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Recurrence equations and their classical continuous, discrete, and q-discrete orthogonal polynomial solutions

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Abstract
Classical orthogonal polynomials are solutions of a second order differential, difference, or q-difference equation for continuous, discrete, or q-discrete variables, respectively. Furthermore, all such systems satisfy a three-term recurrence equation of the form:

$$p_{n+1}(x) = (A_n x + B_n) p_n(x) - C_n p_{n-1}(x),$$

for $n \geq 0$ with $p_{-1} = 0, \ p_0 = 1$. Given a holonomic three-term recurrence equation, we implement in `Maxima` and `Maple` an algorithm which detects its classical orthogonal polynomial solutions for the continuous, discrete, and q-discrete variables when they exist. With our implementations, the results obtained using the Maple implementations by Koepf and Schmersau (Appl Math Comput 128:303–327, 2002) and Koorwinder and Swarttouw (Priv Commun, 1998) are easily recovered. In addition, we obtain new relations that extend beyond those previously established in the literature.

Keywords Three-term recurrence equations · Orthogonal polynomials · Maxima · Maple

Mathematics Subject Classification Primary 33C20 · 33F10; Secondary 30B10 · 68W30

1 Introduction

Orthogonal polynomials form a (potentially) infinite sequence of polynomials $\{p_0(x), p_1(x), p_2(x), \ldots\}$, where each polynomial $p_n(x)$ has degree n and any two distinct polynomials in the sequence are orthogonal to each other.

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In the continuous case, this orthogonality is expressed as:

$$\langle p_n, p_m \rangle = \int_A^B p_n(x) p_m(x) w(x) dx = d_n^2 \delta_{m,n} \tag{1}$$

where $w(x)$ is a non-negative weight function on the interval (A, B) called interval of orthogonality which may be infinite at one or both ends (Foupouagnigni and Koepf 2020),

$$\delta_{m,n} = \begin{cases} 1 & \text{if } m = n, \\ 0 & \text{if } m \neq n. \end{cases}$$

In the discrete case, equation (1) takes the form:

$$\sum_i p_n(x_i) p_m(x_i) w(x_i) = 0, \qquad \text{for all } m \neq n \tag{2}$$

and polynomials $p_n(x)$ that satisfy equation (2), with discrete weight $w(x)$ are referred to as orthogonal polynomials of a discrete variable (Nikiforov and Uvarov 1988; Nikiforov et al. 1991).

Regarding q-orthogonal polynomials, any two distinct polynomials in the sequence are orthogonal with respect to a q-analogue of an inner product. Depending on whether the orthogonality is defined using a q-integral or a q-sum, we distinguish between continuous and discrete q-orthogonal polynomials.

For continuous q-orthogonal polynomials, we have the orthogonality relation:

$$\langle p_n, p_m \rangle = \int_A^B p_n(x) p_m(x) w(x) d_q x = d_n^2 \delta_{m,n}, \tag{3}$$

where $d_q x$ denotes the q-integral measure.

For discrete q-orthogonal polynomials, we have:

$$\sum_i p_n(x_i) p_m(x_i) w(x_i) = d_n^2 \delta_{m,n}, \tag{4}$$

where $\{x_i\}$ is a discrete q-grid, and polynomials $p_n(x)$ satisfying equation (4) are referred to as q-orthogonal polynomials of a discrete variable (Koekoek and Swarttouw 1996).

A fundamental aspect of classical orthogonal polynomials is their adherence to a three-term recurrence equation:

$$p_{n+1}(x) = (A_n x + B_n) p_n(x) - C_n p_{n-1}(x), \tag{5}$$

with explicitly provided coefficients A_n, B_n, and C_n, where $n \geq 0$, $A_n \neq 0$, $p_{-1} = 0$, and $p_0 = 1$ (Olver et al. 2010). Favard's theorem confirms that any solution of the recurrence equation (5) forms an orthogonal polynomial system, ensuring that such equations reliably yield orthogonal polynomial solutions (Chihara 1978).

The main question of this work is, given a holonomic recurrence equation, that is a linear, homogeneous, recurrence equation with polynomial coefficients (Koelink and Van Assche 2003), can it be associated with a classical orthogonal polynomial system? If so, what specific family of classical orthogonal polynomials does it represent?

In previous work, Koornwinder and Swarttouw (1998) implemented an algorithm in Maple (`rec2ortho` package) that identifies orthogonal polynomials from the coefficients of their three-term recurrence relations. Afterwards, (Koepf and Schmersau 2002) developed a rational approach in Maple (`retode` package) to classify various orthogonal polynomial families based on holonomic three-term recurrence equations. `retode` was able to cover a wider range of classical orthogonal polynomials than `rec2ortho`, and returned more information like the weight function, the coefficients $\sigma(x)$, $\tau(x)$ of the defining differential/difference equations and the interval of orthogonality. However, `retode` still had some limitations like needing extra hand computations to identify the specific classical families, and being unable to detect solutions of some three-term recurrence equations, for example, the three-term recurrence equation for the so-called Masjed-Jamei polynomials (Masjed-Jamei 2002),

$$J_n^{(p,q)}(x;a,b,c,d) = \frac{2^n(ad-bc)^n(-p+1-iq/2)_n}{i^n} \times {}_2F_1\left(\begin{matrix}-n,\ n+1-2p\\ -p+1-iq/2\end{matrix};\frac{i(a^2+c^2)}{2(ad-bc)}\left(x-\frac{i(ad-bc)-(ab+cd)}{a^2+c^2}\right)\right), \tag{6}$$

given in Koepf and Tcheutia (2024).

We implement an algorithm for identifying classical orthogonal polynomial solutions of three-term recurrence equations when such solutions exist. These implementations are realized using the computer algebra systems `Maxima` and `Maple` (see Maplesoft 2018; Maxima 2024). We take a less generalized approach in this work, by focusing on the known classical orthogonal families. This focused approach leverages on the assumption that every solution must either be an exact solution or a linear transformation of these families. Our approach immediately recognizes the original polynomial families without any further calculations, a significant improvement compared to the `retode` package. Furthermore, our methodology accommodates a broader range of orthogonal polynomial families (including those obtained through linear transformations) than the `rec2ortho` and `retode` packages, making it more versatile for practical applications.

While our work extends the capabilities of the `retode` package in `Maple`, our implementation in `Maxima` is a novel contribution to the field. This development addresses the important need for accessible computational tools in mathematical research by providing these analytical capabilities in an open-access software environment. Additionally, we are able to derive some novel identities between the classical orthogonal polynomial systems.

In this paper, we cover classical continuous, discrete, and q-discrete orthogonal polynomial solutions of three-term recurrence equations. In Section 2, we revisit the steps taken to derive the coefficients of the three-term recurrence equations. In Section 3, we focus on the inverse problem, starting with a recurrence equation and identifying its classical orthogonal polynomial solution, if one exists, outlining the algorithmic approach to this problem. In Section 4, we present results from our implementations, and in Section 5, we outline the new relations discovered.

2 From orthogonal polynomials to recurrence equations (see Koepf and Schmersau 2002)

A family of classical continuous orthogonal polynomials is a set of polynomials

$$y(x) = p_n(x) = k_n x^n + k_n' x^{n-1} + k_n'' x^{n-2} + \dots \qquad (n \in \mathbb{N}_0, k_n \neq 0) \tag{7}$$

of degree exactly n, which satisfies a differential equation of the form

$$\sigma(x)y''(x) + \tau(x)y'(x) + \lambda_n y(x) = 0, \tag{8}$$

where $\sigma(x) = ax^2+bx+c$ is a polynomial of at most second degree and $\tau(x) = dx+e$, $(d \neq 0)$ a first degree polynomial (Doman 2016). To obtain λ_n, we equate the coefficients of x^n in equation (8). Given that $p_n(x)$ is of exact degree n, this yields:

$$\lambda_n = -n(an + d - a). \tag{9}$$

Additionally, a family of classical discrete orthogonal polynomials is a set of polynomials of degree exactly n given in equation (7), which satisfies a difference equation of the form

$$\sigma(x)\Delta\nabla y(x) + \tau(x)\Delta y(x) + \lambda_n y(x) = 0, \tag{10}$$

where $\Delta y(x) = y(x+1) - y(x)$ is the forward difference operator, and $\nabla y(x) = y(x) - y(x-1)$ is the backward difference operator (Doman 2016). As in the continuous case, $\sigma(x) = ax^2+bx+c$ is a polynomial of at most second degree and $\tau(x) = dx+e$, $(d \neq 0)$ a first degree polynomial. Then we can again obtain the relation (9) from the equation (10) by equating the coefficients of x^n.

Lastly, a family of classical q-orthogonal polynomials is a set of polynomials of degree exactly n given in equation (7), which satifies a q-difference equation of the form

$$\sigma(x)D_q D_{1/q} y(x) + \tau(x)D_q y(x) + \lambda_{q,n} y(x) = 0, \tag{11}$$

where

$$D_q f(x) = \frac{f(qx) - f(x)}{(q-1)x}, \quad q \neq 1,$$

is the q-difference operator. As before, $\sigma(x) = ax^2 + bx + c$ is a polynomial of at most second degree and $\tau(x) = dx + e$, $(d \neq 0)$ a first degree polynomial. To obtain $\lambda_{q,n}$, we again equate the coefficients of x^n in equation (11), and recover:

$$\lambda_{q,n} = -a[n]_{1/q}[n-1]_q - d[n]_q, \tag{12}$$

where $[n]_q = \frac{1-q^n}{1-q}$ is called the q-bracket, and $\lim_{q\to 1}[n]_q = n$. Thus we have:

$$\lambda_{q,n} = \frac{(a+d)q + a - d + q^n(d-a) - aq^{1-n} - dq^{n+1}}{(q-1)^2}. \tag{13}$$

To derive the coefficients of the recurrence equation in terms of the coefficients of $\sigma(x)$ and $\tau(x)$, we do (Koepf and Schmersau 1998):

Step 1. Substitute the orthogonal polynomial given in equation (7) into the differential equation (8), the difference equation (10) or the q-difference equation (11), respectively.

Step 2. For the discrete case, substitute in the expressions $(x+a)^k$ using,
$(x+a)^k = \sum_{p=0}^{k} \binom{k}{p} x^p a^{k-p} = x^k + kx^{k-1}a + \frac{k(k-1)}{2}x^{k-2}a^2 + \ldots$.
For the q-discrete case, substitute in the expressions $(qx)^k = x^k q^k$, and $(x/q)^k = x^k q^{-k}$.

Step 3. Equating the coefficients of the highest power of x, i.e., x^n gives λ_n or $\lambda_{q,n}$, given by the relations (9) or (12).

Step 4. Equating the coefficients of x^{n-1} and x^{n-2}, i.e., the second and third highest powers of x gives k'_n and k''_n, respectively, as rational multiples of k_n.

Step 5. Substitute $p_n(x)$ defined by equation (7), into the proposed recurrence equation given by equation (5) and equate again the three highest coefficients to obtain A_n, B_n and C_n, respectively.

Step 6. Substitute in the values of k'_n and k''_n gotten in step 4 into the equations in step 5 to express the coefficients A_n, B_n, C_n in terms of $a, b, c, d, e, n, k_{n-1}, k_n, k_{n+1}$.

Proposition 2.1 (see Koepf and Schmersau (1998)). *Let $p_n(x) = k_n x^n + k'_n x^{n-1} + k''_n x^{n-2} + \ldots$ be a family of polynomial solutions of the differential equation (8). Then the recurrence equation (5) is valid with:*

$$\frac{k_n}{k_{n+1}} A_n = 1, \tag{14}$$

$$\frac{k_n}{k_{n+1}} B_n = \frac{2bn(an+d-a) - e(2a-d)}{(d+2an)(d-2a+2an)}, \tag{15}$$

$$\begin{aligned} \frac{k_{n-1}}{k_{n+1}} C_n = & -\frac{(an+d-2a)n}{(d-2a+2an)^2(2an-3a+d)(2an-a+d)} \\ & \times ((an+d-2a) - n(4ca-b^2) + 4a^2c - ab^2 + ae^2 \\ & - 4acd + db^2 - bed + d^2c), \end{aligned} \tag{16}$$

in terms of the coefficients a, b, c, d, e of $\sigma(x)$ and $\tau(x)$ given in the differential equation (8).

Let $p_n(x) = k_n x^n + k'_n x^{n-1} + k''_n x^{n-2} + \ldots$ be a family of polynomial solutions of the difference equation (10). Then the recurrence equation (5) is valid with:

$$\frac{k_n}{k_{n+1}} A_n = 1, \tag{17}$$

$$\frac{k_n}{k_{n+1}} B_n = \frac{n(d+2b)(d+an-a) + e(d-2a)}{(2an-2a+d)(d+2an)}, \tag{18}$$

$$\begin{aligned} \frac{k_{n-1}}{k_{n+1}} C_n = & -\frac{(an+d-2a)n}{(d-a+2an)(d+2an-3a)(2an-2a+d)^2} \\ & \times ((n-1)(d+an-a)(and-db-ad+a^2n^2-2a^2n \\ & + 4ca + a^2 + 2ea - b^2) - dbe + d^2c + ae^2), \end{aligned} \tag{19}$$

in terms of the coefficients a, b, c, d, e of $\sigma(x)$ and $\tau(x)$ given in the difference equation (10).

Let $p_n(x) = k_n x^n + k'_n x^{n-1} + k''_n x^{n-2} + \ldots$ be a family of polynomial solutions of the q-difference equation (11). Then the recurrence equation (5) is valid with (where $q^n = N$):

$$\frac{k_n}{k_{n+1}} A_n = 1, \tag{20}$$

$$\begin{aligned} \frac{k_n}{k_{n+1}} B_n = & \left(\frac{1-Nq}{1-q} \times \frac{b - bN + eN - eNq}{a - aN^2 + dN^2 - dN^2q}\right) \\ & - \left(\frac{1-N}{1-q} \times \frac{b - b\frac{N}{q} + e\frac{N}{q} - eN}{a - a\frac{N^2}{q^2} + d\frac{N^2}{q^2} - d\frac{N^2}{q}}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{k_{n-1}}{k_{n+1}} C_n = & \Big((N-1)((dq+a-d)N - aq^2) \\ & \times ((-ae^2q^4 + b^2cdq^2 + abeq - 2acdq - a^2c + 2acd - cd^2) N^4 \\ & + (abeq^2 - 2bdeq^2 + beq^3 - abeq + abq^2 + bdeq) N^3 \\ & + (-2abeq^3 + 2acdq^3 + a^2cq^2 - 2abdq^2 - 2acdq^2 - b^2d^2q + 2ab^2e \end{aligned}$$

Table 1 Normal forms of classical continuous orthogonal polynomials

$\sigma(x)$	$\tau(x)$	$p_n(x)$	Family
1	$-2x$	$H_n(x)$	Hermite
x	$-x+\alpha+1$	$L_n^{(\alpha)}(x)$	Laguerre
x^2	$(\alpha+2)x+2$	$B_n^{(\alpha)}(x)$	Bessel
$(x+1)(x-1)$	$(\alpha+\beta+2)x+\alpha-\beta$	$P_n^{(\alpha,\beta)}(x)$	Jacobi

Table 2 Normal forms of classical discrete orthogonal polynomials

$\sigma(x)$	$\tau(x)$	$\sigma(x)+\tau(x)$	$p_n(x)$	Family
x	$\mu-x$	$\mu(\mu\neq 0)$	$C_n^{(\mu)}(x)$	Charlier
x	$\mu\gamma+(\mu-1)x$	$\mu(\gamma+x)$	$M_n^{(\gamma,\mu)}(x)$	Meixner
$x(1-p)$	$pN-x$	$p(N-x)$	$K_n^{(p)}(x,N)$	Krawtchouk
$x(\beta+N+1-x)$	$N(\alpha+1)-x(2+\beta+\alpha)$	$(x+\alpha+1)(x-N)$	$Q_n^{(\alpha,\beta)}(x,N)$	Hahn

$$+3abq^2+b^2dq+2e^2q^2)N^2-Nabeq^3-a^2cq^4)Nq\Big)/\Big(\left(N^2dq-aq\right)$$
$$\times\left((dq+a-d)\,N^2-aq^3\right)\left((dq+a-d)\,N^2-aq^2\right)^2\Big), \tag{22}$$

in terms of the coefficients a, b, c, d, e of $\sigma(x)$ and $\tau(x)$ given in the q-difference equation (11).

In the `Maxima` and `Maple` files associated to this manuscript, we recover these coefficients A_n, B_n, C_n of the three-term recurrence equations for each of the classical orthogonal polynomial families as given in Koekoek et al. (2010).

3 From recurrence equations to orthogonal polynomials

If a polynomial system is a solution of the differential, difference, or q-difference equation (8), (10), or (11), respectively, it can be categorized based on the zeros of $\sigma(x)$ and $\tau(x)$. These classical continuous, discrete and q-discrete orthogonal polynomials subject to linear transformations can be classified according to Table 1, Table 2, and Table 3. It is proved for example in Castillo and Petronilho (2023) through an equivalence relation that, up to constant factors and affine changes of variables, the four families of polynomials Hermite, Laguerre, Jacobi, and Bessel are the only families of classical orthogonal polynomials.

Proposition 2.1 makes use of a rational approach to obtain the recurrence equation given $\frac{k_{n+1}}{k_n}\in\mathbb{Q}(n)$.

Suppose that a polynomial system satisfies equation (8), (10) or (11), then from Table 1, Table 2, and Table 3, respectively we know the system and its three-term recurrence relation.

However, given an arbitrary holonomic three-term recurrence equation,

$$q_n(x)p_{n+2}(x)+r_n(x)p_{n+1}(x)+s_n(x)p_n(x)=0, \tag{23}$$

$q_n(x), r_n(x), s_n(x)\in\mathbb{Q}[n,x]),$ or $q_n(x), r_n(x), s_n(x)\in\mathbb{Q}[q^n,q,x]$, it can be challenging to determine if an orthogonal polynomial system (of type (7)) that satisfies this equation exists,

Table 3 Normal forms of classical q-orthogonal polynomials

$\sigma(x)$	$\tau(x)$	$p_n(x)$	Family
$\frac{(aq-x)(cq-x)}{q}$	$\frac{abq^2(x-1)-acq^2+aq+cq-x}{q(q-1)}$	$P_n(x;a,b,c;q)$	Big q-Jacobi
$\frac{x(x-1)}{q}$	$\frac{abq^2x-aq-x+1}{q(q-1)}$	$p_n(x;a,b;q)$	Little q-Jacobi
$\frac{x}{q}$	$\frac{q^{\alpha+1}(x+1)-1}{q(q-1)}$	$L_n^{(\alpha)}(x;q)$	q-Laguerre
$-\frac{(x-1)x}{q}$	$\frac{aqx+x-1}{q(q-1)}$	$y_n(x;a;q)$	q-Bessel
$\frac{(x-1)(x-a)}{q}$	$\frac{a-x+1}{q(q-1)}$	$U_n^a(x;q)$	Al-Salam Carlitz I
a	$\frac{x-a-1}{q-1}$	$V_n^a(x;q)$	Al-Salam Carlitz II
$\frac{c(x-bq)}{q}$	$\frac{bcq+qx-c-q}{q(q-1)}$	$M_n(x;b,c;q)$	q-Meixner
$\frac{x(xq^N-1)}{q^{N+1}}$	$-\frac{pq^{N+1}x-pq^{N+1}+xq^N-1}{q^{N+1}(q-1)}$	$K_n(x;p,N;q)$	q-Krawtchouk
$\frac{x}{q}$	$\frac{qx-1}{q(q-1)}$	$S_n(x;q)$	Stieltjes-Wigert
$\frac{(x-\alpha q)(xq^N-1)}{q^{N+1}}$	$\frac{\alpha\beta q^{N+2}(x-1)+q^N(\alpha\beta-x)-\alpha q+1}{q^{N+1}(q-1)}$	$Q_n(x;\alpha,\beta,N;q)$	q-Hahn

especially one that might be a linear transformation of a classical orthogonal polynomial system and to accurately identify which system it is. To address this difficulty, an algorithm has been developed to assist in solving this problem. To solve this inverse problem we use Proposition 2.1, and the coefficients of each of the classical families, given Favard's theorem holds (Marcellán and Álvarez-Nodarse 2001).
Algorithm (see Koepf and Schmersau 1996, 2002): This algorithm begins with the holonomic three-term recurrence equation (23) as input and checks if it has (linear transformations of) classical orthogonal polynomial solutions and returns all data if they exist (the differential/difference/q-difference equation, the weight function, $\frac{k_{n+1}}{k_n}$, the interval of orthogonality).

1. **Input:** A holonomic three-term recurrence equation of the form (23).
2. **Shift:** Shift by

$$N := \begin{cases} 0 & \text{if } q_{n-1}(x) \text{ and } s_n(x) \text{ have no non-negative integer zero.} \\ \max\{n \in \mathbb{N}_0 | n \text{ is a zero of } q_{n-1}(x) \text{ or } s_n(x)\} + 1, & \text{otherwise.} \end{cases}$$

3. **Rewriting:** Rewrite the recurrence equation in the form

$$p_{n+1}(x) = t_n p_n(x) + u_n(x) p_{n-1}(x), \quad t_n(x), u_n(x) \in \mathbb{Q}(n, x) \text{ or } \mathbb{Q}(q^n, q, x).$$

   If either $t_n(x)$ is not a polynomial of degree one in x or $u_n(x)$ is not a constant with respect to x, then return "no classical orthogonal polynomial solution exists" and exit.
4. **Linear transformation:** Rewrite the recurrence equation by the linear transformation $x \mapsto \frac{x-g}{f}$, with unknowns f and g.
5. **Standardization:** Rewrite the new recurrence equation as

$$p_{n+1}(x) = (A_n x + B_n) p_n(x) - C_n p_{n-1}(x), \tag{24}$$

   $A_n, B_n, C_n \in \mathbb{Q}(n)$ or $\mathbb{Q}(q^n, q, x)$, $A_n \neq 0$. Following Proposition 2.1, define

$$\frac{k_{n+1}}{k_n} := A_n = \frac{v_n}{w_n} \qquad (v_n, w_n \in \mathbb{Q}[n] \text{ or } \mathbb{Q}[q^n, q]).$$

6. **Make monic:** We have that $p_n(x) = k_n \tilde{p}_n(x)$, where $\tilde{p}_n(x)$ is the monic family. Rewrite (24) as

$$\tilde{p}_{n+1}(x) = (x + \tilde{B}_n) \tilde{p}_n(x) - \tilde{C}_n \tilde{p}_{n-1}(x),$$

   where

$$\tilde{B}_n := \frac{B_n}{A_n} \in \mathbb{Q}(n) \text{ or } \mathbb{Q}(q^n, q) \text{ and } \tilde{C}_n := \frac{C_n}{A_n A_{n-1}} \in \mathbb{Q}(n) \text{ or } \mathbb{Q}(q^n, q).$$

   Reduce these rational functions to lowest terms. According to Proposition 2.1, in the continuous and discrete cases (with n as the variable), the degree of either the numerator or denominator of $\tilde{B}_n$ should not exceed 2. In the q-discrete case (with $q^n = N$ as the variable), the degree of both the numerator and denominator of $\tilde{B}_n$ cannot exceed 6. Similarly, for $\tilde{C}_n$, the degree of the numerator or denominator must not exceed 4 in the continuous case, 6 in the discrete case, and 8 in the q-discrete case. If any of these conditions are violated, then return "no classical orthogonal polynomial solution exists" and exit.
7. **Polynomial identities:** Set

$$\tilde{B}_n := \frac{k_n}{k_{n+1}} B_n, \quad \tilde{C}_n := \frac{k_{n-1}}{k_{n+1}} C_n,$$

where the right hand sides are obtained for each of the classical families. Clear the fractions and then return them in polynomial form with n or $q^n = N$ as variable. This step is necessary because we know that the set $\{1, n, \ldots, n^k\}$ is a basis of polynomials of degree k of the variable n. A direct consequence is that having $a_0 + a_1 n + \ldots a_k n^k = 0$ implies the coefficients $a_0 = a_1 = \ldots = a_k = 0$ which leads to the next step.

8. **Equating coefficients:** Equate the coefficients of the powers of n (continuous and discrete cases), or N (q-discrete case) in the two resulting equations. This gives a nonlinear system of equations with unknowns f, g and any parameters in the equation. If the system has no solution, then return "no classical orthogonal polynomial solution exists" and exit.
9. **Output:** Return the classical orthogonal polynomial solutions of the differential equation (8), difference equation (10), or q-difference equation (11) given by the solution vectors (f, g) and parameters of the previous step, according to the classifications given in Table 1, Table 2, and Table 3 together with the standardization given by the relations (Eqs.(14), (17) and (20)). Return also the necessary linear transformation $y = fx + g$, and the weight function $w(x)$ given as

$$\frac{w(x)}{C} = \frac{1}{\sigma(x)} \exp\left(\int \frac{\tau(x)}{\sigma(x)} dx\right) \qquad \text{(Continuous case)},$$
$$\frac{w(x+1)}{w(x)} = \frac{\sigma(x) + \tau(x)}{\sigma(x+1)} \qquad \text{(Discrete case)},$$
$$\frac{w(qx)}{w(x)} = \frac{\sigma(x) + (q-1)x\tau(x)}{\sigma(qx)} \qquad (q\text{-Discrete case}),$$

and the supporting interval. Note that the weight function is the solution of the Pearson equation:

$$\frac{d}{dx}(\sigma(x)w(x)) = \tau(x)w(x) \qquad \text{(Continuous case)},$$
$$\Delta(\sigma(x)w(x)) = \tau(x)w(x) \qquad \text{(Discrete case)},$$
$$D_q(\sigma(x)w(x)) = \tau(x)w(x) \qquad (q\text{-Discrete case}),$$

that classical orthogonal polynomials satisfy and satisfies the boundary conditions

$$\lim_{x \to A, x > A} \sigma(x)\omega(x)x^k = \lim_{x \to B, x < B} \sigma(x)\omega(x)x^k = 0, \; k \geq 0.$$

The primary distinction between our algorithm and that presented in Koepf and Schmersau (2002) occurs at Item 7. In our approach, we determine $\tilde{B}_n$ and $\tilde{C}_n$ for each classical orthogonal family categorized in Tables 1, 2 and 3. This enhancement enables us to identify a broader range of relations and solutions than the previous algorithm while simultaneously reducing computational complexity, and avoiding the need for extra hand computations (Nwoku et al. May 2024).

4 Implementation and results

We have implemented the above algorithm in `Maxima` and `Maple` for each of the classical families. The choice of function depends on the type of variable in the given recurrence equation: `REtoDE` is used when the variable is continuous, `REtoDiscrete` when the variable is discrete, and `REtoqDE` when the variable is q-discrete. This distinction corresponds to the underlying lattice on which the orthogonal polynomials are defined.

The function `REtoDE(RE, p[n], x)` for the continuous case is a collection of four functions (`REtoJacobi(RE, p[n], x)`, `REtoLaguerre(RE, p[n], x)`, `REtoHermite(RE, p[n], x)`, `REtoBessel(RE, p[n], x)`), for the Jacobi, Laguerre, Hermite, and Bessel polynomials, respectively. Additionally, `REtoDiscrete(RE, p[n], x)` in the discrete case is also a collection of four functions for the Meixner, Charlier, Krawtchouk, and Hahn polynomials, while `REtoqde(RE, p[n], x, q)` in the q-discrete case is a collection of ten functions for the classical q-orthogonal polynomials listed in Table 3. The functions take as input the recurrence equation `RE` (of type (23)), the polynomial `p[n]` of degree n, and the variable `x`, where `p[n]` is a polynomial of the variable `x` and solution of the three-term recurrence equation `RE`.

The identification of the classical family occurs within the main function, which sequentially tests each family specific algorithm. Each function (`REtoJacobi`, `REtoLaguerre`, `REtoHermite`, etc.) is called in turn within the larger function (`REtoDE`, `REtoDiscrete`, or `REtoqde`). The algorithm checks whether the input recurrence satisfies the defining relations for that family, and the function that exits without errors determines the corresponding classical family. The successful algorithm then returns the associated parameters and differential, difference or q-difference equation.

The `Maxima` and `Maple` packages defining these functions can be downloaded from https://www.mathematik.uni-kassel.de/~koepf/Publikationen, article "Recurrence equations and their classical continuous, discrete, and q-discrete orthogonal polynomial solutions."

Example 1 Consider the recurrence equation for equation (7) in Tcheutia and Koepf (2024)

$$\begin{aligned} &P_{n+1}(x)\,(-p+2n+3)\,\left(p^2x-4npx-6px+4n^2x+12nx+8x-pq-2np\right.\\ &\quad\left.-3p+2n^2+6n+4\right)\\ &\quad+P_n(x)\,(n+1)\,(-p+2n+4)\,(-q-p+n+1)\,(q+n+1)\\ &\quad+P_{n+2}(x)\,(-p+n+2)\,(-p+2n+2)=0, \end{aligned} \tag{25}$$

satisfied by the polynomials, with continuous variable x (see Masjed-Jamei 2002, (2.3))

$$M_n^{(p,q)}(x)=(-1)^n n!\binom{n+q}{n}\,{}_2F_1\left(\begin{matrix}-n,-p+n+1\\ q+1\end{matrix};-x\right).$$

With our `Maxima` implementation we obtain the result by:

```
Mnpq: ((-p+2*n+2)*(-p+n+2))*P[n+2] + ((2*n-p+3)*(4*n^2*x
- 4*n*p*x + p^2*x + 2*n^2 - 2*p*n + 12*x*n - p*q - 6*p*x + 6*n
- 3*p + 8*x + 4))*P[n+1] + ((q + n+1)*(n+1)*(2*n-p+4)
*(n-p-q+1))*P[n] = 0;

REtoDE(Mnpq, P[n], x);
```

With solutions as:
"Warning, parameters have the values":

$$\left[f=-2\,,\,g=-1\,,\,\text{alphaJacobi}=-q-p\,,\,\text{betaJacobi}=q\right],$$

$$\left[f=2\,,\,g=1\,,\,\text{alphaJacobi}=q\,,\,\text{betaJacobi}=-q-p\right].$$

"Warning, several solutions found", "Has a solution as Jacobi".

$$\sigma(x)=x^2+x\,,\,\tau(x)=-px+2x+q+1\,,\,\lambda_n=np-n^2-n\,,$$

$$\mathrm{p}\,(n\,,x) = J_n\,(-2x-1\,,-q-p\,,q)\,,\,\mathrm{w}(x) = e^{-q\log(x+1)-p\log(x+1)+q\log(x)}\,,$$
$$\frac{k_{n+1}}{k_n} = -\frac{(p-2n-2)\,(p-2n-1)}{2\,(p-n-1)}\,,\,I=[-1\,,0]\,,$$

$$\sigma(x) = x^2+x\,,\,\tau(x) = -px+2x+q+1\,,\,\lambda_n = np-n^2-n\,,$$
$$\mathrm{p}\,(n\,,x) = J_n\,(2x+1\,,q\,,-q-p)\,,\,\mathrm{w}(x) = e^{-q\log(x+1)-p\log(x+1)+q\log(x)}\,,$$
$$\frac{k_{n+1}}{k_n} = \frac{(p-2n-2)\,(p-2n-1)}{2\,(p-n-1)}\,,\,I=[-1\,,0]\,,$$

where alphaJacobi, betaJacobi are the parameters α, β for the Jacobi polynomials, $J_n(x,\alpha,\beta) = P_n^{(\alpha,\beta)}(x)$. Therefore we have the classical orthogonal polynomial solutions as: $P_n^{(q,-q-p)}(2x+1)$, and $P_n^{(-q-p,q)}(-(2x+1))$. Hence $M_n^{(p,q)}(x) = C_n P_n^{(q,-q-p)}(2x+1)$, and equating the coefficients of x^n we obtain $C_n = (-1)^n n!$.

We recover here the relation given after equation (10) in Tcheutia and Koepf (2024), i.e., $M_n^{(-\alpha-\beta,\alpha)}\left(\frac{1}{2}(x-1)\right) = (-1)^n n! P_n^{(\alpha,\beta)}(x)$ and an additional relation derived from Jacobi's symmetry relation, $P_n^{(\alpha,\beta)}(-x) = (-1)^n P_n^{(\beta,\alpha)}(x)$.

This shows that equation (25) has classical orthogonal polynomial solutions as linear transformations of the Jacobi polynomials. Unlike with Koepf-Schmersau's `retode` the exact form of the linear transformation, and the polynomial family is shown.

Example 2 Consider the recurrence equation of the Masjed-Jamei polynomials defined by equation (6), with continuous variable x

$$\begin{aligned}
&(n-p+1)(n+2-2p)S(n+2) + (2n+3-2p)\Big(2a^2n^2x - 4a^2npx\\
&+2a^2p^2x + 2c^2n^2x - 4c^2npx + 2c^2p^2x + 6a^2nx - 6a^2px + 2abn^2\\
&-4abnp + 2abp^2 - adpq + bcpq + 6c^2nx - 6c^2px + 2cdn^2 - 4cdnp\\
&+2cdp^2 + 4a^2x + 6abn - 6abp + 4c^2x + 6cdn - 6cdp + 4ab + 4cd\Big)\\
&S(n+1) - (n+1)(-p+2+n)(4n^2 - 8np + 4p^2 + q^2 + 8n - 8p + 4)\\
&(ad-bc)^2S(n) = 0.
\end{aligned} \tag{26}$$

With our `Maple` implementation we obtain the result by:

```
Rjn := (n-p+1)*(n+2-2*p)*S(n+2)+(2*n+3-2*p)*(2*a^2*n^2*x-4*a^2
*n*p*x+2*a^2*p^2*x+2*c^2*n^2*x-4*c^2*n*p*x+2*c^2*p^2*x+6*a^2*n
*x-6*a^2*p*x+2*a*b*n^2-4*a*b*n*p+2*a*b*p^2-a*d*p*q+b*c*p*q+6
*c^2*n*x-6*c^2*p*x+2*c*d*n^2-4*c*d*n*p+2*c*d*p^2+4*a^2*x+6*a*b
*n-6*a*b*p+4*c^2*x+6*c*d*n-6*c*d*p+4*a*b+4*c*d)*S(n+1)-(n+1)*
(-p+2+n)*(4*n^2-8*n*p+4*p^2+q^2+8*n-8*p+4)*(a*d-b*c)^2*S(n) = 0
```

```
REtoDE(Rjn, S(n), x)
```

With solutions as: "Warning, parameters have the values":

$$\left\{\left\{aJ = -\frac{1}{2}\mathrm{i}q - p,\quad bJ = \frac{1}{2}\mathrm{i}q - p,\, f = -\frac{\mathrm{i}(a^2+c^2)}{ad-bc},\quad g = -\frac{\mathrm{i}(ab+cd)}{ad-bc}\right\},\right.$$
$$\left.\left\{aJ = \frac{1}{2}\mathrm{i}q - p,\quad bJ = -\frac{1}{2}\mathrm{i}q - p,\, f = \frac{\mathrm{i}(a^2+c^2)}{ad-bc},\quad g = \frac{\mathrm{i}(ab+cd)}{ad-bc}\right\}\right\}.$$

“Warning, several solutions found”, “Has a solution as Jacobi”.

$$\sigma(x) = a^2x^2 + c^2x^2 + 2abx + 2cdx + b^2 + d^2,$$
$$\tau(x) = -2a^2px - 2c^2px + 2a^2x - 2abp + adq - bcq + 2c^2x - 2cdp + 2ab + 2cd,$$
$$\lambda_n = -n(a^2 + c^2)(n + 1 - 2p),$$
$$S(n, x) = P_n\left(-\tfrac{1}{2}\mathrm{i}q - p, \tfrac{1}{2}\mathrm{i}q - p, -\mathrm{i}\frac{(a^2 + c^2)x}{ad - bc} - \mathrm{i}\frac{ab + cd}{ad - bc}\right),$$
$$w(x) = \left(a^2x^2 + c^2x^2 + 2abx + 2cdx + b^2 + d^2\right)^{-p}$$
$$\times \exp\left(\arctan\left(\frac{a^2x + c^2x + ab + cd}{ad - bc}\right)q\right),$$
$$\frac{k_{n+1}}{k_n} = \frac{2\mathrm{i}(ad - bc)(n - p + 1)(2n + 1 - 2p)}{n + 1 - 2p}, \quad I = \left[-\frac{b + id}{ic + a}, -\frac{id - b}{ic - a}\right],$$

$$\sigma(x) = a^2x^2 + c^2x^2 + 2abx + 2cdx + b^2 + d^2,$$
$$\tau(x) = -2a^2px - 2c^2px + 2a^2x - 2abp + adq - bcq + 2c^2x - 2cdp + 2ab + 2cd,$$
$$\lambda_n = -n(a^2 + c^2)(n + 1 - 2p),$$
$$S(n, x) = P_n\left(\tfrac{1}{2}\mathrm{i}q - p, -\tfrac{1}{2}\mathrm{i}q - p, \mathrm{i}\frac{(a^2 + c^2)x}{ad - bc} + \mathrm{i}\frac{ab + cd}{ad - bc}\right),$$
$$w(x) = \left(a^2x^2 + c^2x^2 + 2abx + 2cdx + b^2 + d^2\right)^{-p}$$
$$\times \exp\left(\arctan\left(\frac{a^2x + c^2x + ab + cd}{ad - bc}\right)q\right),$$
$$\frac{k_{n+1}}{k_n} = \frac{2\mathrm{i}(ad - bc)(n - p + 1)(2n + 1 - 2p)}{n + 1 - 2p}, \quad I = \left[-\frac{id - b}{ic - a}, -\frac{b + id}{ic + a}\right],$$

where aJ, bJ are the parameters α, β for the Jacobi polynomials, $P_n(\alpha,\ \beta,\ x) = P_n^{(\alpha,\beta)}(x)$. Therefore we have the classical orthogonal polynomial solutions as: $P_n^{\left(\frac{1}{2}\mathrm{i}q-p,-\frac{1}{2}\mathrm{i}q-p\right)}\left(\mathrm{i}\frac{(a^2+c^2)x}{ad-bc} + \mathrm{i}\frac{ab+cd}{ad-bc}\right)$, and $P_n^{\left(-\frac{1}{2}\mathrm{i}q-p,\frac{1}{2}\mathrm{i}q-p\right)}\left(-\mathrm{i}\frac{(a^2+c^2)x}{ad-bc} - \mathrm{i}\frac{ab+cd}{ad-bc}\right)$.

Hence $J_n^{(p,q)}(x; a, b, c, d) = C_n P_n^{\left(\frac{1}{2}\mathrm{i}q-p,-\frac{1}{2}\mathrm{i}q-p\right)}\left(\mathrm{i}\frac{(a^2+c^2)x}{ad-bc} + \mathrm{i}\frac{ab+cd}{ad-bc}\right)$, equating the coefficients of x^n we obtain $C_n = \frac{2^n(ad-bc)^n n!}{(i)^n}$. We recover here the relation given as equation (20) in Tcheutia and Koepf (2024) and an additional relation derived from Jacobi's symmetry relation, $P_n^{(\alpha,\beta)}(-x) = (-1)^n P_n^{(\beta,\alpha)}(x)$.

This shows that the polynomial system (6) is a linear transformation of the Jacobi polynomials. However Koepf-Schmersau's `retode` is unable to detect this solution.

While our algorithm successfully determines the orthogonality relationships for most classical families, it encounters limitations when addressing the Masjed-Jamei polynomials (equation (6)). Though Masjed-Jamei established that these polynomials are orthogonal across the entire real line (Masjed-Jamei 2002), our implementation returns complex-valued interval bounds. This is a limitation of our current methodology and highlights an area for further investigation.

Example 3 Consider the recurrence equation given as Example 4 in Koepf and Schmersau (2002), with discrete variable x

$$p_{n+1}(x)\,(2n+3)\,(-4n^2x-12nx-8x-2\alpha n^2+2Nn^2-6\alpha n+6Nn \\ -4\alpha+4N)+p_{n+2}(x)\,(n+2)\,(n-N+1)\,(n+\alpha+2)\,(2n+2)- \\ p_n(x)\,(n+1)\,(n+N+2)\,(n-\alpha+1)\,(2n+4)=0. \tag{27}$$

With our `Maxima` implementation we obtain the result by:

```
RE: ((n+2+alpha)*(2+n)*(2*n+2)*(n-N+1))*p[n+2] + ((3+2*n)
*(-6*n*alpha - 2*n^2*alpha - 4*n^2*x - 12*n*x + 2*n^2*N + 6*n*N
+ 4*N - 4*alpha - 8*x))*p[n+1] - ((1+n)*(n + 1-alpha)*(2*n+4)
*(n+N+2))*p[n] = 0;

REtoDiscrete(RE, p[n], x);
```

We obtain solutions as transformations of the Hahn polynomials, some solutions are: "Warning, parameters have the values":

$$[f=1\,,g=0\,,\text{alphaHahn}=\alpha\,,\text{betaHahn}=-\alpha\,,\text{NHahn}=N],$$
$$\left[f=1\,,g=\alpha\,,\text{alphaHahn}=-\alpha\,,\text{betaHahn}=\alpha\,,\text{NHahn}=N\right],$$
$$\left[f=-1\,,g=N\,,\text{alphaHahn}=-\alpha\,,\text{betaHahn}=\alpha\,,\text{NHahn}=N\right],$$
$$\left[f=-1\,,g=N-\alpha\,,\text{alphaHahn}=\alpha\,,\text{betaHahn}=-\alpha\,,\text{NHahn}=N\right].$$

"Warning, several solutions found", "Has a solution as Hahn".

$$\sigma(x)=-x^2-\alpha x+Nx+x\,,\tau(x)=-2x+N\alpha+N\,,\lambda_n=n^2+n\,,$$
$$\mathrm{p}\,(n\,,x)=Q_n\,(x\,,\alpha\,,-\alpha\,,N)\,,\,\frac{\mathrm{w}\,(x+1)}{\mathrm{w}(x)}=\frac{(x-N)\,(x+\alpha+1)}{(x+1)\,(x+\alpha-N)}$$
$$,\,\frac{k_{n+1}}{k_n}=\frac{2\,(2n+1)}{(n-N)\,(n+\alpha+1)},\,I=[0\,,1\,,2\,,\ldots,N]\,,$$

$$\sigma(x)=-x^2-\alpha x+Nx+x+N\alpha+\alpha\,,\tau(x)=-2x-N\alpha-2\alpha+N\,,$$
$$\lambda_n=n^2+n\,,\mathrm{p}\,(n\,,x)=Q_n\,(x+\alpha\,,-\alpha\,,\alpha\,,N)\,,$$
$$\frac{\mathrm{w}\,(x+1)}{\mathrm{w}(x)}=\frac{(x+1)\,(x+\alpha-N)}{(x-N)\,(x+\alpha+1)}\,,$$
$$\frac{k_{n+1}}{k_n}=\frac{2\,(2n+1)}{(n-N)\,(n+\alpha+1)},\,I=[-\alpha\,,1-\alpha\,,2-\alpha\,,\ldots,N-\alpha]\,,$$

$$\sigma(x)=-x^2-\alpha x+Nx-x+N\alpha+N\,,\tau(x)=2x-N\alpha-N\,,\lambda_n=n^2+n\,,$$
$$\mathrm{p}\,(n\,,x)=Q_n\,(N-x\,,-\alpha\,,\alpha\,,N)\,,\,\frac{\mathrm{w}\,(x+1)}{\mathrm{w}(x)}=\frac{x\,(x+\alpha-N-1)}{(x-N+1)\,(x+\alpha+2)}\,,$$
$$\frac{k_{n+1}}{k_n}=-\frac{2\,(2n+1)}{(n-N)\,(n+\alpha+1)},\,I=[0\,,\ldots,N-2\,,N-1\,,N]\,,$$

$$\sigma(x)=-x^2-\alpha x+Nx-x-\alpha+N\,,\tau(x)=2x+N\alpha+2\alpha-N\,,$$

$$\lambda_n = n^2 + n\,,\, \mathrm{p}\,(n\,, x) = Q_n\,(-x - \alpha + N\,, \alpha\,, -\alpha\,, N)\,,$$
$$\frac{\mathrm{w}\,(x+1)}{\mathrm{w}(x)} = \frac{(x - N - 1)\,(x+\alpha)}{(x+2)\,(x+\alpha-N+1)}\,,\, \frac{k_{n+1}}{k_n} = -\frac{2\,(2n+1)}{(n-N)\,(n+\alpha+1)},$$
$$I = [-\alpha\,, \ldots, -\alpha + N - 2\,, -\alpha + N - 1\,, N - \alpha]\,.$$

We also obtain solutions as transformations of the Meixner polynomials (N, α are considered parameters), some solutions are:
"Warning, parameters have the values":

$$\left[N = -\frac{1}{2}\,, f = 2\,, \alpha = 0\,, g = 0\,, \text{gammaMeixner} = 1\,, \text{muMeixner} = -1\right],$$
$$\left[N = -1\,, f = -2\,, \alpha = -\frac{1}{2}\,, g = -1\,, \text{gammaMeixner} = 1\,, \text{muMeixner} = -1\right].$$

"Warning, several solutions found", "Has a solution as Meixner".

$$\sigma\,(x) = 2x\,,\, \tau\,(x) = -4x - 1\,,\, \lambda_n = 2n\,,\, \mathrm{p}\,(n\,, x) = M_n\,(2x\,, 1\,, -1)\,,$$

where alphaHahn, betaHahn, NHahn are the parameters α, β, N for the Hahn polynomials, and $Q_n(x, \alpha, \beta, N) = Q_n^{(\alpha,\beta)}(x, N)$. gammaMeixner, muMeixner are the parameters γ, μ for the Meixner polynomials, and $M_n(x, \gamma, \mu) = M_n^{(\gamma,\mu)}(x)$.

This shows that this recurrence equation (27) has classical orthogonal polynomial solutions as transformations of Hahn and Meixner polynomials. For the solutions obtained as transformations of Meixner polynomials N, α are considered parameters and these solutions are only possible when N, α are equal to the values obtained above, otherwise no solution exists.

Our implementation detects the additional solutions as Meixner polynomials while the Maple implementation by Koepf and Schmersau (2002) only detects Hahn polynomial solutions.

Example 4 Consider the recurrence equation, with q-discrete variable x

$$\begin{aligned}
&\left(2q^{2n+2}\beta-1\right)\left(2q^{n+2}\beta-1\right)^2\left(-q^{n+1}+q^N\right)S(n+2)-\left(2q^{2n+3}\beta-1\right)\\
&\Big(4q^{N+4n+6}x\beta^2-8q^{N+3n+5}\beta^2+4q^{N+2n+4}\beta^2-2q^{N+2n+4}x\beta-4q^{3n+4}\beta^2\\
&+4q^{N+2n+3}\beta^2+2q^{N+2n+4}\beta-2q^{3n+4}\beta+2q^{N+2n+3}\beta-2q^{N+2n+2}x\beta\\
&+4q^{2n+3}\beta-4q^{N+n+2}\beta+4q^{2n+2}\beta-2\beta q^{n+1}+xq^N-q^{n+1}\Big)S(n+1)\\
&+2\left(q^{n+1}-1\right)^2q^{n+1}\left(2q^{2n+4}\beta-1\right)\left(2q^{N+n+2}\beta-1\right)\beta S(n)=0.
\end{aligned}\tag{28}$$

With our `Maple` implementation we obtain the result by:

```
RE := (2*q^(2*n+2)*beta-1)*(2*q^(n+2)*beta-1)^2*(-q^(n+1)+q^N)
*S(n+2)-(2*q^(2*n+3)*beta-1)*(4*q^(N+4*n+6)*x*beta^2-
8*q^(N+3*n+5)*beta^2+4*q^(N+2*n+4)*beta^2-2*q^(N+2*n+4)*x
*beta-4*q^(3*n+4)*beta^2+4*q^(N+2*n+3)*beta^2+2*q^(N+2*n+4)
*beta-2*q^(3*n+4)*beta+2*q^(N+2*n+3)*beta-2*q^(N+2*n+2)*x*beta
+4*q^(2*n+3)*beta-4*q^(N+n+2)*beta+4*q^(2*n+2)*beta-2*beta
*q^(n+1)+x*q^N-q^(n+1))*S(n+1)+2*(q^(n+1)-1)^2*q^(n+1)
*(2*q^(2*n+4)*beta-1)*(2*q^(N+n+2)*beta-1)*beta*S(n) = 0;
```

```
REtoqde(RE, S(n), x, q);
```

We obtain solutions as linear transformations of the Big q-Jacobi polynomials: "Warning, parameters have the values"

$$\left\{\left\{aB = 1, bB = 2\,\beta, cB = 2\,\beta\,q^{1+N}, f = q^{1+N}, g = 0\right\},\right.$$

$$\left\{aB = q^{-N-1}, bB = 2\,\beta\,q^{1+N}, cB = 2\,\beta, f = 1, g = 0\right\},$$

$$\left\{aB = 2\,\beta, bB = 1, cB = q^{-N-1}, f = 1, g = 0\right\},$$

$$\left.\left\{aB = 2\,\beta\,q^{1+N}, bB = q^{-N-1}, cB = 1, f = q^{1+N}, g = 0\right\}\right\}$$

"Warning, several solutions found", "Has a solution as Big q-Jacobi".

$$\sigma\,(x) = -\,(2\,\beta\,q - x)\left(xq^N - 1\right)\left(q^N\right)^3 q^2,$$

$$\tau\,(x) = \frac{\left(2\,q^N\beta\,q^2x - 2\,q^N\beta\,q^2 + 2\,\beta\,qq^N - xq^N - 2\,\beta\,q + 1\right)q^N}{q-1},$$

$$\lambda_{q,n} = -\frac{(q^n-1)\,(2\,\beta\,q^nq - 1)}{q^n\,(q-1)^2\,q}, S\,(n,x) = P_n\left(1, 2\,\beta, 2\,\beta\,q^{1+N}, q^{1+N}x, q\right),$$

$$\frac{\rho\,(qx)}{\rho\,(x)} = \frac{2\,\left(q^N\right)^3\beta\,q^3x - \left(q^N\right)^3 q^2x^2 - 2\,\left(q^N\right)^2\beta\,q^3 - 2\,q^N\beta\,q^2x^2}{\left(q^N\right)^2 q^3\,(2\,\beta - x)\left(qq^Nx - 1\right)}$$

$$\times\left(q^N\right)^2 q^2x + 2\,q^N\beta\,q^2x - 2\,q^N\beta\,qx + q^Nx^2 + 2\,\beta\,qx - x,$$

$$\frac{k_{n+1}}{k_n} = -\frac{\left(2\,(q^n)^2\,q\beta - 1\right)\left(2\,(q^n)^2\,\beta\,q^2 - 1\right)q^N}{(2\,\beta\,q^nq - 1)^2\left(q^n - q^N\right)q^{1+N}}, I = [2\,\beta\,q, q^{-N}],$$

$$\sigma\,(x) = -\frac{(2\,\beta\,q - x)\left(xq^N - 1\right)}{q^Nq},$$

$$\tau\,(x) = \frac{2\,q^N\beta\,q^2x - 2\,q^N\beta\,q^2 + 2\,\beta\,qq^N - xq^N - 2\,\beta\,q + 1}{qq^N\,(q-1)},$$

$$\lambda_{q,n} = -\frac{(q^n-1)\,(2\,\beta\,q^nq - 1)}{q^n\,(q-1)^2}, S\,(n,x) = P_n\left(q^{-N-1}, 2\,\beta\,q^{1+N}, 2\,\beta, x, q\right),$$

$$\frac{\rho\,(qx)}{\rho\,(x)} = -2\,\frac{(-1+x)\,\beta}{2\,\beta - x}, \frac{k_{n+1}}{k_n} = -\frac{\left(2\,(q^n)^2\,q\beta - 1\right)\left(2\,(q^n)^2\,\beta\,q^2 - 1\right)q^N}{(2\,\beta\,q^nq - 1)^2\left(q^n - q^N\right)},$$

$$I = [2\,\beta\,q, q^{-N}],$$

$$\sigma\,(x) = -\frac{(2\,\beta\,q - x)\left(xq^N - 1\right)}{q^Nq},$$

$$\tau\left(x\right)=\frac{2\,q^N\beta\,q^2x-2\,q^N\beta\,q^2+2\,\beta\,qq^N-xq^N-2\,\beta\,q+1}{qq^N\left(q-1\right)},$$

$$\lambda_{q,n}=-\frac{\left(q^n-1\right)\left(2\,\beta\,q^nq-1\right)}{q^n\left(q-1\right)^2},\,S\left(n,x\right)=P_n\left(2\,\beta,1,q^{-N-1},x,q\right),$$

$$\frac{\rho\left(qx\right)}{\rho\left(x\right)}=-2\,\frac{\left(-1+x\right)\beta}{2\,\beta-x},\ \frac{k_{n+1}}{k_n}=-\frac{\left(2\,\left(q^n\right)^2q\beta-1\right)\left(2\,\left(q^n\right)^2\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^nq-1\right)^2\left(q^n-q^N\right)},$$

$$I=[q^{-N},2\,\beta\,q],$$

$$\sigma\left(x\right)=-\left(2\,\beta\,q-x\right)\left(xq^N-1\right)\left(q^N\right)^3q^2,$$

$$\tau\left(x\right)=\frac{\left(2\,q^N\beta\,q^2x-2\,q^N\beta\,q^2+2\,\beta\,qq^N-xq^N-2\,\beta\,q+1\right)q^N}{q-1},$$

$$\lambda_{q,n}=-\frac{\left(q^n-1\right)\left(2\,\beta\,q^nq-1\right)}{q^n\left(q-1\right)^2q},\,S\left(n,x\right)=P_n\left(2\,\beta\,q^{1+N},q^{-N-1},1,q^{1+N}x,q\right),$$

$$\frac{\rho\left(qx\right)}{\rho\left(x\right)}=\frac{2\,\left(q^N\right)^3\beta\,q^3x-\left(q^N\right)^3q^2x^2-2\,\left(q^N\right)^2\beta\,q^3-2\,q^N\beta\,q^2x^2+\left(q^N\right)^2q^2x}{\left(q^N\right)^2q^3\left(2\,\beta-x\right)\left(qq^Nx-1\right)}$$

$$\times(+2\,q^N\beta\,q^2x-2\,q^N\beta\,qx+q^Nx^2+2\,\beta\,qx-x),$$

$$\frac{k_{n+1}}{k_n}=-\frac{\left(2\,\left(q^n\right)^2q\beta-1\right)\left(2\,\left(q^n\right)^2\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^nq-1\right)^2\left(q^n-q^N\right)q^{1+N}},\,I=[q^{-N},2\,\beta\,q].$$

We obtain solutions as transformations of the q-Hahn polynomials:
"Warning, parameters have the values"

$$\left\{\left\{aH=1,bH=2\,\beta,\,f=q^Nq,\,g=0,\,q^{NH}=\frac{1}{2q^N\beta\,q^2}\right\},\right.$$

$$\left\{aH=\frac{1}{q^Nq},bH=2\,\beta\,qq^N,\,f=1,\,g=0,\,q^{NH}=\frac{1}{2\beta\,q}\right\},$$

$$\left\{aH=2\,\beta,bH=1,\,f=1,\,g=0,\,q^{NH}=q^N\right\},$$

$$\left.\left\{aH=2\,\beta\,qq^N,bH=\frac{1}{q^Nq},\,f=q^Nq,\,g=0,\,q^{NH}=q^{-1}\right\}\right\}$$

"Warning, several solutions found", "Has a solution as q-Hahn".

$$\sigma\left(x\right)=-\left(2\,\beta\,q-x\right)\left(xq^N-1\right)\left(q^N\right)^3\beta\,q^3,$$

$$\tau\left(x\right)=\frac{\left(2\,q^N\beta\,q^2x-2\,q^N\beta\,q^2+2\,\beta\,qq^N-xq^N-2\,\beta\,q+1\right)q^N\beta\,q}{q-1},$$

$$\lambda_{q,n}=-\frac{\beta\,\left(q^n-1\right)\left(2\,\beta\,q^nq-1\right)}{q^n\left(q-1\right)^2},$$

$$S\left(n,x\right)=Q_n\left(1,2\,\beta,-\frac{1}{\ln\left(q\right)}\left(\ln\left(q\right)N-\ln\left(1/2\,\frac{1}{\beta\,q^2}\right)\right),qq^Nx,q\right),$$

$$\frac{\rho\left(qx\right)}{\rho\left(x\right)}=\frac{2\,\left(q^N\right)^3\beta\,q^3x-\left(q^N\right)^3q^2x^2-2\,\left(q^N\right)^2\beta\,q^3-2\,q^N\beta\,q^2x^2+\left(q^N\right)^2q^2x}{\left(q^N\right)^2q^3\left(2\,\beta-x\right)\left(qq^Nx-1\right)}$$

$$\times(+2\,q^N\beta\,q^2x-2\,q^N\beta\,qx+q^Nx^2+2\,\beta\,qx-x),$$

$$\frac{k_{n+1}}{k_n}=-\frac{\left(2\,\left(q^n\right)^2q\beta-1\right)\left(2\,\left(q^n\right)^2\beta\,q^2-1\right)}{\left(2\,\beta\,q^nq-1\right)^2\left(q^n-q^N\right)q},$$

$$I=[0,\frac{1}{q^Nq},\frac{2}{q^Nq},...,-\frac{\ln\left(q\right)N-\ln\left(\frac{1}{2\beta\,q^2}\right)}{\ln\left(q\right)qq^N}],$$

$$\sigma\left(x\right)=-\frac{\left(2\,\beta\,q-x\right)\left(xq^N-1\right)\beta}{q^Nq},$$

$$\tau\left(x\right)=\frac{\left(2\,q^N\beta\,q^2x-2\,q^N\beta\,q^2+2\,\beta\,qq^N-xq^N-2\,\beta\,q+1\right)\beta}{qq^N\left(q-1\right)},$$

$$\lambda_{q,n}=-\frac{\beta\,\left(q^n-1\right)\left(2\,\beta\,q^nq-1\right)}{q^n\left(q-1\right)^2},$$

$$S\left(n,x\right)=Q_n\left(\frac{1}{q^Nq},2\,\beta\,qq^N,\frac{1}{\ln\left(q\right)}\ln\left(1/2\,\frac{1}{\beta\,q}\right),x,q\right),\ \frac{\rho\left(qx\right)}{\rho\left(x\right)}=-2\,\frac{\left(-1+x\right)\beta}{2\,\beta-x},$$

$$\frac{k_{n+1}}{k_n}=-\frac{\left(2\,\left(q^n\right)^2q\beta-1\right)\left(2\,\left(q^n\right)^2\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^nq-1\right)^2\left(q^n-q^N\right)},$$

$$I=[0,1,2,...,\frac{\ln(\frac{1}{2\beta\,q})}{\ln(q)}],$$

$$\sigma\left(x\right)=-\frac{\left(2\,\beta\,q-x\right)\left(xq^N-1\right)}{q^Nq},$$

$$\tau\left(x\right)=\frac{2\,q^N\beta\,q^2x-2\,q^N\beta\,q^2+2\,\beta\,qq^N-xq^N-2\,\beta\,q+1}{qq^N\left(q-1\right)},$$

$$\lambda_{q,n}=-\frac{\left(q^n-1\right)\left(2\,\beta\,q^nq-1\right)}{q^n\left(q-1\right)^2},\ S\left(n,x\right)=Q_n\left(2\,\beta,1,N,x,q\right)$$

$$,\ \frac{\rho\left(qx\right)}{\rho\left(x\right)}=-2\,\frac{\left(-1+x\right)\beta}{2\,\beta-x},\ \frac{k_{n+1}}{k_n}=-\frac{\left(2\,\left(q^n\right)^2q\beta-1\right)\left(2\,\left(q^n\right)^2\beta\,q^2-1\right)q^N}{\left(2\,\beta\,q^nq-1\right)^2\left(q^n-q^N\right)},$$

$$I=[0,1,2,...,N],$$

$$\sigma\left(x\right)=-\left(2\,\beta\,q-x\right)\left(xq^N-1\right)\left(q^N\right)^3q^2,$$

$$\tau\left(x\right)=\frac{\left(2\,q^N\beta\,q^2x-2\,q^N\beta\,q^2+2\,\beta\,qq^N-xq^N-2\,\beta\,q+1\right)q^N}{q-1},$$

$$\lambda_{q,n} = -\frac{(q^n-1)(2\beta q^n q-1)}{q^n (q-1)^2 q}, \; S(n,x) = Q_n\left(2\beta q q^N, \frac{1}{q^N q}, -1, q q^N x, q\right),$$

$$\frac{\rho(qx)}{\rho(x)} = \frac{2\left(q^N\right)^3 \beta q^3 x - \left(q^N\right)^3 q^2 x^2 - 2\left(q^N\right)^2 \beta q^3 - 2 q^N \beta q^2 x^2 + \left(q^N\right)^2 q^2 x}{\left(q^N\right)^2 q^3 (2\beta - x)\left(q q^N x - 1\right)}$$

$$\times(+2 q^N \beta q^2 x - 2 q^N \beta q x + q^N x^2 + 2\beta q x - x),$$

$$\frac{k_{n+1}}{k_n} = -\frac{\left(2\left(q^n\right)^2 q\beta - 1\right)\left(2\left(q^n\right)^2 \beta q^2 - 1\right)}{(2\beta q^n q - 1)^2\left(q^n - q^N\right) q}, \; I = [0, \frac{1}{q^N q}, \frac{2}{q^N q}, \ldots, -\frac{1}{q^N q}],$$

where aB, bB, cB are the parameters a, b, c for the Big q-Jacobi polynomials, and $P_n(a, b, c, x, q) = P_n(x; a, b, c; q)$. aH, bH, NH are the parameters α, β, N for the q-Hahn polynomials, and $Q_n(\alpha, \beta, N, x, q) = Q_n(x; \alpha, \beta, N; q)$.

This shows that this recurrence equation (28) has classical orthogonal polynomial solutions as linear transformations of q-Hahn and Big q-Jacobi polynomials.

5 New relations

With our implementation we are able to identify some novel relations between the classical orthogonal polynomial systems:

1. **Meixner polynomials :**

$$M_n(x; \beta, \frac{1}{c}) = \frac{(1-c)^n}{(1-\frac{1}{c})^n(-1)^n} M_n(x; \beta, c).$$

2. **Krawtchouk polynomials :**

$$K_n(-x+N; 1-p, N) = \frac{(1-\frac{1}{p})^n}{(\frac{1}{p})^n(-1)^n} K_n(x; p, N).$$

3. **Hahn polynomials :**

$$Q_n(-x+N; \beta, \alpha, N) = \frac{(\alpha+1)_n}{(\beta+1)_n(-1)^n} Q_n(x; \alpha, \beta, N).$$

4. **Big q-Jacobi :**

$$P_n(x; a, b, c; q) = P_n\left(x; c, \frac{ab}{c}, a; q\right), \tag{29}$$

$$P_n\left(\frac{bx}{c}; b, a, \frac{ab}{c}; q\right) = \frac{(\frac{b}{c})^n (aq; q)_n (cq; q)_n}{(bq; q)_n (\frac{ab}{c} q; q)_n} P_n(x; a, b, c; q),$$

$$P_n(x; a, b, c; q) = Q_n\left(x; a, b, \frac{\ln(\frac{1}{cq})}{\ln(q)}; q\right). \tag{30}$$

5. **Little q-Jacobi :**

$$p_n(x; a, b; q) = C_n K_n\left(qxb; -abq, \frac{\ln(\frac{1}{bq})}{\ln(q)}; q\right),$$

with $C_n = \dfrac{(bq;q)_n}{(aq;q)_n(-1)^n(bq)^n q^{\binom{n}{2}}}$.

6. **q-Laguerre :**
$$L_n(x;a;q) = \frac{1}{(q;q)_n} M_n(-x;0,-q^{-a};q).$$
7. **Al-Salam Carlitz I :**
$$U_n(x;a;q) = \frac{1}{a^n} U_n\left(\frac{x}{a};\frac{1}{a};q\right).$$
And with respect to the Discrete q-Hermite I, we obtain $h_n(x;q) = U_n(x;-1;q) = (-1)^n U_n(-x;-1;q)$. The first part is given on page 536 of Koekoek et al. (2010).
8. **Al-Salam Carlitz II :**
$$V_n(x;a;q) = \frac{1}{a^n} V_n\left(\frac{x}{a};\frac{1}{a};q\right).$$
And with respect to the Discrete q-Hermite I, we obtain $\tilde{h}_n(x;q) = i^{-n} V_n(ix;-1;q) = (-1)^n V_n(-ix;-1;q)$. The first part is given on page 539 of Koekoek et al. (2010).
9. **q-Meixner :**
$$M_n\left(-\frac{x}{bc};-\frac{1}{c},-\frac{1}{b};q\right) = \frac{(bq;q)_n}{(-\frac{q}{c};q)_n} M_n(x;b,c;q).$$
10. **q-Krawtchouk :**
$$P_n(x;q^{-N-1},-q^N p,0;q) = K_n(x;p,N;q),$$
$$p_n(xq^N;-q^N p,q^{-N-1};q) = \frac{(-1)^n(q^N)^n(q^{-N};q)_n}{(-pq^{N+1};q)_n q^{\binom{n}{2}}} K_n(x;p,N;q).$$
11. **q-Hahn :**
$$Q_n\left(x;\frac{1}{q^{N+1}},\alpha\beta q^{N+1},\frac{\ln(\frac{1}{\alpha q})}{\ln(q)};q\right) = Q_n(x;\alpha,\beta,N;q),$$
$$Q_n\left(q^{N+1}\beta x;\beta,\alpha,\frac{-N\ln(q)-\ln(\frac{1}{\alpha\beta q^2})}{\ln(q)};q\right) = C_n Q_n(x;\alpha,\beta,N;q), \tag{31}$$
$$Q_n(x;\alpha,\beta,N;q) = P_n(x;\alpha,\beta,q^{-N-1};q), \tag{32}$$
with $C_n = \dfrac{q^{nN+\frac{1}{2}n^2+\frac{1}{2}n-\binom{n}{2}}\beta^n(\alpha q;q)_n(q^{-N};q)_n}{(\beta q;q)_n(\alpha\beta q^{N+2};q)_n}$.

Proof We are going to prove relation (31), and the remaining relations are obtained using the same approach. Consider the polynomial systems are defined as (Koekoek et al. 2010, (14.6.1))
$$Q_n(\alpha,\beta,N,x,q) = {}_3\phi_2\left(\begin{matrix} q^{-n},\alpha\beta q^{n+1},x \\ \alpha q,q^{-N}\end{matrix};q,q\right).$$

The output of our implementation in Example 4 suggests that there exists a constant C_n such that (31) is valid. To obtain C_n from equation (31), we equate the coefficients of the highest degree, i.e., x^n. For the left-hand side the coefficient is given as

$\frac{(-1)^n (q^{-n};q)_n (\alpha\beta q^{n+1};q)_n q^{\frac{1}{2}n(3+2N+n)} \beta^n}{(q\beta;q)_n (\beta q^{2+N}\alpha;q)_n (q;q)_n}$, and for the right-hand side we have $C_n \left(\frac{(q^{-n};q)_n (\alpha\beta q^{n+1};q)_n q^n (-1)^n q^{\binom{n}{2}}}{(q\alpha;q)_n (q^{-N};q)_n (q;q)_n} \right)$. Solving for C_n we obtain

$$C_n = \frac{q^{nN+\frac{1}{2}n^2+\frac{1}{2}n-\binom{n}{2}} \beta^n (\alpha q; q)_n (q^{-N}; q)_n}{(\beta q; q)_n (\alpha\beta q^{N+2}; q)_n} .$$

Using this constant, we can easily check that both left and right-hand side of (31) are solution of the same three-term recurrence relation

$$\begin{aligned}
&(\beta q^{2n+2}\alpha - 1)(q^{n+N+3}\alpha\beta - 1)(\beta q^{n+2} - 1)(\beta q^{n+2}\alpha - 1)S(n+2) \\
&+ \beta q\Big(\beta^2 q^{N+4n+6} x\alpha^2 - \beta^2 q^{N+3n+5}\alpha^2 - \beta q^{N+3n+5}\alpha^2 + \beta q^{N+2n+4}\alpha^2 \\
&- \beta q^{N+2n+4} x\alpha - \beta q^{3n+4}\alpha^2 + \beta q^{N+2n+3}\alpha^2 + \beta q^{N+2n+4}\alpha - \beta q^{3n+4}\alpha \\
&+ \beta q^{N+2n+3}\alpha - \beta q^{N+2n+2} x\alpha + \beta q^{2n+3}\alpha - \beta q^{N+n+2}\alpha + \beta q^{2n+2}\alpha \\
&+ q^{2n+3}\alpha - q^{N+n+2}\alpha + q^{2n+2}\alpha - q^{n+1}\alpha + xq^N - q^{n+1}\Big)(\beta q^{2n+3}\alpha - 1) \\
&S(n+1) + (q^{n+1} - 1)\beta^2 (q^{n+1}\alpha - 1) q^{n+3}\alpha(-q^n + q^N) \\
&(\beta q^{2n+4}\alpha - 1)S(n) = 0
\end{aligned} \tag{33}$$

with the same initial conditions. The remaining relations are derived using the same method.

Please note that some of the above automatically generated identities are trivial in some sense. For example, the q-hypergeometric representation of the Big q-Jacobi polynomials (Koekoek et al. 2010, (14.5.1))

$$P_n(x; a, b, c; q) = {}_3\phi_2 \left(\begin{matrix} q^{-n}, abq^{n+1}, x \\ aq, cq \end{matrix} ; q, q \right)$$

shows directly that, if we swap a and c, then the lower parameters are still the same, hence the denominators of the summands agree. If we therefore replace b by $\frac{a}{c}b$, then the upper parameters agree, too, and therefore we have termwise the same sum. This shows that (29) is trivially true. The same applies to (30) and to (32).

However, the other identities given above are non-trivial. □

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