\,x\,\beta-12\,x\,n+4\,N-2\,\alpha+2\,\beta-8\,x\right)\,p(n+1)+(n+1)\,(\beta$$

$$+n+1)\,(4+2\,n+\alpha+\beta)\,(N+\alpha+\beta+n+2)\,p(n)=0,\ 1,$$

$$\frac{\alpha\,N-x\,\alpha-x\,\beta+N-2\,x}{(\alpha+1)\,N}\Bigg] \tag{65}$$

```
> RE1-RE3;
```

$$[0,\ 0,\ 0] \tag{66}$$

(28)

```
> termtohyper(GAMMA(alpha+1)*GAMMA(-N-beta+x)/(GAMMA(n+alpha+1)*
  GAMMA(-n-N+x-beta)),n);
```

$$\frac{\mathrm{pochhammer}(\beta+N+1-x,n)\,(-1)^n}{\mathrm{pochhammer}(\alpha+1,n)} \tag{67}$$

```
> hahnterm4:=pochhammer(beta + N + 1 - x, n)*(-1)^n/pochhammer
  (alpha + 1, n)*hyperterm([-n,-n-N-alpha-beta-1,-N+x],[-N,-n-N-
  beta+x],1,k);
```

$$hahnterm4 := \left(\mathrm{pochhammer}(\beta+N+1-x,n)\,(-1)^n\,\mathrm{pochhammer}(-n,k)\,\mathrm{pochhammer}(-\alpha-\beta-n-N-1,k)\,\mathrm{pochhammer}(x-N,k)\right)\Big/\left(\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(\right. \tag{68}$$

$$-N, k)\ \text{pochhammer}(-n-N+x-\beta, k)\ k!)$$

```
> RE4:=RecurrenceNormalForm2(hahnterm4,k,p(n));
```

$$RE4 := \left[(\alpha+n+2)\ (\alpha+\beta+2n+2)\ (\alpha+\beta+n+2)\ (-1-n+N)\ p(2+n) - (2n \right. \tag{69}$$

$$+3+\alpha+\beta)\ (N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2$$

$$-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n$$

$$-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x)\ p(n+1)+(n+1)\ (\beta$$

$$+n+1)\ (4+2n+\alpha+\beta)\ (N+\alpha+\beta+n+2)\ p(n)=0,\ 1,$$

$$\left. \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)\ N} \right]$$

```
> RE1-RE4;
```

$$[0, 0, 0] \tag{70}$$

(29)

```
> hahnterm5:=pochhammer(-n-beta,n)*pochhammer(-n-N-alpha-beta-1,n)/
  (pochhammer(-N,n)*pochhammer(alpha+1,n))*hyperterm([-n,n+alpha+
  beta+1,N-x+beta+1],[N+alpha+beta+2,beta+1],1,k);
```

$$hahnterm5 := (\text{pochhammer}(-n-\beta, n)\ \text{pochhammer}(-\alpha-\beta-n-N-1, \tag{71}$$

$$n)\ \text{pochhammer}(-n, k)\ \text{pochhammer}(\alpha+\beta+n+1, k)\ \text{pochhammer}(\beta+N+1-x, k))$$

$$/\ (\text{pochhammer}(-N, n)\ \text{pochhammer}(\alpha+1, n)\ \text{pochhammer}(N+\alpha+\beta+2,$$

$$k)\ \text{pochhammer}(\beta+1, k)\ k!)$$

```
> RE5:=RecurrenceNormalForm2(hahnterm5,k,p(n));
```

$$RE5 := \left[(\alpha+n+2)\ (\alpha+\beta+2n+2)\ (\alpha+\beta+n+2)\ (-1-n+N)\ p(2+n) - (2n \right. \tag{72}$$

$$+3+\alpha+\beta)\ (N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2$$

$$-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n$$

$$-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x)\ p(n+1)+(n+1)\ (\beta$$

$$+n+1)\ (4+2n+\alpha+\beta)\ (N+\alpha+\beta+n+2)\ p(n)=0,\ 1,$$

$$\left. \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)\ N} \right]$$

```
> RE1-RE5;
```

$$[0, 0, 0] \tag{73}$$

Reversion 1, (30)

```
> Sumtohyper(subs(k=n-k,hahnterm1),k)
```

$$\left((-1)^n\,\Gamma(n-x)\,\Gamma(\alpha+\beta+2\,n+1)\,\Gamma(\alpha+1)\,\Gamma(-N)\,\mathrm{Hypergeom}\left([N+1-n,\,-n,\,-\alpha-n],\,[-\alpha-\beta-2\,n,\,-n+1+x],\,1\right)\right)\Big/\left(\Gamma(-x)\,\Gamma(\alpha+\beta+n+1)\,\Gamma(\alpha+1+n)\,\Gamma(-N+n)\right) \tag{74}$$

```
> termtohyper((-1)^n*GAMMA(-x + n)*GAMMA(2*n + alpha + beta + 1)*
  GAMMA(alpha + 1)*GAMMA(-N)/(GAMMA(-x)*GAMMA(alpha + beta + n + 1)
  *GAMMA(n + alpha + 1)*GAMMA(n - N)),n);
```

$$\frac{\mathrm{pochhammer}(-x,n)\,\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+1,n\right)\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{1}{2},n\right)(-4)^n}{\mathrm{pochhammer}(-N,n)\,\mathrm{pochhammer}(\alpha+\beta+1,n)\,\mathrm{pochhammer}(\alpha+1,n)} \tag{75}$$

```
> hahnterm6:=(-4)^n*pochhammer(alpha/2 + beta/2 + 1/2, n)*
  pochhammer(-x, n)*pochhammer(alpha/2 + beta/2 + 1, n)/(pochhammer
  (alpha + 1, n)*pochhammer(alpha + beta + 1, n)*pochhammer(-N, n))
  *hyperterm([1 - n + N, -n - alpha, -n], [x - n + 1, -2*n - alpha
  - beta],1,k)
```

$$hahnterm6 := \left((-4)^n\,\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{1}{2},n\right)\mathrm{pochhammer}(-x,n)\,\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+1,n\right)\mathrm{pochhammer}(N+1-n,k)\,\mathrm{pochhammer}(-\alpha-n,k)\,\mathrm{pochhammer}(-n,k)\right)\Big/\left(\mathrm{pochhammer}(-N,n)\,\mathrm{pochhammer}(\alpha+\beta+1,n)\,\mathrm{pochhammer}(\alpha+1,n)\,\mathrm{pochhammer}(-n+1+x,k)\,\mathrm{pochhammer}(-\alpha-\beta-2\,n,k)\,k!\right) \tag{76}$$

```
> RE6:=RecurrenceNormalForm2(hahnterm6,k,p(n));
```

$$RE6 := \Big[(\alpha+n+2)\,(\alpha+\beta+2\,n+2)\,(\alpha+\beta+n+2)\,(-1-n+N)\,p(2+n) - (2\,n+3+\alpha+\beta)\,(N\,\alpha^2+N\,\alpha\,\beta+2\,N\,\alpha\,n+2\,N\,\beta\,n+2\,N\,n^2-\alpha^2\,n-\alpha^2\,x-2\,\alpha\,\beta\,x-\alpha\,n^2-4\,\alpha\,n\,x+\beta^2\,n-\beta^2\,x+\beta\,n^2-4\,\beta\,n\,x-4\,n^2\,x+3\,\alpha\,N+3\,N\,\beta+6\,N\,n-\alpha^2-3\,\alpha\,n-6\,x\,\alpha+\beta^2+3\,\beta\,n-6\,x\,\beta-12\,x\,n+4\,N-2\,\alpha+2\,\beta-8\,x)\,p(n+1)+(n+1)\,(\beta+n+1)\,(4+2\,n+\alpha+\beta)\,(N+\alpha+\beta+n+2)\,p(n)=0,\ 1,\ \frac{\alpha\,N-x\,\alpha-x\,\beta+N-2\,x}{(\alpha+1)\,N}\Big] \tag{77}$$

```
> RE1-RE6;
```

$$[0, 0, 0] \tag{78}$$

Reversion 2, (31)

```
> Sumtohyper(subs(k=n-k,hahnterm2),k)
```

$$\left(\Gamma(\alpha+x+n+1)\,\Gamma(x-N+n)\,\Gamma(\alpha+1)\,\Gamma(-N)\,(-1)^n\,\Gamma(n-x)\,\Gamma(\beta+N+1-x+n)\,\Gamma(N+1-x-n)\,\Gamma(-\alpha-x-n)\,\text{Hypergeom}\left([-n,\alpha+x+1,x-N],[-n-N+x-\beta,-n+1+x],1\right)\right)\Big/\left(\Gamma(\alpha+x+1)\,\Gamma(x-N)\,\Gamma(\alpha+1+n)\,\Gamma(-N+n)\,\Gamma(-x)\,\Gamma(\beta+N+1-x)\,\Gamma(N+1-x)\,\Gamma(-\alpha-x)\right) \tag{79}$$

```
> termtohyper((-1)^n*GAMMA(n + alpha + x + 1)*GAMMA(n + x - N)*
  GAMMA(alpha + 1)*GAMMA(-N)*GAMMA(-x + n)*GAMMA(n + beta + N + 1 -
  x)*GAMMA(N + 1 - x - n)*GAMMA(-alpha - x - n)/(GAMMA(alpha + x +
  1)*GAMMA(x - N)*GAMMA(n + alpha + 1)*GAMMA(n - N)*GAMMA(-x)*GAMMA
  (beta + N + 1 - x)*GAMMA(N + 1 - x)*GAMMA(-alpha - x)),n);
```

$$\frac{\text{pochhammer}(-x,n)\,\text{pochhammer}(\beta+N+1-x,n)\,(-1)^n}{\text{pochhammer}(-N,n)\,\text{pochhammer}(\alpha+1,n)} \tag{80}$$

```
> hahnterm7:=(-1)^n*pochhammer(-x, n)*pochhammer(beta + N + 1 - x,
  n)/(pochhammer(alpha + 1, n)*pochhammer(-N, n))*hyperterm([x - N,
  alpha + x + 1, -n], [-n - beta - N + x, x - n + 1],1,k)
```

$$hahnterm7 := \left((-1)^n\,\text{pochhammer}(-x,n)\,\text{pochhammer}(\beta+N+1-x,n)\,\text{pochhammer}(x-N,k)\,\text{pochhammer}(\alpha+x+1,k)\,\text{pochhammer}(-n,k)\right)\Big/\left(\text{pochhammer}(-N,n)\,\text{pochhammer}(\alpha+1,n)\,\text{pochhammer}(-n-N+x-\beta,k)\,\text{pochhammer}(-n+1+x,k)\,k!\right) \tag{81}$$

```
> RE7:=RecurrenceNormalForm2(hahnterm7,k,p(n));
```

$$RE7 := \Big[(\alpha+n+2)\,(\alpha+\beta+2n+2)\,(\alpha+\beta+n+2)\,(-1-n+N)\,p(2+n)-(2n+3+\alpha+\beta)\,(N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x)\,p(n+1)+(n+1)\,(\beta+n+1)\,(4+2n+\alpha+\beta)\,(N+\alpha+\beta+n+2)\,p(n)=0,\ 1,\ \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)\,N}\Big] \tag{82}$$

```
> RE1-RE7;
```

$$[0,0,0] \tag{83}$$

Reversion 3, (32)

```
> Sumtohyper(subs(k=n-k,hahnterm3),k)
```

$$\left(\left((-1)^n\right)^2\,\Gamma(\alpha+1)\,\Gamma(\alpha+\beta+2n+1)\,\Gamma(x-N+n)\,\Gamma(-N)\,\text{Hypergeom}\left([N+1-n,-n,-n-\beta],[-\alpha-\beta-2n,N+1-x-n],1\right)\right)\Big/\left(\Gamma(\alpha+1+n)\,\Gamma(\alpha+\beta+n+1)\,\Gamma(x-N)\,\Gamma(-N+n)\right) \tag{84}$$

```
> termtohyper(GAMMA(alpha + 1)*GAMMA(2*n + alpha + beta + 1)*GAMMA
  (x - N + n)*GAMMA(-N)/(GAMMA(alpha + 1 + n)*GAMMA(alpha + beta +
  n + 1)*GAMMA(x - N)*GAMMA(-N+n)),n);
```

$$\frac{\text{pochhammer}(x-N,n)\ \text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+1,n\right)\ \text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{1}{2},n\right)\ 4^n}{\text{pochhammer}(-N,n)\ \text{pochhammer}(\alpha+\beta+1,n)\ \text{pochhammer}(\alpha+1,n)} \tag{85}$$

```
> hahnterm8:=pochhammer(alpha/2 + beta/2 + 1, n)*pochhammer(x - N,
  n)*pochhammer(alpha/2 + beta/2 + 1/2, n)*4^n/(pochhammer(alpha +
  1, n)*pochhammer(-N, n)*pochhammer(alpha + beta + 1, n))*
  hyperterm([-n - beta, N + 1 - n, -n], [N + 1 - x - n, -2*n -
  alpha - beta],1,k)
```

$$hahnterm8 := \left(\text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+1,n\right)\ \text{pochhammer}(x-N,n)\ \text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{1}{2},n\right)\ 4^n\ \text{pochhammer}(-n-\beta,k)\ \text{pochhammer}(N+1-n,k)\ \text{pochhammer}(-n,k)\right) \Big/ \left(\text{pochhammer}(-N,n)\ \text{pochhammer}(\alpha+\beta+1,n)\ \text{pochhammer}(\alpha+1,n)\ \text{pochhammer}(N+1-x-n,k)\ \text{pochhammer}(-\alpha-\beta-2n,k)\ k!\right) \tag{86}$$

```
> RE8:=RecurrenceNormalForm2(hahnterm8,k,p(n));
```

$$RE8 := \left[(\alpha+n+2)\ (\alpha+\beta+2n+2)\ (\alpha+\beta+n+2)\ (-1-n+N)\ p(2+n) - (2n+3+\alpha+\beta)\ (N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x)\ p(n+1)+(n+1)\ (\beta+n+1)\ (4+2n+\alpha+\beta)\ (N+\alpha+\beta+n+2)\ p(n)=0,\ 1,\ \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)\ N}\right] \tag{87}$$

```
> RE1-RE8;
```

$$[0,\ 0,\ 0] \tag{88}$$

Reversion 4, (33)

```
> Sumtohyper(subs(k=n-k,hahnterm4),k)
```

$$\left(\Gamma(\beta+N+1-x+n)\ ((-1)^n)^2\ \Gamma(\alpha+1)\ \Gamma(-\alpha-\beta-1-N)\ \Gamma(x-N+n)\ \Gamma(-N)\ \Gamma(-n-N+x-\beta)\ \text{Hypergeom}([N+1-n,-n,\beta+N+1-x],[N+\alpha+\beta+2,N+1-x-n],1)\right) \Big/ \left(\Gamma(\beta+N+1-x)\ \Gamma(\alpha+1+n)\ \Gamma(-\alpha-\beta-n-N-1)\ \Gamma(x-N)\ \Gamma(-N+n)\ \Gamma(x-\beta-N)\right) \tag{89}$$

```
> termtohyper(GAMMA(beta + N + 1 - x + n)*GAMMA(alpha + 1)*GAMMA(-
```

```
alpha - beta - 1 - N)*GAMMA(x - N + n)*GAMMA(-N)*GAMMA(-n - N + x
- beta)/(GAMMA(beta + N + 1 - x)*GAMMA(alpha + 1 + n)*GAMMA(-
alpha - beta - n - N - 1)*GAMMA(x - N)*GAMMA(-N + n)*GAMMA(x -
beta - N)),n);
```

$$\frac{\text{pochhammer}(x-N,n)\ \text{pochhammer}(N+\alpha+\beta+2,n)}{\text{pochhammer}(-N,n)\ \text{pochhammer}(\alpha+1,n)} \tag{90}$$

```
> hahnterm9:=pochhammer(x - N, n)*pochhammer(N + alpha + beta + 2,
n)/(pochhammer(-N, n)*pochhammer(alpha + 1, n))*hyperterm([N + 1
- n, -n, beta + N + 1 - x], [N + alpha + beta + 2, N + 1 - x -
n],1,k)
```

$$hahnterm9 := \left(\text{pochhammer}(N+\alpha+\beta+2,n)\ \text{pochhammer}(x-N,n)\ \text{pochhammer}(N+1-n,k)\ \text{pochhammer}(-n,k)\ \text{pochhammer}(\beta+N+1-x,k)\right) \Big/ \left(\text{pochhammer}(-N,n)\ \text{pochhammer}(\alpha+1,n)\ \text{pochhammer}(N+\alpha+\beta+2,k)\ \text{pochhammer}(N+1-x-n,k)\ k!\right) \tag{91}$$

```
> RE9:=RecurrenceNormalForm2(hahnterm9,k,p(n));
```

$$RE9 := \left[(\alpha+n+2)\ (\alpha+\beta+2n+2)\ (\alpha+\beta+n+2)\ (-1-n+N)\ p(2+n) - (2n+3+\alpha+\beta)\ (N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x)\ p(n+1)+(n+1)\ (\beta+n+1)\ (4+2n+\alpha+\beta)\ (N+\alpha+\beta+n+2)\ p(n)=0,\ 1,\ \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)N}\right] \tag{92}$$

```
> RE1-RE9;
```

$$[0,\ 0,\ 0] \tag{93}$$

Reversion 5, (34)

```
> Sumtohyper(subs(k=n-k,hahnterm5),k)
```

$$\left(\Gamma(-\beta)\ \Gamma(-\alpha-\beta-1-N)\ \Gamma(-N)\ \Gamma(\alpha+1)\ (-1)^n\ \Gamma(\alpha+\beta+2n+1)\ \Gamma(\beta+N+1-x+n)\ \Gamma(N+\alpha+\beta+2)\ \Gamma(\beta+1)\ \text{Hypergeom}([-n,-\alpha-\beta-n-N-1,-n-\beta],[-\alpha-\beta-2n,-n-N+x-\beta],1)\right) \Big/ \left(\Gamma(-n-\beta)\ \Gamma(-\alpha-\beta-n-N-1)\ \Gamma(-N+n)\ \Gamma(\alpha+1+n)\ \Gamma(\alpha+\beta+n+1)\ \Gamma(\beta+N+1-x)\ \Gamma(N+\alpha+\beta+n+2)\ \Gamma(\beta+n+1)\right) \tag{94}$$

```
> termtohyper(GAMMA(-beta)*GAMMA(-alpha - beta - 1 - N)*GAMMA(-N)*
GAMMA(alpha + 1)*(-1)^n*GAMMA(alpha + beta + 2*n + 1)*GAMMA(beta
+ N + 1 - x + n)*GAMMA(N + alpha + beta + 2)*GAMMA(beta + 1)/
(GAMMA(-n - beta)*GAMMA(-alpha - beta - n - N - 1)*GAMMA(-N + n)*
GAMMA(alpha + 1 + n)*GAMMA(alpha + beta + n + 1)*GAMMA(beta + N +
```

```
  1 - x)*GAMMA(N + alpha + beta + n + 2)*GAMMA(beta + n + 1)),n);
```

$$\left(\text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+1,n\right)\text{pochhammer}(\beta+N+1-x,n)\,\text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{1}{2},n\right)(-4)^n\right)\Big/\left(\text{pochhammer}(-N,n)\,\text{pochhammer}(\alpha+\beta+1,n)\,\text{pochhammer}(\alpha+1,n)\right) \tag{95}$$

```
> hahnterm10:=pochhammer(alpha/2 + beta/2 + 1, n)*pochhammer(beta +
  N + 1 - x, n)*pochhammer(alpha/2 + beta/2 + 1/2, n)*(-4)^n/
  (pochhammer(-N, n)*pochhammer(alpha + beta + 1, n)*pochhammer
  (alpha + 1, n))*hyperterm([-n, -alpha - beta - n - N - 1, -n -
  beta], [-alpha - beta - 2*n, -n - N + x - beta],1,k)
```

$$hahnterm10 := \left(\text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+1,n\right)\text{pochhammer}(\beta+N+1-x,n)\,\text{pochhammer}\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{1}{2},n\right)(-4)^n\,\text{pochhammer}(-n,k)\,\text{pochhammer}(-\alpha-\beta-n-N-1,k)\,\text{pochhammer}(-n-\beta,k)\right)\Big/\left(\text{pochhammer}(-N,n)\,\text{pochhammer}(\alpha+\beta+1,n)\,\text{pochhammer}(\alpha+1,n)\,\text{pochhammer}(-\alpha-\beta-2n,k)\,\text{pochhammer}(-n-N+x-\beta,k)\,k!\right) \tag{96}$$

```
> RE10:=RecurrenceNormalForm2(hahnterm10,k,p(n));
```

$$RE10 := \left[(\alpha+n+2)(\alpha+\beta+2n+2)(\alpha+\beta+n+2)(-1-n+N)\,p(2+n)-(2n+3+\alpha+\beta)\left(N\alpha^2+N\alpha\beta+2N\alpha n+2N\beta n+2Nn^2-\alpha^2 n-\alpha^2 x-2\alpha\beta x-\alpha n^2-4\alpha n x+\beta^2 n-\beta^2 x+\beta n^2-4\beta n x-4n^2 x+3\alpha N+3N\beta+6Nn-\alpha^2-3\alpha n-6x\alpha+\beta^2+3\beta n-6x\beta-12xn+4N-2\alpha+2\beta-8x\right)p(n+1)+(n+1)(\beta+n+1)(4+2n+\alpha+\beta)(N+\alpha+\beta+n+2)\,p(n)=0,\ 1,\ \frac{\alpha N-x\alpha-x\beta+N-2x}{(\alpha+1)N}\right] \tag{97}$$

```
> RE1-RE10;
```

$$[0,\ 0,\ 0] \tag{98}$$