

showtime: true\$

Evaluation took 0.0000 seconds (0.0000 elapsed) using 0 bytes.

file_search_maxima : cons(sconcat("C:\maxima-5.47.0\share\###.{lisp,mac,mc}"), f)

Evaluation took 0.0000 seconds (0.0000 elapsed) using 0 bytes.

load("retode");

Evaluation took 0.0469 seconds (0.0477 elapsed) using 6.434 MB.

C:retode.mac

1 Example 1

We consider the three term recurrence

relation satisfied by the polynomials (7) in

Daniel D. Tcheutia and Wolfram Koepf.

Properties of some finite families of classical
orthogonal polynomials.

Mnpq: ((-p+2·n+2)·(-p+n+2))·P[n+2] + ((2·n-p+3)·(4·n^2·x - 4·n·p·x + p^2·x + 2·n
+ (q + n+1)·(n+1)·(2·n-p+4)·(n-p-q+1))·P[n] = 0;

Evaluation took 0.0000 seconds (0.0004 elapsed) using 95.484 KB.

$$\begin{aligned} P_{n+1} (-p+2 n+3) (p^2 x-4 n p x-6 p x+4 n^2 x+12 n x+8 \\ x-p q-2 n p-3 p+2 n^2+6 n+4)+(n+1) P_n (-p+2 n+4) \\ (-q-p+n+1) (q+n+1)+P_{n+2} (-p+n+2) (-p+2 n+2)=0 \end{aligned}$$

REtoDE(Mnpq, P[n], x);

Warning, parameters have the values [

[f=-2, g=-1, alphaJacobi=-q-p, betaJacobi=q],

[f=2, g=1, alphaJacobi=q, betaJacobi=-q-p]]

Warning, several solutions found

This recurrence equation has no classical orthogonal polynomial solution

This recurrence equation has no classical orthogonal polynomial solution

This recurrence equation has no classical orthogonal polynomial solution

Evaluation took 0.5469 seconds (0.5806 elapsed) using 151.182 MB.

$$\begin{aligned} & [\text{Has a solution as Jacobi}, [[[\sigma(x) = x^2 + x, \tau(x) = -(p x) + 2 x \\ & + q + 1, \lambda_n = n p - n^2 - n, p(n, x) = J_n(-(2 x) - 1, -q - p, q), w(x) = \\ & -(q \log(x+1)) - p \log(x+1) + q \log(x) \frac{k_{n+1}}{k_n} = - \\ & \%e \left(\frac{(p-2 n-2) (p-2 n-1)}{2 (p-n-1)} \right), l = [-1, 0]], [\sigma(x) = x^2 + x, \tau(x) = -(p x) + 2 \\ & x + q + 1, \lambda_n = n p - n^2 - n, p(n, x) = J_n(2 x + 1, q, -q - p), w(x) = \\ & -(q \log(x+1)) - p \log(x+1) + q \log(x) \frac{k_{n+1}}{k_n} = - \\ & \%e \left(\frac{(p-2 n-2) (p-2 n-1)}{2 (p-n-1)} \right), l = [-1, 0]]]]] \end{aligned}$$

Example 4 from article: Wolfram Koepf and Dieter Schmersau. Recurrence equations and their classical orthogonal polynomial solutions.

$$\text{RE: } ((n+2+\alpha) \cdot (2+n) \cdot (2 \cdot n + 2) \cdot (n - N + 1)) \cdot p[n+2] + ((3+2 \cdot n) \cdot (-6 \cdot n \cdot \alpha - 2 \cdot n^2 \cdot \alpha - ((1+n) \cdot (n + 1 - \alpha) \cdot (2 \cdot n + 4) \cdot (n + N + 2)) \cdot p[n] = 0;$$

Evaluation took 0.0000 seconds (0.0002 elapsed) using 31.750 KB.

$$(2 n + 3) p_{n+1} \left(-\left(4 n^2 x\right) - 12 n x - 8 x - 2 \alpha n^2 + 2 N n^2 - 6 \alpha n + 6 N n - 4 \alpha + 4 N \right) + (n + 2) (n - N + 1) (n + \alpha + 2) (2 n + 2) p_{n+2} - (n + 1) (n + N + 2) (n - \alpha + 1) (2 n + 4) p_n = 0$$

REtoDiscrete(*RE, p[n], x***);**

Warning, parameters have the values $[[N = -\left(\frac{1}{2}\right), f = 2, \alpha = 0, g = 0, muMeixner = -1, gammaMeixner = 1], [N = -\left(\frac{3}{2}\right), f = 2, \alpha = 0, g = 1, muMeixner = -1, gammaMeixner = 1], [N = -\left(\frac{1}{2}\right), f = -2, \alpha = 0, g = -1, muMeixner = -1, gammaMeixner = 1], [N = -\left(\frac{3}{2}\right), f = -2, \alpha = 0, g = -2, muMeixner = -1, gammaMeixner = 1], [N = -1, f = 2, \alpha = \frac{1}{2}, g = 1, muMeixner = -1, gammaMeixner = 1], [N = -1, f = 2, \alpha = -\left(\frac{1}{2}\right), g = 0, muMeixner = -1, gammaMeixner = 1], [N = -1, f = -2, \alpha = \frac{1}{2}, g = -2, muMeixner = -1, gammaMeixner = 1], [N = -1, f = -2, \alpha = -\left(\frac{1}{2}\right), g = -1, muMeixner = -1, gammaMeixner = 1]]$

Warning, several solutions found

This recurrence equation has no classical orthogonal polynomial solution

Warning, parameters have the values [

$$\begin{aligned} & \left[N = -\left(\frac{1}{2}\right), f = 2, \alpha = 0, g = 0, NKraw = -1, pKraw = \frac{1}{2} \right], \\ & \left[N = -\left(\frac{3}{2}\right), f = 2, \alpha = 0, g = 1, NKraw = -1, pKraw = \frac{1}{2} \right], \\ & \left[N = -\left(\frac{1}{2}\right), f = -2, \alpha = 0, g = -1, NKraw = -1, pKraw = \frac{1}{2} \right], \\ & \left[N = -\left(\frac{3}{2}\right), f = -2, \alpha = 0, g = -2, NKraw = -1, pKraw = \frac{1}{2} \right], \end{aligned}$$

$$\begin{aligned} & \left[N = -1, f = 2, \alpha = \frac{1}{2}, g = 1, NKraw = -1, pKraw = \frac{1}{2} \right], \\ & \left[N = -1, f = 2, \alpha = -\left(\frac{1}{2}\right), g = 0, NKraw = -1, pKraw = \frac{1}{2} \right], \\ & \left[N = -1, f = -2, \alpha = \frac{1}{2}, g = -2, NKraw = -1, pKraw = \frac{1}{2} \right], [N = -1, f \\ & = -2, \alpha = -\left(\frac{1}{2}\right), g = -1, NKraw = -1, pKraw = \frac{1}{2}] \end{aligned}$$

Warning, several solutions found

Warning, parameters have the values [[f=1,g=0,alphaHahn=-N-1,betaHahn=N+1,NHahn=-alpha-1],[f=1,g=0,alphaHahn=alpha,betaHahn=-alpha,NHahn=N],[f=1,g=alpha-N-1,alphaHahn=N+1,betaHahn=-N-1,NHahn=alpha-1],[f=1,g=alpha-N-1,alphaHahn=-alpha,betaHahn=alpha,NHahn=-N-2],[f=1,g=-N-1,alphaHahn=N+1,betaHahn=-N-1,NHahn=-alpha-1],[f=1,g=-N-1,alphaHahn=alpha,betaHahn=-alpha,NHahn=-N-2],[f=1,g=alpha,alphaHahn=-N-1,betaHahn=N+1,NHahn=alpha-1],[f=1,g=alpha,alphaHahn=-alpha,betaHahn=alpha,NHahn=N],[f=-1,g=-1,alphaHahn=N+1,betaHahn=-N-1,NHahn=alpha-1],[f=-1,g=-1,alphaHahn=-alpha,betaHahn=alpha,NHahn=-N-2],[f=-1,g=N-alpha,alphaHahn=-N-1,betaHahn=N+1,NHahn=-alpha-1],[f=-1,g=N-alpha,alphaHahn=alpha,betaHahn=-alpha,NHahn=N],[f=-1,g=N,alphaHahn=-N-1,betaHahn=N+1,NHahn=alpha-1],[f=-1,g=-alpha-1,alphaHahn=N+1,betaHahn=-N-1,NHahn=-alpha-1],[f=-1,g=-alpha-1,alphaHahn=alpha,betaHahn=-alpha,NHahn=-N-2]]

Warning, several solutions found

Evaluation took 0.9375 seconds (0.9463 elapsed) using 298.584 MB.

$$\begin{aligned} & [\text{Has a solution as Meixner}, [[[\sigma(x) = 2x, \tau(x) = -(4x) - 1, \lambda_n \\ & = 2n, p(n,x) = M_n(2x, 1, -1), \frac{w(x+1)}{w(x)} = -\left(\frac{2x+1}{2(x+1)}\right), \frac{k_{n+1}}{k_n} = \\ & \frac{2}{n+1}, I = \left[0, \frac{1}{2}, 1, \dots, \infty \right]], [\sigma(x) = 2x+1, \tau(x) = -(4x)-3, \lambda_n = 2n, \\ & p(n,x) = M_n(2x+1, 1, -1), \frac{w(x+1)}{w(x)} = -\left(\frac{2(x+1)}{2x+3}\right), \frac{k_{n+1}}{k_n} = \\ & \frac{2(2n+1)}{(n+1)(2n+3)}, I = \left[-\left(\frac{1}{2}\right), 0, \frac{1}{2}, \dots, \infty \right]], [\sigma(x) = -(2x)-1, \tau(x) = 4 \\ & x+1, \lambda_n = 2n, p(n,x) = M_n(-(2x)-1, 1, -1), \frac{w(x+1)}{w(x)} = -\left(\frac{2x}{2x+3}\right), \\ & \frac{k_{n+1}}{k_n} = -\left(\frac{2}{n+1}\right), I = \left[-\infty, \dots, -\left(\frac{3}{2}\right), -1, -\left(\frac{1}{2}\right) \right]], [\sigma(x) = -(2x)-2, \end{aligned}$$

$$\begin{aligned}
& \tau(x) = 4x + 3, \lambda_n = 2n, p(n, x) = M_n(-(2x) - 2, 1, -1), \frac{w(x+1)}{w(x)} = - \\
& \left(\frac{2x+1}{2(x+2)} \right), \frac{k_{n+1}}{k_n} = - \left(\frac{2(2n+1)}{(n+1)(2n+3)} \right), I = \left[-\infty, \dots, -2, -\left(\frac{3}{2} \right), -1 \right] \\
& , [\sigma(x) = 2x + 1, \tau(x) = -(4x) - 3, \lambda_n = 2n, p(n, x) = M_n(2x+1, 1, -1) \\
& , \frac{w(x+1)}{w(x)} = - \left(\frac{2(x+1)}{2x+3} \right), \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n+1)(2n+3)}, I = \\
& \left[-\left(\frac{1}{2} \right), 0, \frac{1}{2}, \dots, \infty \right], [\sigma(x) = 2x, \tau(x) = -(4x) - 1, \lambda_n = 2n, p(n, x) = \\
& M_n(2x, 1, -1), \frac{w(x+1)}{w(x)} = - \left(\frac{2x+1}{2(x+1)} \right), \frac{k_{n+1}}{k_n} = \frac{2}{n+1}, I = \\
& \left[0, \frac{1}{2}, 1, \dots, \infty \right], [\sigma(x) = -(2x) - 2, \tau(x) = 4x + 3, \lambda_n = 2n, p(n, x) = \\
& M_n(-(2x) - 2, 1, -1), \frac{w(x+1)}{w(x)} = - \left(\frac{2x+1}{2(x+2)} \right), \frac{k_{n+1}}{k_n} = - \\
& \left(\frac{2(2n+1)}{(n+1)(2n+3)} \right), I = \left[-\infty, \dots, -2, -\left(\frac{3}{2} \right), -1 \right], [\sigma(x) = -(2x) - 1, \\
& \tau(x) = 4x + 1, \lambda_n = 2n, p(n, x) = M_n(-(2x) - 1, 1, -1), \frac{w(x+1)}{w(x)} = - \\
& \left(\frac{2x}{2x+3} \right), \frac{k_{n+1}}{k_n} = - \left(\frac{2}{n+1} \right), I = \left[-\infty, \dots, -\left(\frac{3}{2} \right), -1, -\left(\frac{1}{2} \right) \right]], \\
& \text{Has a solution as Krawtchouk, } [[[\sigma(x) = 2x, \tau(x) = -(4x) - 1, \lambda_n = 2n \\
& , p(n, x) = K_n(2x, \frac{1}{2}, -1), \frac{w(x+1)}{w(x)} = - \left(\frac{2x+1}{2(x+1)} \right), \frac{k_{n+1}}{k_n} = \frac{2}{n+1}, I \\
& = \left[0, \frac{1}{2}, 1, \dots, -\left(\frac{1}{2} \right) \right]], [\sigma(x) = 2x + 1, \tau(x) = -(4x) - 3, \lambda_n = 2n, \\
& p(n, x) = K_n(2x+1, \frac{1}{2}, -1), \frac{w(x+1)}{w(x)} = - \left(\frac{2(x+1)}{2x+3} \right), \frac{k_{n+1}}{k_n} = \\
& \frac{2(2n+1)}{(n+1)(2n+3)}, I = \left[-\left(\frac{1}{2} \right), 0, \frac{1}{2}, \dots, -1 \right]], [\sigma(x) = -(2x) - 1, \tau(x) = \\
& 4x + 1, \lambda_n = 2n, p(n, x) = K_n(-(2x) - 1, \frac{1}{2}, -1), \frac{w(x+1)}{w(x)} = - \\
& \left(\frac{2x}{2x+3} \right), \frac{k_{n+1}}{k_n} = - \left(\frac{2}{n+1} \right), I = \left[0, \dots, -\left(\frac{3}{2} \right), -1, -\left(\frac{1}{2} \right) \right]], [\sigma(x) = - \\
& (2x) - 2, \tau(x) = 4x + 3, \lambda_n = 2n, p(n, x) = K_n(-(2x) - 2, \frac{1}{2}, -1), \\
& \frac{w(x+1)}{w(x)} = - \left(\frac{2x+1}{2(x+2)} \right), \frac{k_{n+1}}{k_n} = - \left(\frac{2(2n+1)}{(n+1)(2n+3)} \right), I =
\end{aligned}$$

$$\begin{aligned}
& \left[-\left(\frac{1}{2} \right), \dots, -2, -\left(\frac{3}{2} \right), -1 \right], [\sigma(x) = 2x + 1, \tau(x) = -(4x) - 3, \lambda_n = 2n] \\
& , p(n, x) = K_n \left(2x + 1, \frac{1}{2}, -1 \right), \frac{w(x+1)}{w(x)} = -\left(\frac{2(x+1)}{2x+3} \right), \frac{k_{n+1}}{k_n} = \\
& \frac{2(2n+1)}{(n+1)(2n+3)}, I = \left[-\left(\frac{1}{2} \right), 0, \frac{1}{2}, \dots, -1 \right], [\sigma(x) = 2x, \tau(x) = -(4x) \\
& - 1, \lambda_n = 2n], p(n, x) = K_n \left(2x, \frac{1}{2}, -1 \right), \frac{w(x+1)}{w(x)} = -\left(\frac{2x+1}{2(x+1)} \right), \\
& \frac{k_{n+1}}{k_n} = \frac{2}{n+1}, I = \left[0, \frac{1}{2}, 1, \dots, -\left(\frac{1}{2} \right) \right], [\sigma(x) = -(2x) - 2, \tau(x) = 4x \\
& + 3, \lambda_n = 2n], p(n, x) = K_n \left(-(2x) - 2, \frac{1}{2}, -1 \right), \frac{w(x+1)}{w(x)} = - \\
& \left(\frac{2x+1}{2(x+2)} \right), \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n+1)(2n+3)} \right), I = \\
& \left[-\left(\frac{1}{2} \right), \dots, -2, -\left(\frac{3}{2} \right), -1 \right], [\sigma(x) = -(2x) - 1, \tau(x) = 4x + 1, \lambda_n = 2n] \\
& , p(n, x) = K_n \left(-(2x) - 1, \frac{1}{2}, -1 \right), \frac{w(x+1)}{w(x)} = -\left(\frac{2x}{2x+3} \right), \frac{k_{n+1}}{k_n} = - \\
& \left(\frac{2}{n+1} \right), I = \left[0, \dots, -\left(\frac{3}{2} \right), -1, -\left(\frac{1}{2} \right) \right]], \text{Has a solution as Hahn}, [[[\\
& \sigma(x) = -x^2 - \alpha x + N x + x, \tau(x) = -(2x) + N \alpha + N, \lambda_n = n^2 + n, \\
& p(n, x) = Q_n(x, -N-1, N+1, -\alpha - 1), \frac{w(x+1)}{w(x)} = \\
& \frac{(x-N)(x+\alpha+1)}{(x+1)(x+\alpha-N)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [0, 1, 2, \dots, -\alpha - 1]], [\sigma(x) = -x^2 - \alpha x + N x + x, \tau(x) = -(2x) \\
& + N \alpha + N, \lambda_n = n^2 + n], p(n, x) = Q_n(x, \alpha, -\alpha, N), \frac{w(x+1)}{w(x)} \\
& = \frac{(x-N)(x+\alpha+1)}{(x+1)(x+\alpha-N)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [0, 1, 2, \dots, N]], [\sigma(x) = -x^2 - \alpha x + N x + x, \tau(x) = -(2x) + N \alpha \\
& + N, \lambda_n = n^2 + n], p(n, x) = Q_n(x+\alpha-N-1, N+1, -N-1, \alpha-1), \\
& \frac{w(x+1)}{w(x)} = \frac{(x-N)(x+\alpha+1)}{(x+1)(x+\alpha-N)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [-\alpha+N+1, -\alpha+N+2, -\alpha+N+3, \dots, N]], [\sigma(x) = -x^2 - \\
& \alpha x + N x + x, \tau(x) = -(2x) + N \alpha + N, \lambda_n = n^2 + n], p(n, x) =
\end{aligned}$$

$$\begin{aligned}
& Q_n(x + \alpha - N - 1, -\alpha, \alpha, -N - 2), \frac{w(x+1)}{w(x)} = \\
& \frac{(x-N)(x+\alpha+1)}{(x+1)(x+\alpha-N)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [-\alpha + N + 1, -\alpha + N + 2, -\alpha + N + 3, \dots, -\alpha - 1]], [\sigma(x) = \\
& -x^2 - \alpha x + N x + x + N \alpha + \alpha, \tau(x) = -(2x) - N \alpha - 2 \\
& \alpha + N, \lambda_n = n^2 + n, p(n, x) = Q_n(x - N - 1, N + 1, -N - 1, -\alpha - 1), \\
& \frac{w(x+1)}{w(x)} = \frac{(x+1)(x+\alpha-N)}{(x-N)(x+\alpha+1)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [N + 1, N + 2, N + 3, \dots, N - \alpha]], [\sigma(x) = -x^2 - \alpha x + N x + x + N \\
& \alpha + \alpha, \tau(x) = -(2x) - N \alpha - 2 \alpha + N, \lambda_n = n^2 + n, p(n, x) \\
& = Q_n(x - N - 1, \alpha, -\alpha, -N - 2), \frac{w(x+1)}{w(x)} = \\
& \frac{(x+1)(x+\alpha-N)}{(x-N)(x+\alpha+1)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [N + 1, N + 2, N + 3, \dots, -1]], [\sigma(x) = -x^2 - \alpha x + N x + x + N \alpha + \\
& \alpha, \tau(x) = -(2x) - N \alpha - 2 \alpha + N, \lambda_n = n^2 + n, p(n, x) = \\
& Q_n(x + \alpha, -N - 1, N + 1, \alpha - 1), \frac{w(x+1)}{w(x)} = \\
& \frac{(x+1)(x+\alpha-N)}{(x-N)(x+\alpha+1)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [-\alpha, 1 - \alpha, 2 - \alpha, \dots, -1]], [\sigma(x) = -x^2 - \alpha x + N x + x + \\
& N \alpha + \alpha, \tau(x) = -(2x) - N \alpha - 2 \alpha + N, \lambda_n = n^2 + n, \\
& p(n, x) = Q_n(x + \alpha, -\alpha, \alpha, N), \frac{w(x+1)}{w(x)} = \\
& \frac{(x+1)(x+\alpha-N)}{(x-N)(x+\alpha+1)}, \frac{k_{n+1}}{k_n} = \frac{2(2n+1)}{(n-N)(n+\alpha+1)}, I = \\
& [-\alpha, 1 - \alpha, 2 - \alpha, \dots, N - \alpha], [\sigma(x) = -x^2 - \alpha x + N \\
& x - x - \alpha + N, \tau(x) = 2x + N \alpha + 2 \alpha - N, \lambda_n = n^2 + n, p(n, x) = \\
& Q_n(-x - 1, N + 1, -N - 1, \alpha - 1), \frac{w(x+1)}{w(x)} = \\
& \frac{(x-N-1)(x+\alpha)}{(x+2)(x+\alpha-N+1)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = \\
& [-\alpha, \dots, -3, -2, -1]], [\sigma(x) = -x^2 - \alpha x + N x - x - \alpha + N,
\end{aligned}$$

$$\begin{aligned}
& \tau(x) = 2x + N\alpha + 2\alpha - N, \lambda_n = n^2 + n, p(n, x) = \\
& Q_n(-x-1, -\alpha, \alpha, -N-2), \frac{w(x+1)}{w(x)} = \\
& \frac{(x-N-1)(x+\alpha)}{(x+2)(x+\alpha-N+1)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = \\
& [N+1, \dots, -3, -2, -1], [\sigma(x) = -x^2 - \alpha x + N x - x - \alpha + N, \\
& \tau(x) = 2x + N\alpha + 2\alpha - N, \lambda_n = n^2 + n, p(n, x) = \\
& Q_n(-x-\alpha+N, -N-1, N+1, -\alpha-1), \frac{w(x+1)}{w(x)} = \\
& \frac{(x-N-1)(x+\alpha)}{(x+2)(x+\alpha-N+1)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = \\
& [N+1, \dots, -\alpha+N-2, -\alpha+N-1, N-\alpha], [\sigma(x) = -x^2 - \\
& \alpha x + N x - x - \alpha + N, \tau(x) = 2x + N\alpha + 2\alpha - N, \lambda_n = n^2 + \\
& n, p(n, x) = Q_n(-x-\alpha+N, \alpha, -\alpha, N), \frac{w(x+1)}{w(x)} = \\
& \frac{(x-N-1)(x+\alpha)}{(x+2)(x+\alpha-N+1)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = \\
& [-\alpha, \dots, -\alpha+N-2, -\alpha+N-1, N-\alpha], [\sigma(x) = -x^2 - \\
& \alpha x + N x - x + N\alpha + N, \tau(x) = 2x - N\alpha - N, \lambda_n = n^2 + n, \\
& p(n, x) = Q_n(N-x, -N-1, N+1, \alpha-1), \frac{w(x+1)}{w(x)} = \\
& \frac{x(x+\alpha-N-1)}{(x-N+1)(x+\alpha+2)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = \\
& [-\alpha+N+1, \dots, N-2, N-1, N], [\sigma(x) = -x^2 - \alpha x + N x - x + N \\
& \alpha + N, \tau(x) = 2x - N\alpha - N, \lambda_n = n^2 + n, p(n, x) = \\
& Q_n(N-x, -\alpha, \alpha, N), \frac{w(x+1)}{w(x)} = \frac{x(x+\alpha-N-1)}{(x-N+1)(x+\alpha+2)}, \\
& \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = [0, \dots, N-2, N-1, N], [\sigma(x) = - \\
& x^2 - \alpha x + N x - x + N\alpha + N, \tau(x) = 2x - N\alpha - N, \lambda_n = n^2 + n, \\
& p(n, x) = Q_n(-x-\alpha-1, N+1, -N-1, -\alpha-1), \frac{w(x+1)}{w(x)} = \\
& \frac{x(x+\alpha-N-1)}{(x-N+1)(x+\alpha+2)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I =
\end{aligned}$$

$$\begin{aligned}
& [0, \dots, -\alpha - 3, -\alpha - 2, -\alpha - 1]], [\sigma(x) = -x^2 - \alpha x + N x \\
& - x + N \alpha + N, \tau(x) = 2 x - N \alpha - N, \lambda_n = n^2 + n, p(n, x) = \\
& Q_n(-x - \alpha - 1, \alpha, -\alpha, -N - 2), \frac{w(x+1)}{w(x)} = \\
& \frac{x(x + \alpha - N - 1)}{(x - N + 1)(x + \alpha + 2)}, \frac{k_{n+1}}{k_n} = -\left(\frac{2(2n+1)}{(n-N)(n+\alpha+1)}\right), I = \\
& [-\alpha + N + 1, \dots, -\alpha - 3, -\alpha - 2, -\alpha - 1]]]
\end{aligned}$$