```
> restart;
> read "REtoqDE.mpl";
```

*Package "q-Hypergeometric Summation", Maple V-2019*

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*Package "Hypergeometric Summation", Maple V - Maple 2019*

*Copyright 1998-2019, Wolfram Koepf, University of Kassel* **(1)**

# Example 2

```
> jnsum := 2^n*(a*d-b*c)^n*pochhammer(-p+1-I*q*(1/2), n)*hyperterm(
  [-n, n+1-2*p], [-p+1-I*q*(1/2)], I*(a^2+c^2)*(x-(I*(a*d-b*c)-a*b-
  c*d)/(a^2+c^2))/(2*(a*d-b*c)), j)/I^n;
```

$$jnsum := \frac{1}{\mathrm{pochhammer}\left(-p+1-\frac{I q}{2}, j\right) j!\, I^n} \left(2^n (a d - b c)^n \,\mathrm{pochhammer}\left(-p+1-\frac{I q}{2}, n\right) \mathrm{pochhammer}(-n, j)\, \mathrm{pochhammer}(n+1-2 p, j) \left(\frac{I (a^2+c^2)\left(x - \frac{I (a d - b c) - a b - c d}{a^2+c^2}\right)}{2 a d - 2 b c}\right)^j\right) \quad (2)$$

```
> Rjn := sumrecursion(jnsum, j, S(n), recursion = up);
```

$$Rjn := (1+n-p)(n+2-2 p) S(n+2) + (3+2 n-2 p)\left(2 a^2 x n^2 - 4 a^2 x p n + 2 a^2 p^2 x + 2 c^2 x n^2 - 4 c^2 x p n + 2 c^2 p^2 x + 6 a^2 x n - 6 a^2 x p + 2 a b n^2 - 4 a b p n + 2 a b p^2 - a d p q + b c p q + 6 c^2 x n - 6 c^2 x p + 2 c d n^2 - 4 c d p n + 2 c d p^2 + 4 a^2 x + 6 a b n - 6 a b p + 4 c^2 x + 6 c d n - 6 c d p + 4 a b + 4 c d\right) S(n+1) - (n+1)(-p+2+n)\left(4 n^2 - 8 n p + 4 p^2 + q^2 + 8 n - 8 p + 4\right)(a d - b c)^2 S(n) = 0 \quad (3)$$

```
> REtoDE(Rjn, S(n), x);
```

"Warning, parameters have the values", $\left\{aJ = \frac{RootOf(\_Z^2+1)\, q}{2} - p,\ bJ = -\frac{RootOf(\_Z^2+1)\, q}{2} - p,\ f = \frac{RootOf(\_Z^2+1)(a^2+c^2)}{a d - b c},\ g = \frac{RootOf(\_Z^2+1)(a b + c d)}{a d - b c}\right\}$

$\Big[\Big[$"Has a solution as Jacobi", $\Big[\sigma(x) = a^2 x^2 + c^2 x^2 + 2 a b x + 2 c d x + b^2 + d^2, \tau(x) = -2 a^2 x p$ **(4)**

$$- 2 c^2 x p + 2 a^2 x - 2 a b p + a d q - b c q + 2 c^2 x - 2 c d p + 2 a b + 2 c d, \lambda_n = -n \left(a^2 + c^2\right) (n + 1 - 2 p), S(n, x) = P_n\left(\frac{RootOf(\_Z^2+1)\, q}{2} - p, -\frac{RootOf(\_Z^2+1)\, q}{2} - p, \frac{RootOf(\_Z^2+1)\,(a^2+c^2)\, x}{a d - b c} + \frac{RootOf(\_Z^2+1)\,(a b + c d)}{a d - b c}\right), w(x)$$

$$= e^{q \arctan\left(\frac{a^2 x + c^2 x + a b + c d}{a d - b c}\right)} \left(\left(a^2 + c^2\right) x^2 + (2 a b + 2 c d) x + b^2 + d^2\right)^{-p}, \frac{k_{n+1}}{k_n}$$

$$= \frac{2\, RootOf(\_Z^2+1)\,(a d - b c)\,(1 + n - p)\,(2 n + 1 - 2 p)}{n + 1 - 2 p}, I$$

$$= \left[\frac{\left(-1 - \frac{RootOf(\_Z^2+1)\,(a b + c d)}{a d - b c}\right)(a d - b c)}{RootOf(\_Z^2+1)\,(a^2+c^2)}, \frac{\left(1 - \frac{RootOf(\_Z^2+1)\,(a b + c d)}{a d - b c}\right)(a d - b c)}{RootOf(\_Z^2+1)\,(a^2+c^2)}\right]\Bigg]\Bigg]\Bigg]$$

# Example 4

```
> qhsum := subs([alpha = 2*beta, beta = 1], qphihyperterm([q^(-n),
  alpha*beta*q^(n+1), x], [alpha*q, q^(-N)], q, q, j));
```

$$qhsum := \frac{qpochhammer(q^{-n}, q, j)\, qpochhammer(2 \beta q^{n+1}, q, j)\, qpochhammer(x, q, j)\, q^j}{qpochhammer(2 \beta q, q, j)\, qpochhammer(q^{-N}, q, j)\, qpochhammer(q, q, j)} \quad \textbf{(5)}$$

```
> RE := qsumrecursion(qhsum, q, j, S(n), recursion = up);
```

$$RE := \left(-q^{n+1} + q^N\right)\left(2 q^{2n+2} \beta - 1\right)\left(2 q^{n+2} \beta - 1\right)^2 S(n+2) - \left(2 q^{2n+3} \beta - 1\right)\left(4 q^{N+4n+6} x \beta^2 - 8 q^{N+3n+5} \beta^2 + 4 q^{N+2n+4} \beta^2 - 2 q^{N+2n+4} x \beta - 4 q^{3n+4} \beta^2 + 4 q^{N+2n+3} \beta^2 + 2 q^{N+2n+4} \beta - 2 q^{3n+4} \beta + 2 q^{N+2n+3} \beta - 2 q^{N+2n+2} x \beta + 4 q^{2n+3} \beta - 4 q^{N+n+2} \beta + 4 q^{2n+2} \beta - 2 \beta q^{n+1} + x q^N - q^{n+1}\right) S(n+1) + 2 q^{n+1} \left(q^{n+1} - 1\right)^2 \left(2 q^{2n+4} \beta - 1\right) \beta \left(2 q^{N+n+2} \beta - 1\right) S(n) = 0 \quad \textbf{(6)}$$

```
> REtoqde(RE, S(n), x, q);
```

"Warning, parameters have the values", $\{\{aB = 1, bB = 2\beta, cB = 2 q^{1+N} \beta, f = q^{1+N}, g = 0\}, \{aB = q^{-1-N}, bB = 2 q^{1+N} \beta, cB = 2\beta, f = 1, g = 0\}, \{aB = 2\beta, bB = 1, cB = q^{-1-N}, f = 1, g = 0\}, \{aB = 2 q^{1+N} \beta, bB = q^{-1-N}, cB = 1, f = q^{1+N}, g = 0\}\}$

"Warning, several solutions found"

"Warning, parameters have the values", $\left\{\left\{aB=1, bB=2\,\beta, f=q\,q^N, g=0, q^{NB}=\frac{1}{2\,q^N\,\beta\,q^2}\right\}, \left\{aB=\frac{1}{q\,q^N}, bB=2\,q^N\,\beta\,q, f=1, g=0, q^{NB}=\frac{1}{2\,q\,\beta}\right\}, \{aB=2\,\beta, bB=1, f=1, g=0, q^{NB}=q^N\}, \left\{aB=2\,q^N\,\beta\,q, bB=\frac{1}{q\,q^N}, f=q\,q^N, g=0, q^{NB}=\frac{1}{q}\right\}\right\}$

"Warning, several solutions found"

$$\left[\left["\text{Has a solution as Big q-Jacobi}", \left[\left[\sigma(x)=-(2\,\beta\,q-x)\,(x\,q^N-1)\,(q^N)^3\,q^2, \tau(x)=\frac{(2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1)\,q^N}{q-1}, \lambda_{q,n}=-\frac{(q^n-1)\,(2\,\beta\,q^n\,q-1)}{q^n\,(q-1)^2\,q}, S(n,x)=P_n(1, 2\,\beta, 2\,q^{1+N}\,\beta, q^{1+N}\,x, q), \frac{\rho(q\,x)}{\rho(x)}=\frac{1}{(q^N)^2\,q^3\,(2\,\beta-x)\,(q\,q^N\,x-1)}\left(2\,(q^N)^3\,\beta\,q^3\,x-(q^N)^3\,q^2\,x^2-2\,\beta\,q^3\,(q^N)^2-2\,q^N\,\beta\,q^2\,x^2+(q^N)^2\,q^2\,x+2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q\,x+q^N\,x^2+2\,\beta\,q\,x-x\right), \frac{k_{n+1}}{k_n}=\frac{(2\,(q^n)^2\,q\,\beta-1)\,(2\,(q^n)^2\,\beta\,q^2-1)\,q^N}{(2\,\beta\,q^n\,q-1)^2\,(q^N-q^n)\,q^{1+N}}, \mathrm{I}=\left[2\,\beta\,q, \frac{q}{q^{1+N}}\right]\right]\right], \left[\sigma(x)=-\frac{(2\,\beta\,q-x)\,(x\,q^N-1)}{q^N\,q}, \tau(x)=\frac{2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1}{q\,(q-1)\,q^N}, \lambda_{q,n}=-\frac{(q^n-1)\,(2\,\beta\,q^n\,q-1)}{q^n\,(q-1)^2}, S(n,x)=P_n(q^{-1-N}, 2\,q^{1+N}\,\beta, 2\,\beta, x, q), \frac{\rho(q\,x)}{\rho(x)}=-\frac{2\,\beta\,(x-1)}{2\,\beta-x}, \frac{k_{n+1}}{k_n}=\frac{(2\,(q^n)^2\,q\,\beta-1)\,(2\,(q^n)^2\,\beta\,q^2-1)\,q^N}{(2\,\beta\,q^n\,q-1)^2\,(q^N-q^n)}, \mathrm{I}=[2\,\beta\,q, q^{-1-N}\,q]\right], \tag{7}$$

$$\left[\sigma(x) = -\frac{(2\,\beta\,q - x)\,(x\,q^N - 1)}{q^N\,q}, \tau(x)\right.$$

$$= \frac{2\,q^N\,\beta\,q^2\,x - 2\,q^N\,\beta\,q^2 + 2\,q^N\,\beta\,q - x\,q^N - 2\,\beta\,q + 1}{q\,(q-1)\,q^N}, \lambda_{q,\,n} = -\frac{(q^n - 1)\,(2\,\beta\,q^n\,q - 1)}{q^n\,(q-1)^2},$$

$$S(n,x) = P_n\left(2\,\beta, 1, q^{-1-N}, x, q\right), \frac{\rho(q\,x)}{\rho(x)} = -\frac{2\,\beta\,(x-1)}{2\,\beta - x}, \frac{k_{n+1}}{k_n}$$

$$= \frac{\left(2\,(q^n)^2\,q\,\beta - 1\right)\left(2\,(q^n)^2\,\beta\,q^2 - 1\right)q^N}{(2\,\beta\,q^n\,q - 1)^2\,(q^N - q^n)}, \mathrm{I} = \left[q^{-1-N}\,q, 2\,\beta\,q\right]\Bigg], \left[\sigma(x) = -(2\,\beta\,q\right.$$

$$-x)\,(x\,q^N - 1)\,(q^N)^3\,q^2, \tau(x)$$

$$= \frac{(2\,q^N\,\beta\,q^2\,x - 2\,q^N\,\beta\,q^2 + 2\,q^N\,\beta\,q - x\,q^N - 2\,\beta\,q + 1)\,q^N}{q-1}, \lambda_{q,\,n} =$$

$$-\frac{(q^n - 1)\,(2\,\beta\,q^n\,q - 1)}{q^n\,(q-1)^2\,q}, S(n,x) = P_n\left(2\,q^{1+N}\,\beta, q^{-1-N}, 1, q^{1+N}\,x, q\right), \frac{\rho(q\,x)}{\rho(x)}$$

$$= \frac{1}{(q^N)^2\,q^3\,(2\,\beta - x)\,(q\,q^N\,x - 1)}\left(2\,(q^N)^3\,\beta\,q^3\,x - (q^N)^3\,q^2\,x^2 - 2\,\beta\,q^3\,(q^N)^2\right.$$

$$\left.-2\,q^N\,\beta\,q^2\,x^2 + (q^N)^2\,q^2\,x + 2\,q^N\,\beta\,q^2\,x - 2\,q^N\,\beta\,q\,x + q^N\,x^2 + 2\,\beta\,q\,x - x\right), \frac{k_{n+1}}{k_n}$$

$$= \frac{\left(2\,(q^n)^2\,q\,\beta - 1\right)\left(2\,(q^n)^2\,\beta\,q^2 - 1\right)q^N}{(2\,\beta\,q^n\,q - 1)^2\,(q^N - q^n)\,q^{1+N}}, \mathrm{I} = \left[\frac{q}{q^{1+N}}, 2\,\beta\,q\right]\Bigg]\Bigg]\Bigg],$$

$$\left[\text{"Has a solution as q-Hahn"}, \left[\left[\sigma(x) = -(2\,\beta\,q - x)\,(x\,q^N - 1)\,(q^N)^3\,\beta\,q^3, \tau(x)\right.\right.\right.$$

$$= \frac{(2\,q^N\,\beta\,q^2\,x - 2\,q^N\,\beta\,q^2 + 2\,q^N\,\beta\,q - x\,q^N - 2\,\beta\,q + 1)\,\beta\,q^N\,q}{q-1}, \lambda_{q,\,n} =$$

$$-\frac{\beta\,(q^n-1)\,(2\,\beta\,q^n\,q-1)}{q^n\,(q-1)^2},\ S(n,x)=Q_n\left(1,2\,\beta,-\frac{N\ln(q)-\ln\left(\frac{1}{2\,\beta\,q^2}\right)}{\ln(q)},q\,q^N x,q\right),$$

$$\frac{\rho(q\,x)}{\rho(x)}=\frac{1}{(q^N)^2\,q^3\,(2\,\beta-x)\,(q\,q^N x-1)}\Big(2\,(q^N)^3\,\beta\,q^3\,x-(q^N)^3\,q^2\,x^2$$

$$-2\,\beta\,q^3\,(q^N)^2-2\,q^N\,\beta\,q^2\,x^2+(q^N)^2\,q^2\,x+2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q\,x+q^N\,x^2+2\,\beta\,q\,x-x\Big),$$

$$\frac{k_{n+1}}{k_n}=\frac{\left(2\,(q^n)^2\,q\,\beta-1\right)\left(2\,(q^n)^2\,\beta\,q^2-1\right)}{(2\,\beta\,q^n\,q-1)^2\,(q^N-q^n)\,q},\ \mathrm{I}=\left[0,\frac{1}{q\,q^N},\frac{2}{q\,q^N},\text{"..."},\right.$$

$$\left.\left.-\frac{N\ln(q)-\ln\left(\frac{1}{2\,\beta\,q^2}\right)}{\ln(q)\,q\,q^N}\right]\right],\ \left[\sigma(x)=-\frac{(2\,\beta\,q-x)\,(x\,q^N-1)\,\beta}{q\,q^N},\tau(x)\right.$$

$$=\frac{\beta\,(2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1)}{q\,q^N\,(q-1)},\ \lambda_{q,n}=$$

$$-\frac{\beta\,(q^n-1)\,(2\,\beta\,q^n\,q-1)}{q^n\,(q-1)^2},\ S(n,x)=Q_n\left(\frac{1}{q\,q^N},2\,q^N\,\beta\,q,\frac{\ln\left(\frac{1}{2\,q\,\beta}\right)}{\ln(q)},x,q\right),\ \frac{\rho(q\,x)}{\rho(x)}=$$

$$-\frac{2\,\beta\,(x-1)}{2\,\beta-x},\ \frac{k_{n+1}}{k_n}=\frac{\left(2\,(q^n)^2\,q\,\beta-1\right)\left(2\,(q^n)^2\,\beta\,q^2-1\right)q^N}{(2\,\beta\,q^n\,q-1)^2\,(q^N-q^n)},\ \mathrm{I}=\left[0,1,2,\text{"..."},\right.$$

$$\left.\left.\frac{\ln\left(\frac{1}{2\,q\,\beta}\right)}{\ln(q)}\right]\right],\ \left[\sigma(x)=-\frac{(2\,\beta\,q-x)\,(x\,q^N-1)}{q^N\,q},\tau(x)\right.$$

$$=\frac{2\,q^N\,\beta\,q^2\,x-2\,q^N\,\beta\,q^2+2\,q^N\,\beta\,q-x\,q^N-2\,\beta\,q+1}{q\,(q-1)\,q^N},\ \lambda_{q,n}=-\frac{(q^n-1)\,(2\,\beta\,q^n\,q-1)}{q^n\,(q-1)^2},$$

$$S(n,x)=Q_n(2\,\beta,1,N,x,q),\ \frac{\rho(q\,x)}{\rho(x)}=-\frac{2\,\beta\,(x-1)}{2\,\beta-x},\ \frac{k_{n+1}}{k_n}$$

$$=\frac{\left(2\,(q^n)^2\,q\,\beta-1\right)\left(2\,(q^n)^2\,\beta\,q^2-1\right)q^N}{(2\,\beta\,q^n\,q-1)^2\,(q^N-q^n)},\ \mathrm{I}=[0,1,2,\text{"..."},N]\Big],\ \Big[\sigma(x)=-(2\,\beta\,q$$

$$-x)\,(x\,q^N-1)\,(q^N)^3\,q^2,\tau(x)$$

$$= \frac{\left(2\, q^{N} \beta\, q^{2}\, x - 2\, q^{N} \beta\, q^{2} + 2\, q^{N} \beta\, q - x\, q^{N} - 2\, \beta\, q + 1\right) q^{N}}{q - 1}, \lambda_{q,\, n} =$$

$$-\frac{\left(q^{n} - 1\right) \left(2\, \beta\, q^{n}\, q - 1\right)}{q^{n} \left(q - 1\right)^{2} q}, S(n, x) = Q_{n}\left(2\, q^{N} \beta\, q, \frac{1}{q\, q^{N}}, -1, q\, q^{N} x, q\right), \frac{\rho(q\, x)}{\rho(x)}$$

$$= \frac{1}{\left(q^{N}\right)^{2} q^{3} \left(2\, \beta - x\right) \left(q\, q^{N} x - 1\right)} \left(2\, \left(q^{N}\right)^{3} \beta\, q^{3}\, x - \left(q^{N}\right)^{3} q^{2}\, x^{2} - 2\, \beta\, q^{3} \left(q^{N}\right)^{2}\right.$$

$$\left. - 2\, q^{N} \beta\, q^{2}\, x^{2} + \left(q^{N}\right)^{2} q^{2}\, x + 2\, q^{N} \beta\, q^{2}\, x - 2\, q^{N} \beta\, q\, x + q^{N} x^{2} + 2\, \beta\, q\, x - x\right), \frac{k_{n+1}}{k_{n}}$$

$$= \frac{\left(2\, \left(q^{n}\right)^{2} q\, \beta - 1\right) \left(2\, \left(q^{n}\right)^{2} \beta\, q^{2} - 1\right)}{\left(2\, \beta\, q^{n}\, q - 1\right)^{2} \left(q^{N} - q^{n}\right) q}, \mathrm{I} = \left[0, \frac{1}{q\, q^{N}}, \frac{2}{q\, q^{N}}, \text{"..."}, -\frac{1}{q\, q^{N}}\right]\Bigg]\Bigg]\Bigg]\Bigg]$$

# Meixner

```
> msum := proc (b, c, x, j) hyperterm([-n, -x], [beta], 1-1/c, j)
  end proc;
```

$$msum := \mathbf{proc}(b, c, x, j)\ hyperterm([-n, -x], [\mathrm{beta}], 1 - 1/c, j)\ \mathbf{end\ proc} \quad \mathbf{(8)}$$

```
> RM1 := sumrecursion(msum(b, c, x, j), j, S(n), recursion = up);
```

$$RM1 := c\left(\beta + n + 1\right) S(n + 2) - \left(\beta\, c + c\, n + x\, c + c + n - x + 1\right) S(n + 1) + (n + 1)\, S(n) = 0 \quad \mathbf{(9)}$$

```
> RM2 := sumrecursion(msum(beta, 1/c, -x-beta, j), j, S(n),
  recursion = up);
```

$$RM2 := \left(\beta + n + 1\right) S(n + 2) - \left(\beta\, c + c\, n + x\, c + c + n - x + 1\right) S(n + 1) + (n + 1)\, c\, S(n) = 0 \quad \mathbf{(10)}$$

```
> `recursion/compare`(RM1, RM2, S(n));
```

*Recursions are NOT identical!* **(11)**

```
> msum(beta, 1/c, -x-beta, n);
```

$$\frac{\mathrm{pochhammer}(-n, n)\, \mathrm{pochhammer}(x + \beta, n)\, (1 - c)^{n}}{\mathrm{pochhammer}(\beta, n)\, n!} \quad \mathbf{(12)}$$

```
> denumM := msum(beta, 1/c, -x-beta, n)/pochhammer(x+beta, n);
```

$$denumM := \frac{\mathrm{pochhammer}(-n, n)\, (1 - c)^{n}}{\mathrm{pochhammer}(\beta, n)\, n!} \quad \mathbf{(13)}$$

```
> msum(b, c, x, n);
```

$$\frac{\mathrm{pochhammer}(-n, n)\, \mathrm{pochhammer}(-x, n) \left(1 - \frac{1}{c}\right)^{n}}{\mathrm{pochhammer}(\beta, n)\, n!} \quad \mathbf{(14)}$$

```
> numM := msum(b, c, x, n)*(-1)^n/pochhammer(-x, n);
```

$$numM := \frac{\mathrm{pochhammer}(-n, n) \left(1 - \frac{1}{c}\right)^{n} (-1)^{n}}{\mathrm{pochhammer}(\beta, n)\, n!} \quad \mathbf{(15)}$$

```
> CM := denumM/numM;
```

$$CM := \frac{(1-c)^n}{\left(1-\frac{1}{c}\right)^n (-1)^n} \tag{16}$$

```
> NRM1 := sumrecursion(CM*msum(b, c, x, j), j, S(n), recursion =
  up);
```

$$NRM1 := (\beta+n+1) S(n+2) - (\beta c + c n + x c + c + n - x + 1) S(n+1) + (n+1) c S(n) = 0 \tag{17}$$

```
> `recursion/compare`(NRM1, RM2, S(n));
```

*Recursions are identical.* (18)

```
> simplify([seq(CM*add(msum(b, c, x, j), j = 0 .. n), n = 0 .. 3)-
  seq(add(msum(beta, 1/c, -x-beta, j), j = 0 .. n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \tag{19}$$

## Krawtchouk

```
> ksum := proc (p, N, x, j) hyperterm([-n, -x], [-N], 1/p, j) end
  proc;
```

$$ksum := \mathbf{proc}(p, N, x, j)\ hyperterm([-n, -x], [-N], 1/p, j)\ \mathbf{end\ proc} \tag{20}$$

```
> RK1 := sumrecursion(ksum(p, N, x, j), j, S(n), recursion = up);
```

$$RK1 := -p(-n-1+N) S(n+2) + (N p - 2 n p + n - 2 p - x + 1) S(n+1) + (n+1)(p-1) S(n) = 0 \tag{21}$$

```
> RK2 := sumrecursion(ksum(1-p, N, -x+N, j), j, S(n), recursion =
  up);
```

$$RK2 := -(p-1)(-n-1+N) S(n+2) + (N p - 2 n p + n - 2 p - x + 1) S(n+1) + (n+1) p S(n) = 0 \tag{22}$$

```
> `recursion/compare`(RK1, RK2, S(n));
```

*Recursions are NOT identical!* (23)

```
> ksum(p, N, x, n);
```

$$\frac{\mathrm{pochhammer}(-n, n)\ \mathrm{pochhammer}(-x, n) \left(\frac{1}{p}\right)^n}{\mathrm{pochhammer}(-N, n)\ n!} \tag{24}$$

```
> denumK := ksum(p, N, x, n)*(-1)^n/pochhammer(-x, n);
```

$$denumK := \frac{\mathrm{pochhammer}(-n, n) \left(\frac{1}{p}\right)^n (-1)^n}{\mathrm{pochhammer}(-N, n)\ n!} \tag{25}$$

```
> ksum(1-p, N, -x+N, n);
```

$$\frac{\mathrm{pochhammer}(-n, n)\ \mathrm{pochhammer}(x-N, n) \left(\frac{1}{-p+1}\right)^n}{\mathrm{pochhammer}(-N, n)\ n!} \tag{26}$$

```
> numK := ksum(1-p, N, -x+N, n)/pochhammer(x-N, n);
```

(27)

$$numK := \frac{\mathrm{pochhammer}(-n, n)\left(\frac{1}{-p+1}\right)^n}{\mathrm{pochhammer}(-N, n)\, n!} \tag{27}$$

```
> CK := numK/denumK;
```

$$CK := \frac{\left(\frac{1}{-p+1}\right)^n}{\left(\frac{1}{p}\right)^n (-1)^n} \tag{28}$$

```
> NRK1 := sumrecursion(CK*ksum(p, N, x, j), j, S(n), recursion = up);
```

$$NRK1 := -(p-1)(-n-1+N)\,S(n+2) + (N p - 2 n p + n - 2 p - x + 1)\,S(n+1) + (n+1)\,p\,S(n) = 0 \tag{29}$$

```
> `recursion/compare`(NRK1, RK2, S(n));
```

*Recursions are identical.* **(30)**

```
> simplify([seq(CK*add(ksum(p, N, x, j), j = 0 .. n), n = 0 .. 3)-seq(add(ksum(1-p, N, -x+N, j), j = 0 .. n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \tag{31}$$

## Hahn

```
> hsum := proc (alpha, beta, N, x, j) hyperterm([-n, n+alpha+beta+1, -x], [alpha+1, -N], 1, j) end proc;
```

$hsum :=$ **proc**(alpha, beta, $N$, $x$, $j$) **(32)**

$\quad hyperterm([-n, n + \mathrm{alpha} + \mathrm{beta} + 1, -x], [\mathrm{alpha} + 1, -N], 1, j)$

**end proc**

```
> RH1 := sumrecursion(hsum(alpha, beta, N, x, j), j, S(n), recursion = up);
```

$$RH1 := (2+\alpha+n)(2+2n+\alpha+\beta)(n+2+\alpha+\beta)(-n-1+N)\,S(n+2) - (2n+3+\alpha+\beta)\big(N\alpha^2 + N\alpha\beta + 2N\alpha n + 2N\beta n + 2Nn^2 - \alpha^2 n - \alpha^2 x - 2\alpha\beta x - \alpha n^2 - 4\alpha n x + \beta^2 n - \beta^2 x + \beta n^2 - 4\beta n x - 4n^2 x + 3\alpha N + 3N\beta + 6Nn - \alpha^2 - 3n\alpha - 6\alpha x + \beta^2 + 3n\beta - 6\beta x - 12xn + 4N - 2\alpha + 2\beta - 8x\big)\,S(n+1) + (n+1)(\beta+n+1)(2n+4+\alpha+\beta)(2+n+\alpha+\beta+N)\,S(n) = 0 \tag{33}$$

```
> RH2 := sumrecursion(hsum(beta, alpha, N, -x+N, j), j, S(n), recursion = up);
```

$$RH2 := (2+\beta+n)(2+2n+\alpha+\beta)(n+2+\alpha+\beta)(-n-1+N)\,S(n+2) + (2n+3+\alpha+\beta)\big(N\alpha^2 + N\alpha\beta + 2N\alpha n + 2N\beta n + 2Nn^2 - \alpha^2 n - \alpha^2 x - 2\alpha\beta x - \alpha n^2 - 4\alpha n x + \beta^2 n - \beta^2 x + \beta n^2 - 4\beta n x - 4n^2 x + 3\alpha N + 3N\beta + 6Nn - \alpha^2 - 3n\alpha - 6\alpha x + \beta^2 + 3n\beta - 6\beta x - 12xn + 4N - 2\alpha + 2\beta - 8x\big)\,S(n+1) + (n+1)(\alpha+1+n)(2n+4+\alpha+\beta)(2+n+\alpha+\beta+N)\,S(n) = 0 \tag{34}$$

```
> `recursion/compare`(RH1, RH2, S(n));
```

*Recursions are NOT identical!* **(35)**

```
> hsum(alpha, beta, N, x, n);
```

$$\frac{\mathrm{pochhammer}(-n, n)\ \mathrm{pochhammer}(n+\alpha+\beta+1, n)\ \mathrm{pochhammer}(-x, n)}{\mathrm{pochhammer}(\alpha+1, n)\ \mathrm{pochhammer}(-N, n)\ n!} \quad \textbf{(36)}$$

```
> denumH := hsum(alpha, beta, N, x, n)*(-1)^n/pochhammer(-x, n);
```

$$denumH := \frac{\mathrm{pochhammer}(-n, n)\ \mathrm{pochhammer}(n+\alpha+\beta+1, n)\ (-1)^n}{\mathrm{pochhammer}(\alpha+1, n)\ \mathrm{pochhammer}(-N, n)\ n!} \quad \textbf{(37)}$$

```
> hsum(beta, alpha, N, -x+N, n);
```

$$\frac{\mathrm{pochhammer}(-n, n)\ \mathrm{pochhammer}(n+\alpha+\beta+1, n)\ \mathrm{pochhammer}(x-N, n)}{\mathrm{pochhammer}(\beta+1, n)\ \mathrm{pochhammer}(-N, n)\ n!} \quad \textbf{(38)}$$

```
> numH21 := hsum(beta, alpha, N, -x+N, n)/pochhammer(x-N, n);
```

$$numH21 := \frac{\mathrm{pochhammer}(-n, n)\ \mathrm{pochhammer}(n+\alpha+\beta+1, n)}{\mathrm{pochhammer}(\beta+1, n)\ \mathrm{pochhammer}(-N, n)\ n!} \quad \textbf{(39)}$$

```
> CH21 := simplify(numH21/denumH);
```

$$CH21 := \frac{\mathrm{pochhammer}(\alpha+1, n)\ (-1)^{-n}}{\mathrm{pochhammer}(\beta+1, n)} \quad \textbf{(40)}$$

```
> NRH1 := sumrecursion(CH21*hsum(alpha, beta, N, x, j), j, S(n),
  recursion = up);
```

$$NRH1 := (2+\beta+n)\ (2+2n+\alpha+\beta)\ (n+2+\alpha+\beta)\ (-n-1+N)\ S(n+2) + (2n + 3+\alpha+\beta)\ \big(N\alpha^2 + N\alpha\beta + 2N\alpha n + 2N\beta n + 2Nn^2 - \alpha^2 n - \alpha^2 x - 2\alpha\beta x - \alpha n^2 - 4\alpha n x + \beta^2 n - \beta^2 x + \beta n^2 - 4\beta n x - 4n^2 x + 3\alpha N + 3N\beta + 6Nn - \alpha^2 - 3n\alpha - 6\alpha x + \beta^2 + 3n\beta - 6\beta x - 12xn + 4N - 2\alpha + 2\beta - 8x\big)\ S(n+1) + (n+1)\ (\alpha + 1+n)\ (2n+4+\alpha+\beta)\ (2+n+\alpha+\beta+N)\ S(n) = 0 \quad \textbf{(41)}$$

```
> `recursion/compare`(NRH1, RH2, S(n));
```

*Recursions are identical.* **(42)**

```
> simplify([seq(CH21*add(hsum(alpha, beta, N, x, j), j = 0 .. n), n
  = 0 .. 3)-seq(add(hsum(beta, alpha, N, -x+N, j), j = 0 .. n), n =
  0 .. 3)]);
```

$$[0, 0, 0, 0] \quad \textbf{(43)}$$

## Big q-Jacobi

```
> BJsummand := proc (a, b, c, x, q, j) qphihyperterm([q^(-n), a*b*
  q^(n+1), x], [a*q, c*q], q, q, j) end proc;
```

*BJsummand* := **proc**$(a, b, c, x, q, j)$ **(44)**

*qphihyperterm*$([q\hat{}(-n), a*b*q\hat{}(n+1), x], [a*q, c*q], q, q, j)$

**end proc**

```
> RB1 := qsumrecursion(BJsummand(a, b, c, x, q, j), q, j, S(n),
  recursion = up);
```

$$RB1 := -(q^{n+2}a-1)\ (q^{n+2}c-1)\ (q^{2n+2}ab-1)\ (q^{n+2}ab-1)\ S(n+2) - (q^{2n+3}ab - 1)\ (q^{4n+6}a^2xb^2 - q^{3n+5}a^2b^2 - q^{3n+5}a^2bc - q^{3n+5}a^2b - q^{3n+5}abc + q^{2n+4}a^2b + q^{2n+4}abc - q^{2n+4}axb + q^{2n+3}a^2b + q^{2n+3}abc + q^{2n+4}ab + q^{2n+4}ac \quad \textbf{(45)}$$

$+ q^{2n+3} a b - q^{2n+2} a x b + q^{2n+3} a c - q^{n+2} a b - q^{n+2} a c - q^{n+2} a - q^{n+2} c + x) S(n + 1) + q^{n+2} a (q^{n+1} - 1) (a b q^{n+1} - c) (q^{n+1} b - 1) (q^{2n+4} a b - 1) S(n) = 0$

```
> RB2 := qsumrecursion(BJsummand(b, a, a*b/c, b*x/c, q, j), q, j, S(n), recursion = up);
```

$$RB2 := -(q^{n+2} b - 1) (q^{2n+2} a b - 1) (q^{n+2} a b - c) (q^{n+2} a b - 1) S(n+2) - (q^{2n+3} a b - 1) b (q^{4n+6} a^2 x b^2 - q^{3n+5} a^2 b^2 - q^{3n+5} a^2 b c - q^{3n+5} a^2 b - q^{3n+5} a b c + q^{2n+4} a^2 b + q^{2n+4} a b c - q^{2n+4} a x b + q^{2n+3} a^2 b + q^{2n+3} a b c + q^{2n+4} a b + q^{2n+4} a c + q^{2n+3} a b - q^{2n+2} a x b + q^{2n+3} a c - q^{n+2} a b - q^{n+2} a c - q^{n+2} a - q^{n+2} c + x) S(n+1) + q^{n+2} a (q^{n+1} - 1) b^2 (q^{n+1} c - 1) (q^{2n+4} a b - 1) (q^{n+1} a - 1) S(n) = 0 \tag{46}$$

```
> RB3 := qsumrecursion(BJsummand(c, a*b/c, a, x, q, j), q, j, S(n), recursion = up);
```

$$RB3 := -(q^{n+2} a - 1) (q^{n+2} c - 1) (q^{2n+2} a b - 1) (q^{n+2} a b - 1) S(n+2) - (q^{2n+3} a b - 1) (q^{4n+6} a^2 x b^2 - q^{3n+5} a^2 b^2 - q^{3n+5} a^2 b c - q^{3n+5} a^2 b - q^{3n+5} a b c + q^{2n+4} a^2 b + q^{2n+4} a b c - q^{2n+4} a x b + q^{2n+3} a^2 b + q^{2n+3} a b c + q^{2n+4} a b + q^{2n+4} a c + q^{2n+3} a b - q^{2n+2} a x b + q^{2n+3} a c - q^{n+2} a b - q^{n+2} a c - q^{n+2} a - q^{n+2} c + x) S(n+1) + q^{n+2} a (q^{n+1} - 1) (a b q^{n+1} - c) (q^{n+1} b - 1) (q^{2n+4} a b - 1) S(n) = 0 \tag{47}$$

```
> RB4 := qsumrecursion(BJsummand(a*b/c, c, b, b*x/c, q, j), q, j, S(n), recursion = up);
```

$$RB4 := -(q^{n+2} b - 1) (q^{2n+2} a b - 1) (q^{n+2} a b - c) (q^{n+2} a b - 1) S(n+2) - (q^{2n+3} a b - 1) b (q^{4n+6} a^2 x b^2 - q^{3n+5} a^2 b^2 - q^{3n+5} a^2 b c - q^{3n+5} a^2 b - q^{3n+5} a b c + q^{2n+4} a^2 b + q^{2n+4} a b c - q^{2n+4} a x b + q^{2n+3} a^2 b + q^{2n+3} a b c + q^{2n+4} a b + q^{2n+4} a c + q^{2n+3} a b - q^{2n+2} a x b + q^{2n+3} a c - q^{n+2} a b - q^{n+2} a c - q^{n+2} a - q^{n+2} c + x) S(n+1) + q^{n+2} a (q^{n+1} - 1) b^2 (q^{n+1} c - 1) (q^{2n+4} a b - 1) (q^{n+1} a - 1) S(n) = 0 \tag{48}$$

```
>
> `recursion/compare`(RB1, simplify(RB2), S(n));
```

*Recursions are NOT identical!* (49)

```
> `recursion/compare`(RB1, simplify(RB3), S(n));
```

*Recursions are identical.* (50)

```
> `recursion/compare`(RB1, simplify(RB4), S(n));
```

*Recursions are NOT identical!* (51)

```
> `recursion/compare`(RB2, simplify(RB3), S(n));
```

*Recursions are NOT identical!* (52)

```
> `recursion/compare`(RB2, simplify(RB4), S(n));
```

*Recursions are identical.* (53)

```
> `recursion/compare`(RB3, simplify(RB4), S(n));
```

*Recursions are NOT identical!* (54)

Obtaining Cn

```
> BJsummand(a, b, c, x, q, n);
```

$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ \mathit{qpochhammer}(x, q, n)\ q^{n}}{\mathit{qpochhammer}(a\,q, q, n)\ \mathit{qpochhammer}(c\,q, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{55}$$

```
> BJsummand(b, a, a*b/c, b*x/c, q, n);
```

$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ \mathit{qpochhammer}\left(\frac{b\,x}{c}, q, n\right)\ q^{n}}{\mathit{qpochhammer}(b\,q, q, n)\ \mathit{qpochhammer}\left(\frac{a\,b\,q}{c}, q, n\right)\ \mathit{qpochhammer}(q, q, n)} \tag{56}$$

```
> num21 := BJsummand(b, a, a*b/c, b*x/c, q, n)*(-1)^n*q^binomial(n,
  2)*(b/c)^n/qpochhammer(b*x/c, q, n);
```

$$\mathit{num21} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ q^{n}\ (-1)^{n}\ q^{\binom{n}{2}}\ \left(\frac{b}{c}\right)^{n}}{\mathit{qpochhammer}(b\,q, q, n)\ \mathit{qpochhammer}\left(\frac{a\,b\,q}{c}, q, n\right)\ \mathit{qpochhammer}(q, q, n)} \tag{57}$$

```
> denumbj := BJsummand(a, b, c, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$\mathit{denumbj} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ q^{n}\ (-1)^{n}\ q^{\binom{n}{2}}}{\mathit{qpochhammer}(a\,q, q, n)\ \mathit{qpochhammer}(c\,q, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{58}$$

```
> C21 := simplify(num21/denumbj);
```

$$\mathit{C21} := \frac{\left(\frac{b}{c}\right)^{n}\ \mathit{qpochhammer}(a\,q, q, n)\ \mathit{qpochhammer}(c\,q, q, n)}{\mathit{qpochhammer}(b\,q, q, n)\ \mathit{qpochhammer}\left(\frac{a\,b\,q}{c}, q, n\right)} \tag{59}$$

Checking initial conditions

```
> qsimpcomb([seq(C21*add(BJsummand(a, b, c, x, q, j), j = 0 .. n),
  n = 0 .. 3)-seq(add(BJsummand(b, a, a*b/c, b*x/c, q, j), j = 0 ..
  n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \tag{60}$$

Comparing recurrence equations with new relation

```
> NRB1 := qsumrecursion(C21*BJsummand(a, b, c, x, q, j), q, j, S
  (n), recursion = up);
```

$$\begin{aligned}\mathit{NRB1} := {} & -(q^{n+2}\,b-1)\,(q^{2n+2}\,a\,b-1)\,(q^{n+2}\,a\,b-c)\,(q^{n+2}\,a\,b-1)\,S(n+2) \\ & -(q^{2n+3}\,a\,b-1)\,b\,(q^{4n+6}\,a^{2}\,x\,b^{2}-q^{3n+5}\,a^{2}\,b^{2}-q^{3n+5}\,a^{2}\,b\,c-q^{3n+5}\,a^{2}\,b \\ & -q^{3n+5}\,a\,b\,c+q^{2n+4}\,a^{2}\,b+q^{2n+4}\,a\,b\,c-q^{2n+4}\,a\,x\,b+q^{2n+3}\,a^{2}\,b+q^{2n+3}\,a\,b\,c \\ & +q^{2n+4}\,a\,b+q^{2n+4}\,a\,c+q^{2n+3}\,a\,b-q^{2n+2}\,a\,x\,b+q^{2n+3}\,a\,c-q^{n+2}\,a\,b-q^{n+2}\,a\,c \\ & -q^{n+2}\,a-q^{n+2}\,c+x)\,S(n+1)+q^{n+2}\,a\,(q^{n+1}-1)\,b^{2}\,(q^{n+1}\,c-1)\,(q^{2n+4}\,a\,b \\ & -1)\,(q^{n+1}\,a-1)\,S(n)=0\end{aligned} \tag{61}$$

```
> `recursion/compare`(NRB1, simplify(RB2), S(n));
```

*Recursions are identical.* **(62)**

```
> BJsummand(c, a*b/c, a, x, q, n);
```

**(63)**

$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ \mathit{qpochhammer}(x, q, n)\ q^n}{\mathit{qpochhammer}(a\,q, q, n)\ \mathit{qpochhammer}(c\,q, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{63}$$

```
> num31 := BJsummand(c, a*b/c, a, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$\mathit{num31} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{\mathit{qpochhammer}(a\,q, q, n)\ \mathit{qpochhammer}(c\,q, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{64}$$

```
> C31 := num31/denumbj;
```

$$C31 := 1 \tag{65}$$

```
> BJsummand(a*b/c, c, b, b*x/c, q, n);
```

$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ \mathit{qpochhammer}\left(\frac{b\,x}{c}, q, n\right)\ q^n}{\mathit{qpochhammer}(b\,q, q, n)\ \mathit{qpochhammer}\left(\frac{a\,b\,q}{c}, q, n\right)\ \mathit{qpochhammer}(q, q, n)} \tag{66}$$

```
> num41 := BJsummand(a*b/c, c, b, b*x/c, q, n)*(-1)^n*q^binomial(n,
  2)*(b/c)^n/qpochhammer(b*x/c, q, n);
```

$$\mathit{num41} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a\,b\,q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}\left(\frac{b}{c}\right)^n}{\mathit{qpochhammer}(b\,q, q, n)\ \mathit{qpochhammer}\left(\frac{a\,b\,q}{c}, q, n\right)\ \mathit{qpochhammer}(q, q, n)} \tag{67}$$

```
> C41 := num41/denumbj;
```

$$C41 := \frac{\left(\frac{b}{c}\right)^n\ \mathit{qpochhammer}(a\,q, q, n)\ \mathit{qpochhammer}(c\,q, q, n)}{\mathit{qpochhammer}(b\,q, q, n)\ \mathit{qpochhammer}\left(\frac{a\,b\,q}{c}, q, n\right)} \tag{68}$$

```
> C21/C41;
```

$$1 \tag{69}$$

```
> NRB3 := qsumrecursion(C21*BJsummand(c, a*b/c, a, x, q, j), q, j,
  S(n), recursion = up);
```

$$\begin{aligned} NRB3 := &-(q^{n+2}\,b-1)\,(q^{2n+2}\,a\,b-1)\,(q^{n+2}\,a\,b-c)\,(q^{n+2}\,a\,b-1)\,S(n+2) \\ &-(q^{2n+3}\,a\,b-1)\,b\,(q^{4n+6}\,a^2\,x\,b^2-q^{3n+5}\,a^2\,b^2-q^{3n+5}\,a^2\,b\,c-q^{3n+5}\,a^2\,b \\ &-q^{3n+5}\,a\,b\,c+q^{2n+4}\,a^2\,b+q^{2n+4}\,a\,b\,c-q^{2n+4}\,a\,x\,b+q^{2n+3}\,a^2\,b+q^{2n+3}\,a\,b\,c \\ &+q^{2n+4}\,a\,b+q^{2n+4}\,a\,c+q^{2n+3}\,a\,b-q^{2n+2}\,a\,x\,b+q^{2n+3}\,a\,c-q^{n+2}\,a\,b-q^{n+2}\,a\,c \\ &-q^{n+2}\,a-q^{n+2}\,c+x)\,S(n+1)+q^{n+2}\,a\,(q^{n+1}-1)\,b^2\,(q^{n+1}\,c-1)\,(q^{2n+4}\,a\,b \\ &-1)\,(q^{n+1}\,a-1)\,S(n)=0 \end{aligned} \tag{70}$$

```
> `recursion/compare`(RB2, simplify(NRB3), S(n));
```

*Recursions are identical.* (71)

# Little q-Jacobi

Liitle q-Jacobi to q-Krawtchouk

```
> LJsummand := proc (a, b, x, q, j) qphihyperterm([q^(-n), a*b*q^
  (n+1)], [a*q], q, q*x, j) end proc;
```

$$LJsummand := \mathbf{proc}(a, b, x, q, j)\quad qphihyperterm([q\verb|^|(-n), a*b*q\verb|^|(n+1)], [a*q], q, q*x, j)\quad \mathbf{end\ proc} \tag{72}$$

```
> RL := qsumrecursion(LJsummand(a, b, x, q, j), q, j, S(n),
  recursion = up);
```

$$RL := q^{n+1} (q^{n+2} a - 1) (q^{2n+2} a b - 1) (q^{n+2} a b - 1) S(n+2) + (q^{2n+3} a b - 1) (q^{4n+6} a^2 x b^2 - q^{3n+4} a^2 b - q^{3n+4} a b - q^{2n+4} a x b + q^{2n+3} a b - q^{2n+2} a x b + q^{2n+2} a b + q^{2n+3} a + q^{2n+2} a - q^{n+1} a - q^{n+1} + x) S(n+1) + q^{n+1} a (q^{n+1} - 1) (q^{n+1} b - 1) (q^{2n+4} a b - 1) S(n) = 0 \tag{73}$$

```
> qksum := proc (p, N, x, q, j) qphihyperterm([q^(-n), x, -p*q^n],
  [q^(-N), 0], q, q, j) end proc;
```

$$qksum := \mathbf{proc}(p, N, x, q, j)\quad qphihyperterm([q\verb|^|(-n), x, -p*q\verb|^|n], [q\verb|^|(-N), 0], q, q, j)\quad \mathbf{end\ proc} \tag{74}$$

```
> RK := qsumrecursion(qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b, q,
  j), q, j, S(n), recursion = up);
```

$$RK := \left(q^{n+1} - q^{\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)}}\right) (q^{2n+2} a b - 1) (q^{n+2} a b - 1) S(n+2) - \left(q^{4n + \frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 6} a^2 x b^3 - q^{3n + \frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 4} a^2 b^2 - q^{\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 2n + 4} a x b^2 - q^{\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 2n + 2} a x b^2 - q^{3n+3} a b + q^{\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 2n + 3} a b + q^{\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 2n + 2} a b + q^{2n+2} a b + q^{2n+1} a b - q^{n+1+\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)}} a b + q^{\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)}} x b - q^n\right) (q^{2n+3} a b - 1) q S(n+1) + q^{2n+2} a (q^{n+1} - 1) b \left(q^{n + \frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)} + 2} a b - 1\right) (q^{2n+4} a b - 1) S(n) = 0 \tag{75}$$

```
> `recursion/compare`(RL, simplify(RK), S(n));
```

*Recursions are NOT identical!* (76)

```
> qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(q x b, q, n)\ qpochhammer(a b q^n q, q, n)\ q^n}{qpochhammer\left(q^{-\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)}}, q, n\right) qpochhammer(q, q, n)} \tag{77}$$

```
> denumLK := qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b, q, n)*(-1)^n*
  (b*q)^n*q^binomial(n, 2)/qpochhammer(q*x*b, q, n);
```

$$denumLK := \frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(a b q^n q, q, n)\ q^n (-1)^n (b q)^n q^{\binom{n}{2}}}{qpochhammer\left(q^{-\frac{\ln\left(\frac{1}{b q}\right)}{\ln(q)}}, q, n\right) qpochhammer(q, q, n)} \tag{78}$$

```
> LJsummand(a, b, x, q, n);
```
$$\frac{qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(a\, b\, q^{n+1}, q, n\right)\, (q\, x)^{n}}{qpochhammer(a\, q, q, n)\, qpochhammer(q, q, n)} \tag{79}$$

```
> numLB := LJsummand(a, b, x, q, n)*q^n/(q*x)^n;
```
$$numLB := \frac{qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(a\, b\, q^{n+1}, q, n\right)\, q^{n}}{qpochhammer(a\, q, q, n)\, qpochhammer(q, q, n)} \tag{80}$$

```
>
> CLK := simplify(numLB/denumLK);
```
$$CLK := \frac{qpochhammer(b\, q, q, n)\, (-1)^{-n}\, (b\, q)^{-n}\, q^{-\frac{n\,(n-1)}{2}}}{qpochhammer(a\, q, q, n)} \tag{81}$$

```
> NRK := qsumrecursion(CLK*qksum(-a*b*q, ln(1/(b*q))/ln(q), q*x*b,
  q, j), q, j, S(n), recursion = up);
```
$$NRK := \left(q^{n+1} - q^{\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}}\right) q^{n+1} \left(q^{n+2} a - 1\right) b \left(q^{2n+2} a\, b - 1\right) \left(q^{n+2} a\, b - 1\right) \left(q^{n+1} a - 1\right) S(n+2) + \left(q^{4n+\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+6} a^{2} x\, b^{3} - q^{3n+\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+4} a^{2} b^{2} - q^{\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+2n+4} a\, x\, b^{2} - q^{\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+2n+2} a\, x\, b^{2} - q^{3n+3} a\, b + q^{\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+2n+3} a\, b + q^{\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+2n+2} a\, b + q^{2n+2} a\, b + q^{2n+1} a\, b - q^{n+1+\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}} a\, b + q^{\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}} x\, b - q^{n}\right) \left(q^{2n+3} a\, b - 1\right) \left(q^{n+2} b - 1\right) \left(q^{n+1} a - 1\right) S(n+1) + q^{n} a \left(q^{n+1} - 1\right) \left(q^{n+2} b - 1\right) \left(q^{n+\frac{\ln\left(\frac{1}{b\,q}\right)}{\ln(q)}+2} a\, b - 1\right) \left(q^{n+1} b - 1\right) \left(q^{2n+4} a\, b - 1\right) S(n) = 0 \tag{82}$$

```
> `recursion/compare`(RL, simplify(NRK), S(n));
```
*Recursions are identical.* **(83)**

```
>
> simplify(qsimpcomb([seq(CLK*add(qksum(-a*b*q, ln(1/(b*q))/ln(q),
  q*x*b, q, j), j = 0 .. n), n = 0 .. 5)-seq(add(LJsummand(a, b, x,
  q, j), j = 0 .. n), n = 0 .. 5)]));
```
$$[0, 0, 0, 0, 0, 0] \tag{84}$$

## q-Laguerre

```
> qlsum := proc (a, x, q, j) qpochhammer(q^(a+1), q, n)*
  qphihyperterm([q^(-n)], [q^(a+1)], q, -q^(n+a+1)*x, j)
  /qpochhammer(q, q, n) end proc;
```
*qlsum* := **proc**$(a, x, q, j)$ **(85)**

$qpochhammer(q\wedge(a+1), q, n) * qphihyperterm([q\wedge(-n)], [q\wedge(a+1)], q, -q\wedge(n+a+1) * x, j) / qpochhammer(q, q, n)$

**end proc**

```
> Rll := qsumrecursion(qlsum(a, x, q, j), q, j, S(n), recursion = up);
```

$$Rll := \left(q^{n+2}-1\right) S(n+2) + \left(-q^{2n+a+3} x - q^{n+2+a} - q^{n+2} + q + 1\right) S(n+1) + \left(q^{n+a+1} - 1\right) q\, S(n) = 0 \quad (86)$$

```
> qmsum := proc (b, c, x, q, j) qphihyperterm([q^(-n), x], [b*q], q, -q^(n+1)/c, j) end proc;
```

$qmsum$ := **proc**$(b, c, x, q, j)$ **(87)**

$qphihyperterm([q\hat{}(-n), x], [b*q], q, -q\hat{}(n+1)/c, j)$

**end proc**

```
> RM := qsumrecursion(qmsum(0, -q^(-a), -x, q, j), q, j, S(n), recursion = up);
```

$$RM := S(n+2) + \left(q^{2n+a+3} x + q^{n+2+a} + q^{n+2} - q - 1\right) S(n+1) + \left(q^{n+a+1} - 1\right) q \left(q^{n+1} - 1\right) S(n) = 0 \quad (88)$$

```
> `recursion/compare`(Rll, simplify(RM), S(n));
```

*Recursions are NOT identical!* **(89)**

```
> qlsum(a, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) \left(-q^{n+a+1} x\right)^n (-1)^n q^{\frac{n(n-1)}{2}}}{qpochhammer(q, q, n)^2} \quad (90)$$

```
> numqL := qlsum(a, x, q, n)*(-q^(n+a+1))^n/(-q^(n+a+1)*x)^n;
```

$$numqL := \frac{qpochhammer\left(q^{-n}, q, n\right) (-1)^n q^{\frac{n(n-1)}{2}} \left(-q^{n+a+1}\right)^n}{qpochhammer(q, q, n)^2} \quad (91)$$

```
> qmsum(0, -q^(-a), -x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer(-x, q, n) \left(\frac{q^{n+1}}{q^{-a}}\right)^n}{qpochhammer(q, q, n)} \quad (92)$$

```
> denumqLM := qmsum(0, -q^(-a), -x, q, n)*q^binomial(n, 2)/qpochhammer(-x, q, n);
```

$$denumqLM := \frac{qpochhammer\left(q^{-n}, q, n\right) \left(\frac{q^{n+1}}{q^{-a}}\right)^n q^{\binom{n}{2}}}{qpochhammer(q, q, n)} \quad (93)$$

```
> CqLM := qsimpcomb(numqL/denumqLM);
```

$$CqLM := \frac{1}{qpochhammer(q, q, n)} \quad (94)$$

```
> NRM := qsumrecursion(CqLM*qmsum(0, -q^(-a), -x, q, j), q, j, S(n), recursion = up);
```

$$NRM := \left(q^{n+2}-1\right) S(n+2) + \left(-q^{2n+a+3} x - q^{n+2+a} - q^{n+2} + q + 1\right) S(n+1) + \left(q^{n+a+1} - 1\right) q\, S(n) = 0 \quad (95)$$

```
> `recursion/compare`(Rll, simplify(NRM), S(n));
```

*Recursions are identical.* **(96)**

```
> qsimpcomb([seq(CqLM*add(qmsum(0, -q^(-a), -x, q, j), j = 0 .. n),
```

```
n = 0 .. 5)-seq(add(qlsum(a, x, q, j), j = 0 .. n), n = 0 .. 5)])
;
```

$$[0, 0, 0, 0, 0, 0] \tag{97}$$

## Al-Salam Carlitz I

```
> A1summand := proc (a, x, q, j) (-a)^n*q^binomial(n, 2)*
  qphihyperterm([q^(-n), 1/x], [0], q, q*x/a, j) end proc;
```

*A1summand* := **proc**$(a, x, q, j)$ (98)

$(-a)\hat{\ }n * q\hat{\ }\mathrm{binomial}(n, 2) * qphihyperterm([q\hat{\ }(-n), 1/x], [0], q, q*x/a, j)$

**end proc**

```
> RA1 := qsumrecursion(A1summand(a, x, q, j), q, j, S(n), recursion
  = up);
```

$$RA1 := S(n+2) + \left(q^{n+1} a + q^{n+1} - x\right) S(n+1) + q^n a \left(q^{n+1} - 1\right) S(n) = 0 \tag{99}$$

```
> RA2 := qsumrecursion(A1summand(1/a, x/a, q, j), q, j, S(n),
  recursion = up);
```

$$RA2 := a\, S(n+2) + \left(q^{n+1} a + q^{n+1} - x\right) S(n+1) + q^n \left(q^{n+1} - 1\right) S(n) = 0 \tag{100}$$

```
> `recursion/compare`(RA1, simplify(RA2), S(n));
```

*Recursions are NOT identical!* (101)

```
> A1summand(a, x, q, n);
```

$$\frac{(-a)^n q^{\binom{n}{2}} qpochhammer(q^{-n}, q, n)\, qpochhammer\left(\frac{1}{x}, q, n\right) \left(\frac{q x}{a}\right)^n}{qpochhammer(q, q, n)} \tag{102}$$

```
> denuma1 := simplify(A1summand(a, x, q, n)*(q/a)^n/(qpochhammer
  (1/x, q, n)*(x*q/a)^n));
```

$$denuma1 := \frac{(-a)^n q^{\frac{n(n-1)}{2}} qpochhammer(q^{-n}, q, n) \left(\frac{q}{a}\right)^n}{qpochhammer(q, q, n)} \tag{103}$$

```
> A1summand(1/a, x/a, q, n);
```

$$\frac{\left(-\frac{1}{a}\right)^n q^{\binom{n}{2}} qpochhammer(q^{-n}, q, n)\, qpochhammer\left(\frac{a}{x}, q, n\right) (q x)^n}{qpochhammer(q, q, n)} \tag{104}$$

```
> numa1 := simplify(A1summand(1/a, x/a, q, n)*q^n/(qpochhammer(a/x,
  q, n)*(q*x)^n));
```

$$numa1 := \frac{q^{\frac{n(n+1)}{2}} \left(-\frac{1}{a}\right)^n qpochhammer(q^{-n}, q, n)}{qpochhammer(q, q, n)} \tag{105}$$

```
> CA1 := qsimpcomb(numa1/denuma1);
```

$$CA1 := \frac{1}{a^n} \tag{106}$$

```
> qsimpcomb([seq(CA1*add(A1summand(a, x, q, j), j = 0 .. n), n = 0
  .. 3)-seq(add(A1summand(1/a, x/a, q, j), j = 0 .. n), n = 0 .. 3)
  ]);
```

$$[0, 0, 0, 0] \tag{107}$$

```
> NRA1 := qsumrecursion(CA1*A1summand(a, x, q, j), q, j, S(n), recursion = up);
```

$$NRA1 := a\,S(n+2) + \left(q^{n+1} a + q^{n+1} - x\right) S(n+1) + q^{n} \left(q^{n+1} - 1\right) S(n) = 0 \qquad (108)$$

```
> `recursion/compare`(NRA1, simplify(RA2), S(n));
```

*Recursions are identical.* (109)

Discrete q-Hermite I

```
> qsumrecursion(A1summand(-1, -x, q, j), q, j, S(n), recursion = up);
```

$$-S(n+2) - x\,S(n+1) + q^{n}\left(q^{n+1} - 1\right) S(n) = 0 \qquad (110)$$

```
> qsumrecursion(A1summand(-1, x, q, j), q, j, S(n), recursion = up);
```

$$-S(n+2) + x\,S(n+1) + q^{n}\left(q^{n+1} - 1\right) S(n) = 0 \qquad (111)$$

```
> `recursion/compare`(qsumrecursion(A1summand(-1, -x, q, j), q, j, S(n), recursion = up), qsumrecursion(A1summand(-1, x, q, j), q, j, S(n), recursion = up), S(n));
```

*Recursions are NOT identical!* (112)

```
> A1summand(-1, -x, q, n);
```

$$\frac{q^{\binom{n}{2}}\, qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(-\frac{1}{x}, q, n\right) (q\,x)^{n}}{qpochhammer(q, q, n)} \qquad (113)$$

```
> numd1 := A1summand(-1, -x, q, n)*q^n/(qpochhammer(-1/x, q, n)*(q*x)^n);
```

$$numd1 := \frac{q^{\binom{n}{2}}\, qpochhammer\left(q^{-n}, q, n\right) q^{n}}{qpochhammer(q, q, n)} \qquad (114)$$

```
> A1summand(-1, x, q, n);
```

$$\frac{q^{\binom{n}{2}}\, qpochhammer\left(q^{-n}, q, n\right)\, qpochhammer\left(\frac{1}{x}, q, n\right) (-q\,x)^{n}}{qpochhammer(q, q, n)} \qquad (115)$$

```
> denumd1 := A1summand(-1, x, q, n)*(-1)^n*q^n/(qpochhammer(1/x, q, n)*(-q*x)^n);
```

$$denumd1 := \frac{q^{\binom{n}{2}}\, qpochhammer\left(q^{-n}, q, n\right) (-1)^{n} q^{n}}{qpochhammer(q, q, n)} \qquad (116)$$

```
> Cd1 := simplify(numd1/denumd1);
```

$$Cd1 := (-1)^{-n} \qquad (117)$$

```
> `recursion/compare`(qsumrecursion(Cd1*A1summand(-1, -x, q, j), q, j, S(n), recursion = up), qsumrecursion(A1summand(-1, x, q, j), q, j, S(n), recursion = up), S(n));
```

*Recursions are identical.* (118)

```
> qsimpcomb([seq(Cd1*add(A1summand(-1, -x, q, j), j = 0 .. n), n = 0 .. 3)-seq(add(A1summand(-1, x, q, j), j = 0 .. n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \qquad (119)$$

# Al-Salam Carlitz II

```
> A2summand := proc (a, x, q, j) (-a)^n*q^(-binomial(n, 2))*
  qphihyperterm([q^(-n), x], [], q, q^n/a, j) end proc;
```

*A2summand* := **proc**$(a, x, q, j)$ **(120)**

$(-a)\wedge n * q\wedge(-\text{binomial}(n,2)) * qphihyperterm([q\wedge(-n), x], [\ ], q, q\wedge n/a, j)$

**end proc**

```
> RA21 := qsumrecursion(CA1*A2summand(a, x, q, j), q, j, S(n),
  recursion = up);
```

$$RA21 := -q^{2n+1}\, a\, S(n+2) + q^n \left(q^{n+1} x - a - 1\right) S(n+1) + \left(q^{n+1} - 1\right) S(n) = 0 \tag{121}$$

```
> RA22 := qsumrecursion(A2summand(1/a, x/a, q, j), q, j, S(n),
  recursion = up);
```

$$RA22 := -q^{2n+1}\, a\, S(n+2) + q^n \left(q^{n+1} x - a - 1\right) S(n+1) + \left(q^{n+1} - 1\right) S(n) = 0 \tag{122}$$

```
> `recursion/compare`(RA21, simplify(RA22), S(n));
```

*Recursions are identical.* **(123)**

```
> A2summand(a, x, q, n);
```

$$\frac{(-a)^n\, q^{-\binom{n}{2}}\, qpochhammer(q^{-n}, q, n)\, qpochhammer(x, q, n) \left(\frac{q^n}{a}\right)^n}{(-1)^n\, q^{\frac{n(n-1)}{2}}\, qpochhammer(q, q, n)} \tag{124}$$

```
> denuma2 := simplify(A2summand(a, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n));
```

$$denuma2 := \frac{(-a)^n\, q^{-\frac{n(n-1)}{2}}\, qpochhammer(q^{-n}, q, n) \left(\frac{q^n}{a}\right)^n}{qpochhammer(q, q, n)} \tag{125}$$

```
> A2summand(1/a, x/a, q, n);
```

$$\frac{\left(-\frac{1}{a}\right)^n q^{-\binom{n}{2}}\, qpochhammer(q^{-n}, q, n)\, qpochhammer\left(\frac{x}{a}, q, n\right) (a\, q^n)^n}{(-1)^n\, q^{\frac{n(n-1)}{2}}\, qpochhammer(q, q, n)} \tag{126}$$

```
> numa2 := simplify(A2summand(1/a, x/a, q, n)*(-1)^n*(1/a)^n*
  q^binomial(n, 2)/qpochhammer(x/a, q, n));
```

$$numa2 := \frac{\left(-\frac{1}{a}\right)^n q^{-\frac{n(n-1)}{2}}\, qpochhammer(q^{-n}, q, n)\, (a\, q^n)^n \left(\frac{1}{a}\right)^n}{qpochhammer(q, q, n)} \tag{127}$$

```
> CA2 := qsimpcomb(numa2/denuma2);
```

$$CA2 := \frac{1}{a^n} \tag{128}$$

```
> `recursion/compare`(qsumrecursion(Cd1*A2summand(-1, -I*x, q, j),
  q, j, S(n), recursion = up), qsumrecursion(A2summand(-1, I*x, q,
  j), q, j, S(n), recursion = up), S(n));
```

*Recursions are identical.* **(129)**

```
> qsimpcomb([seq(CA2*add(A2summand(a, x, q, j), j = 0 .. n), n = 0
  .. 3)-seq(add(A2summand(1/a, x/a, q, j), j = 0 .. n), n = 0 .. 3)
```

```
]);
```

$$[0, 0, 0, 0] \tag{130}$$

Discrete q-Hermite II

```
> qsimpcomb([seq(Cd1*add(A2summand(-1, -I*x, q, j), j = 0 .. n), n
  = 0 .. 3)-seq(add(A2summand(-1, I*x, q, j), j = 0 .. n), n = 0 ..
  3)]);
```

$$[0, 0, 0, 0] \tag{131}$$

## q-Meixner

```
> qMsummand := proc (b, c, x, q, j) qphihyperterm([q^(-n), x], [b*
  q], q, -q^(n+1)/c, j) end proc;
```

$qMsummand := \mathbf{proc}(b, c, x, q, j)$ **(132)**

$\quad qphihyperterm([q\text{^}(-n), x], [b * q], q, -q\text{^}(n+1)/c, j)$

**end proc**

```
> RM1 := qsumrecursion(qMsummand(b, c, x, q, j), q, j, S(n),
  recursion = up);
```

$$RM1 := (q^{n+2}b-1)\,c\,S(n+2) + (-q^{2n+3}x - q^{n+2}bc - q^{n+2}c + q^{n+2} + cq + c)\,S(n+1) + q\,(q^{n+1}+c)\,(q^{n+1}-1)\,S(n) = 0 \tag{133}$$

```
> RM2 := qsumrecursion(qMsummand(-1/c, -1/b, -x/(b*c), q, j), q, j,
  S(n), recursion = up);
```

$$RM2 := (q^{n+2}+c)\,S(n+2) + (q^{2n+3}x + q^{n+2}bc + q^{n+2}c - q^{n+2} - cq - c)\,S(n+1) + q\,(q^{n+1}-1)\,c\,(q^{n+1}b-1)\,S(n) = 0 \tag{134}$$

```
> `recursion/compare`(RM1, simplify(RM2), S(n));
```

$$\mathit{Recursions\ are\ NOT\ identical!} \tag{135}$$

```
> qMsummand(b, c, x, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\; qpochhammer(x, q, n)\left(-\frac{q^{n+1}}{c}\right)^{n}}{qpochhammer(b\,q, q, n)\; qpochhammer(q, q, n)} \tag{136}$$

```
> denumqm := qMsummand(b, c, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

$$denumqm := \frac{qpochhammer(q^{-n}, q, n)\left(-\frac{q^{n+1}}{c}\right)^{n}(-1)^{n}\,q^{\binom{n}{2}}}{qpochhammer(b\,q, q, n)\; qpochhammer(q, q, n)} \tag{137}$$

```
> qMsummand(-1/c, -1/b, -x/(b*c), q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\; qpochhammer\left(-\frac{x}{b\,c}, q, n\right)(q^{n+1}b)^{n}}{qpochhammer\left(-\frac{q}{c}, q, n\right)\; qpochhammer(q, q, n)} \tag{138}$$

```
> numqm := qMsummand(-1/c, -1/b, -x/(b*c), q, n)*(-1)^n*(-1/(b*c))
  ^n*q^binomial(n, 2)/qpochhammer(-x/(b*c), q, n);
```

**(139)**

$$numqm := \frac{qpochhammer(q^{-n}, q, n)\ (q^{n+1} b)^n\ (-1)^n \left(-\frac{1}{b\,c}\right)^n q^{\binom{n}{2}}}{qpochhammer\left(-\frac{q}{c}, q, n\right) qpochhammer(q, q, n)} \tag{139}$$

```
> CqM := simplify(numqm/denumqm);
```

$$CqM := \frac{(q^{n+1} b)^n \left(-\frac{1}{b\,c}\right)^n \left(-\frac{q^{n+1}}{c}\right)^{-n} qpochhammer(b\,q, q, n)}{qpochhammer\left(-\frac{q}{c}, q, n\right)} \tag{140}$$

```
> qsimpcomb((b*q^(n+1))^n*(-1/(b*c))^n*(-q^(n+1)/c)^(-n));
```

$$1 \tag{141}$$

```
> NRM1 := qsumrecursion(CqM*qMsummand(b, c, x, q, j), q, j, S(n),
  recursion = up);
```

$$NRM1 := (q^{n+2} + c)\ S(n+2) + (q^{2n+3} x + q^{n+2} b\,c + q^{n+2} c - q^{n+2} - c\,q - c)\ S(n+1) + q\ (q^{n+1} - 1)\ c\ (q^{n+1} b - 1)\ S(n) = 0 \tag{142}$$

```
> `recursion/compare`(NRM1, simplify(RM2), S(n));
```

*Recursions are identical.* (143)

```
> qsimpcomb([seq(CqM*add(qMsummand(b, c, x, q, j), j = 0 .. n), n =
  0 .. 3)-seq(add(qMsummand(-1/c, -1/b, -x/(b*c), q, j), j = 0 ..
  n), n = 0 .. 3)]);
```

$$[0, 0, 0, 0] \tag{144}$$

## q-Krawtchouk

Checking Big q-Jacobi and q-Krawtchouck

```
> qksum := proc (p, N, x, q, j) qphihyperterm([q^(-n), x, -p*q^n],
  [q^(-N), 0], q, q, j) end proc;
```

**(145)**

```
qksum := proc(p, N, x, q, j)
    qphihyperterm([q^(-n), x, -p*q^n], [q^(-N), 0], q, q, j)
end proc
```

```
> RK := qsumrecursion(qksum(p, N, x, q, j), q, j, S(n), recursion =
  up);
```

$$RK := -(q^{2n+1} p + 1)\ (q^{n+1} - q^N)\ (p\,q^{n+1} + 1)\ S(n+2) - (q^{2n+2} p + 1)\ (q^{4n+N+4} x\,p^2 - q^{3n+N+3} p^2 + q^{N+2n+3} x\,p + q^{3n+3} p - q^{N+2n+3} p - q^{N+2n+2} p + q^{2n+N+1} x\,p - q^{2n+2} p - q^{2n+1} p + q^{n+N+1} p - q^{n+1} + x\,q^N)\ S(n+1) + (q^{n+N+1} p + 1)\ q^{2n+1}\ (q^{2n+3} p + 1)\ p\ (q^{n+1} - 1)\ S(n) = 0 \tag{146}$$

```
> qksum(p, N, x, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(x, q, n)\ qpochhammer(-p\,q^n, q, n)\ q^n}{qpochhammer(q^{-N}, q, n)\ qpochhammer(q, q, n)} \tag{147}$$

```
> denumqK := qksum(p, N, x, q, n)*(-1)^n*q^binomial(n, 2)
  /qpochhammer(x, q, n);
```

**(148)**

$$denumqK := \frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(-p\, q^n, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{qpochhammer(q^{-N}, q, n)\ qpochhammer(q, q, n)} \tag{148}$$

```
> BJsummand(q^(-N-1), -q^N*p, 0, x, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(-q^{-1-N}\, q^N\, p\, q^{n+1}, q, n)\ qpochhammer(x, q, n)\ q^n}{qpochhammer(q^{-1-N}\, q, q, n)\ qpochhammer(q, q, n)} \tag{149}$$

```
> numBK := BJsummand(q^(-N-1), -q^N*p, 0, x, q, n)*(-1)^n*
  q^binomial(n, 2)/qpochhammer(x, q, n);
```

$$numBK := \frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(-q^{-1-N}\, q^N\, p\, q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{qpochhammer(q^{-1-N}\, q, q, n)\ qpochhammer(q, q, n)} \tag{150}$$

```
> CBK := simplify(numBK/denumqK);
```

$$CBK := 1 \tag{151}$$

```
> RBK := qsumrecursion(BJsummand(q^(-N-1), -q^N*p, 0, x, q, j), q,
  j, S(n), recursion = up);
```

$$RBK := (q^{2n+1} p + 1)\ (-q^{n+1} + q^N)\ (p\, q^{n+1} + 1)\ S(n+2) - (q^{2n+2} p + 1)\ (q^{4n+N+4} x\, p^2 - q^{3n+N+3} p^2 + q^{N+2n+3} x\, p + q^{3n+3} p - q^{N+2n+3} p - q^{N+2n+2} p + q^{2n+N+1} x\, p - q^{2n+2} p - q^{2n+1} p + q^{n+N+1} p - q^{n+1} + x\, q^N)\ S(n+1) + (q^{n+N+1} p + 1)\ q^{2n+1}\ (q^{2n+3} p + 1)\ p\ (q^{n+1} - 1)\ S(n) = 0 \tag{152}$$

```
> `recursion/compare`(RBK, RK, S(n));
```

$$\textit{Recursions are identical.} \tag{153}$$

```
> simplify([seq(add(BJsummand(q^(-N-1), -q^N*p, 0, x, q, j), j = 0
  .. n), n = 0 .. 5)-seq(add(qksum(p, N, x, q, j), j = 0 .. n), n =
  0 .. 5)]);
```

$$[0, 0, 0, 0, 0, 0] \tag{154}$$

Little q-Jacobi and q-Krawtchouk

```
> RLK := qsumrecursion(LJsummand(-q^N*p, q^(-N-1), x*q^N, q, j), q,
  j, S(n), recursion = up);
```

$$RLK := -q^{n+1}\ (q^{2n+1} p + 1)\ (q^{N+n+2} p + 1)\ (p\, q^{n+1} + 1)\ S(n+2) - (q^{2n+2} p + 1)\ (q^{4n+N+4} x\, p^2 - q^{3n+N+3} p^2 + q^{N+2n+3} x\, p + q^{3n+3} p - q^{N+2n+3} p - q^{N+2n+2} p + q^{2n+N+1} x\, p - q^{2n+2} p - q^{2n+1} p + q^{n+N+1} p - q^{n+1} + x\, q^N)\ S(n+1) + q^{n+1}\ (q^{2n+3} p + 1)\ p\ (q^{n+1} - 1)\ (q^n - q^N)\ S(n) = 0 \tag{155}$$

```
> `recursion/compare`(RLK, RK, S(n));
```

$$\textit{Recursions are NOT identical!} \tag{156}$$

```
> LJsummand(-q^N*p, q^(-N-1), x*q^N, q, n);
```

$$\frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(-q^{-1-N}\, q^N\, p\, q^{n+1}, q, n)\ (q\, q^N x)^n}{qpochhammer(-q^N p\, q, q, n)\ qpochhammer(q, q, n)} \tag{157}$$

```
> numLK := LJsummand(-q^N*p, q^(-N-1), x*q^N, q, n)*(q^(1+N))^n/(q*
  x*q^N)^n;
```

$$numLK := \frac{qpochhammer(q^{-n}, q, n)\ qpochhammer(-q^{-1-N}\, q^N\, p\, q^{n+1}, q, n)\ (q^{1+N})^n}{qpochhammer(-q^N p\, q, q, n)\ qpochhammer(q, q, n)} \tag{158}$$

```
> CKL := simplify(numLK/denumqK);
```

$$CKL := \frac{(-1)^{-n} q^{-\frac{n(n+1)}{2}} (q^{1+N})^n \, qpochhammer(q^{-N}, q, n)}{qpochhammer(-q^{1+N} p, q, n)} \tag{159}$$

```
> NRK := qsumrecursion((-1)^n*(q^N)^n*qpochhammer(q^(-N), q, n)*
  qksum(p, N, x, q, j)/(qpochhammer(-p*q^(1+N), q, n)*q^binomial(n,
  2)), q, j, S(n), recursion = up);
```

$$NRK := -q^{n+1} (q^{2n+1} p + 1) (q^{N+n+2} p + 1) (p q^{n+1} + 1) S(n+2) - (q^{2n+2} p + 1) (q^{4n+N+4} x p^2 - q^{3n+N+3} p^2 + q^{N+2n+3} x p + q^{3n+3} p - q^{N+2n+3} p - q^{N+2n+2} p + q^{2n+N+1} x p - q^{2n+2} p - q^{2n+1} p + q^{n+N+1} p - q^{n+1} + x q^N) S(n+1) + q^{n+1} (q^{2n+3} p + 1) p (q^{n+1} - 1) (q^n - q^N) S(n) = 0 \tag{160}$$

```
> `recursion/compare`(RLK, NRK, S(n));
```

*Recursions are identical.* **(161)**

```
> simplify([seq(CKL*add(qksum(p, N, x, q, j), j = 0 .. n), n = 0 ..
  3)-seq(add(LJsummand(-q^N*p, q^(-N-1), x*q^N, q, j), j = 0 .. n),
  n = 0 .. 3)]);
```

$$\Big[0, \Big(-qpochhammer(-q^{1+N} p, q, 1)\, qpochhammer(q, q, 1) - qpochhammer\Big(\frac{1}{q}, q, 1\Big) (q\, q^N qpochhammer(x, q, 1) + q^{1+N} x)\, qpochhammer(-q p, q, 1) - qpochhammer(q^{-N}, q, 1)\, qpochhammer(q, q, 1)\, q^N\Big) \Big/ \big(qpochhammer(-q^{1+N} p, q, 1)\, qpochhammer(q, q, 1)\big), \Big(qpochhammer(q, q, 2)\, qpochhammer\Big(\frac{1}{q^2}, q, 1\Big) \big(qpochhammer(x, q, 1)\, qpochhammer(q^{-N}, q, 2)\, qpochhammer(-q^{1+N} p, q, 1)\, q^{2N} - qpochhammer(q^{-N}, q, 1)\, qpochhammer(-q^{1+N} p, q, 2)\, x\, q^{1+N}\big)\, qpochhammer(-p q^2, q, 1) - \Big(qpochhammer\Big(\frac{1}{q^2}, q, 2\Big) (q^{2+2N} x^2 - qpochhammer(x, q, 2)\, q^{1+2N})\, qpochhammer(-p q^2, q, 2) - qpochhammer(q, q, 2)\, (qpochhammer(q^{-N}, q, \tag{162}$$

$$2)\, q^{-1+2N} - qpochhammer(-q^{1+N} p, q, 2))\Big)\, qpochhammer(-q^{1+N} p, q,$$

$$1)\, qpochhammer(q, q, 1)\, qpochhammer(q^{-N}, q, 1)\Big) \Big/ \big(qpochhammer(-q^{1+N} p, q,$$

$$2)\, qpochhammer(q^{-N}, q, 1)\, qpochhammer(q, q, 2)\, qpochhammer(-q^{1+N} p, q,$$

$$1)\, qpochhammer(q, q, 1)), \Big(-qpochhammer(q, q, 2)\, qpochhammer(q, q,$$

$$3)\, qpochhammer\left(\frac{1}{q^3}, q, 1\right) qpochhammer(q^{-N}, q, 2)\, qpochhammer(-q^{1+N} p, q,$$

$$2)\, (q^{1+N}\, qpochhammer(q^{-N}, q, 1)\, qpochhammer(-q^{1+N} p, q, 3)\, x + qpochhammer(x, q,$$

$$1)\, qpochhammer(q^{-N}, q, 3)\, q^{-2+3N}\, qpochhammer(-q^{1+N} p, q, 1))\, qpochhammer($$

$$-p\, q^3, q, 1) - qpochhammer(-q^{1+N} p, q, 1)\, qpochhammer(q^{-N}, q, 1)\, qpochhammer(q,$$

$$q, 1)\, \Big(qpochhammer(q, q, 3)\, qpochhammer\left(\frac{1}{q^3}, q, 2\right) (q^{2+2N}\, qpochhammer(q^{-N}, q,$$

$$2)\, qpochhammer(-q^{1+N} p, q, 3)\, x^2 + qpochhammer(x, q, 2)\, qpochhammer(q^{-N}, q,$$

$$3)\, q^{-1+3N}\, qpochhammer(-q^{1+N} p, q, 2))\, qpochhammer(-p\, q^3, q, 2)$$

$$+ \Big(qpochhammer\left(\frac{1}{q^3}, q, 3\right) (q^{3+3N} x^3 + qpochhammer(x, q, 3)\, q^{3N})\, qpochhammer($$

$$-p\, q^3, q, 3) + qpochhammer(q, q, 3)\, (qpochhammer(q^{-N}, q, 3)\, q^{-3+3N}$$

$$+ qpochhammer(-q^{1+N} p, q, 3))\Big)\, qpochhammer(q, q, 2)\, qpochhammer(q^{-N}, q,$$

$$2)\, qpochhammer(-q^{1+N} p, q, 2)\Big)\Big) \Big/ \big(qpochhammer(-q^{1+N} p, q,$$

$$3)\, qpochhammer(q^{-N}, q, 1)\, qpochhammer(q^{-N}, q, 2)\, qpochhammer(q, q,$$

$$3)\, qpochhammer(-q^{1+N} p, q, 1)\, qpochhammer(q, q, 1)\, qpochhammer(q, q,$$

$$2)\, qpochhammer(-q^{1+N} p, q, 2))\,]$$

# q-Hahn

```
> qhahnsummand := proc (alpha, beta, N, x, q, k) qphihyperterm([q^(-n), alpha*beta*q^(n+1), x], [alpha*q, q^(-N)], q, q, k) end proc;
```

*qhahnsummand* := **proc**(alpha, beta, $N$, $x$, $q$, $k$)
    *qphihyperterm*([q^( − n), alpha * beta * q^(n + 1), x], [alpha * q, q^( − N)], q, q, k)
**end proc** **(163)**

```
> RE1 := qsumrecursion(qhahnsummand(alpha, beta, N, x, q, k), q, k, S(n), recursion = up);
```

$$RE1 := (\alpha q^{n+2}\beta-1)(-q^{n+1}+q^{N})(\alpha q^{n+2}-1)(\alpha q^{2n+2}\beta-1)S(n+2) - (\alpha q^{2n+3}\beta-1)\big(\alpha^2 q^{N+4n+6}x\beta^2-\alpha^2 q^{N+3n+5}\beta^2-\alpha^2 q^{N+3n+5}\beta+\alpha^2 q^{N+2n+4}\beta - \alpha q^{N+2n+4}x\beta-\alpha^2 q^{3n+4}\beta+\alpha^2 q^{N+2n+3}\beta+\alpha q^{N+2n+4}\beta-\alpha q^{3n+4}\beta + \alpha q^{N+2n+3}\beta-\alpha q^{N+2n+2}x\beta+\alpha q^{2n+3}\beta-\alpha q^{N+n+2}\beta+\alpha q^{2n+2}\beta+\alpha q^{2n+3} - \alpha q^{N+n+2}+\alpha q^{2n+2}-\alpha q^{n+1}+x q^{N}-q^{n+1}\big)S(n+1)+\alpha q^{n+1}(q^{n+1} - 1)(\alpha q^{2n+4}\beta-1)(\alpha q^{N+n+2}\beta-1)(\beta q^{n+1}-1)S(n)=0 \quad \textbf{(164)}$$

```
> RE2 := qsumrecursion(qhahnsummand(beta, alpha, -(ln(q)*N-ln(1/(alpha*beta*q^2)))/ln(q), q^N*beta*q*x, q, k), q, k, S(n), recursion = up);
```

$$RE2 := (\alpha q^{n+2}\beta-1)\left(q^{n+N+1}-q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}}\right)(\alpha q^{2n+2}\beta-1)(q^{n+2}\beta-1)S(n+2) \quad \textbf{(165)}$$
$$+q\left(\alpha^2 q^{N+4n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+6}x\beta^3-\alpha q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+N+2n+4}x\beta^2-\alpha^2 q^{3n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+4}\beta^2 - \alpha q^{3n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+4}\beta^2-\alpha q^{3n+N+3}\beta^2-\alpha q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+N+2n+2}x\beta^2-\alpha q^{3n+N+3}\beta + \alpha q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+3}\beta^2+\alpha q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+2}\beta^2+\alpha q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+3}\beta+\alpha q^{N+2n+2}\beta + \alpha q^{2n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+2}\beta+\alpha q^{2n+N+1}\beta+q^{N+2n+2}\beta+q^{2n+N+1}\beta-\alpha q^{n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+1}\beta + q^{\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+N}x\beta-q^{n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+1}\beta-q^{n+N}\beta-q^{n+N}\right)(\alpha q^{2n+3}\beta-1)S(n+1)$$
$$+q^{n+1}(q^{n+1}-1)(\alpha q^{n+1}-1)\left(-\alpha q^{n+\frac{\ln\left(\frac{1}{\alpha\beta q^2}\right)}{\ln(q)}+2}\beta+q^{N}\right)\beta(\alpha q^{2n+4}\beta-1)S(n) = 0$$

```
> RE2 := simplify(RE2);
```

**(166)**

$$RE2 := \left(\alpha q^{n+2}\beta-1\right)\left(q^{n+N+1}-\frac{1}{\alpha\beta q^2}\right)\left(\alpha q^{2n+2}\beta-1\right)\left(q^{n+2}\beta-1\right)S(n+2) + \frac{1}{\alpha}\Big(\big(-\beta\alpha(x-\alpha-1)q^{N+2n+2}+\beta\alpha(\alpha+1)q^{2n+N+1}-\beta\alpha^2(\beta+1)q^{3n+N+3} + \alpha^2 q^{4n+N+4}\beta^2 x-\alpha\beta(\alpha+1)q^{2+3n}+\alpha(\beta+1)q^{2n+1}-q^{N+2n}\beta x\alpha-\alpha(\beta+1)q^{n+N}+(-\alpha-1)q^{n-1}+\alpha(\beta+1)q^{2n}+q^{N-2}x\big)S(n+1)\left(\alpha q^{2n+3}\beta-1\right)q\Big) + \left(\alpha q^{n+1}-1\right)\left(q^N-q^n\right)\beta\left(\alpha q^{2n+4}\beta-1\right)S(n)\,q^n q\left(q^n q-1\right)=0 \tag{166}$$

```
> `recursion/compare`(RE1, RE2, S(n));
```

*Recursions are NOT identical!* **(167)**

```
> RE3 := qsumrecursion(qhahnsummand(1/(q*q^N), alpha*beta*q*q^N, ln
  (1/(alpha*q))/ln(q), x, q, k), q, k, S(n), recursion = up);
```

$$RE3 := -\left(\alpha q^{n+2}\beta-1\right)\left(q^{n+1}-q^{\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}}\right)\left(\alpha q^{2n+2}\beta-1\right)\left(q^{n+1}-q^N\right)S(n+2) - \Bigg(\alpha^2 q^{4n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+6}x\beta^2-\alpha^2 q^{3n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+5}\beta^2-\alpha q^{N+2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+4}x\beta - \alpha q^{3n+N+4}\beta-\alpha q^{3n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+4}\beta+\alpha q^{N+2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+4}\beta+\alpha q^{N+2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+3}\beta - \alpha q^{N+2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2}x\beta-\alpha q^{3n+3}\beta+\alpha q^{N+2n+3}\beta+\alpha q^{2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+3}\beta + \alpha q^{N+2n+2}\beta+\alpha q^{2n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2}\beta-\alpha q^{n+N+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2}\beta+q^{2n+2}+q^{2n+1}-q^{n+N+1} - q^{n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+1}+q^{\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+N}x-q^n\Bigg)\left(\alpha q^{2n+3}\beta-1\right)S(n+1)+q^n\left(q^{n+1}-1\right)\left(\alpha q^{n+\frac{\ln\left(\frac{1}{\alpha q}\right)}{\ln(q)}+2}\beta-1\right)\left(\alpha q^{2n+4}\beta-1\right)\left(\alpha q^{N+n+2}\beta-1\right)S(n)=0 \tag{168}$$

```
> RE3 := simplify(RE3);
```

$$RE3 := -\left(\alpha q^{n+2}\beta-1\right)\left(q^{n+1}-\frac{1}{\alpha q}\right)\left(\alpha q^{2n+2}\beta-1\right)\left(q^{n+1}-q^N\right)S(n+2) - \frac{1}{\alpha}\Big(S(n+1)\big(-\beta\alpha(x-\alpha-1)q^{N+2n+3}+\beta\alpha(\alpha+1)q^{N+2n+2}-\beta\alpha^2(\beta+1)q^{3n+N+4}+\alpha^2 q^{N+5+4n}x\beta^2-q^{2n+N+1}x\beta\alpha+\alpha(\beta+1)q^{2n+1}+\alpha(\beta+1)q^{2n+2}-\alpha\beta(\alpha+1)q^{3n+3}-\alpha(\beta+1)q^{n+N+1}+q^{-1+N}x-(\alpha+1)q^n\big)\left(\alpha q^{2n+3}\beta-1\right)\Big)+\left(\beta q^{n+1}-1\right)\left(\alpha q^{2n+4}\beta-1\right)\left(\alpha q^{N+n+2}\beta-1\right)S(n)\left(q^{2n+1}\right.$$

**(169)**

$- q^{n}) = 0$

```
> `recursion/compare`(RE1, RE3, S(n));
```

*Recursions are identical.* **(170)**

```
> `recursion/compare`(RE2, RE3, S(n));
```

*Recursions are NOT identical!* **(171)**

```
> RE4 := qsumrecursion(qhahnsummand(alpha*beta*q*q^N, 1/(q*q^N), ln(1/(beta*q))/ln(q), q^N*beta*q*x, q, k), q, k, S(n), recursion = up);
```

$$RE4 := \left(\alpha q^{n+2}\beta - 1\right)\left(\beta\alpha q^{3+n+N} - 1\right)\left(q^{n+1} - q^{\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}}\right)\left(\alpha q^{2n+2}\beta - 1\right)S(n+2) \quad (172)$$
$$+ q\left(\alpha q^{2n+3}\beta - 1\right)\left(\alpha^{2} q^{4n+N+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+6} x\beta^{3} - \alpha^{2} q^{3n+N+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+5}\beta^{2}\right.$$
$$- \alpha^{2} q^{3n+N+4}\beta^{2} + \alpha^{2} q^{N+2n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+4}\beta^{2} - \alpha q^{N+2n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+4} x\beta^{2}$$
$$- \alpha^{2} q^{3n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+4}\beta^{2} + \alpha^{2} q^{N+2n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+3}\beta^{2} - \alpha q^{N+2n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+2} x\beta^{2}$$
$$+ \alpha q^{N+2n+3}\beta - \alpha q^{3n+3}\beta + \alpha q^{2n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+3}\beta + \alpha q^{N+2n+2}\beta - \alpha q^{n+N+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+2}\beta$$
$$+ \alpha q^{2n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+2}\beta + \alpha q^{2n+2}\beta - q^{n+N+1}\beta\alpha + q^{2n+1}\beta\alpha - \alpha q^{n+1+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}}\beta$$
$$\left. + q^{\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+N} x\beta - q^{n}\right) S(n+1) + \alpha q^{n+2}\left(q^{n+1} - 1\right)\left(\alpha q^{n+\frac{\ln\left(\frac{1}{q\beta}\right)}{\ln(q)}+2}\beta - 1\right)\left(q^{N}\right.$$
$$\left. - q^{n}\right)\beta\left(\alpha q^{2n+4}\beta - 1\right)S(n) = 0$$

```
> RE4 := simplify(RE4);
```

$$RE4 := \left(\alpha q^{n+2}\beta - 1\right)\left(\beta\alpha q^{3+n+N} - 1\right)\left(q^{n+1} - \frac{1}{q\beta}\right)\left(\alpha q^{2n+2}\beta - 1\right)S(n+2) + S(n \quad (173)$$
$$+ 1)\left(-\beta\alpha\left(x - \alpha - 1\right)q^{N+2n+3} + \beta\alpha\left(\alpha + 1\right)q^{N+2n+2} - \beta\alpha^{2}\left(\beta + 1\right)q^{3n+N+4}\right.$$
$$+ \alpha^{2} q^{N+5+4n} x\beta^{2} - q^{2n+N+1} x\beta\alpha + \alpha\left(\beta + 1\right)q^{2n+1} + \alpha\left(\beta + 1\right)q^{2n+2} - \alpha\beta\left(\alpha\right.$$
$$\left. + 1\right)q^{3n+3} - \alpha\left(\beta + 1\right)q^{n+N+1} + q^{-1+N} x - \left(\alpha + 1\right)q^{n}\Big)\, q\left(\alpha q^{2n+3}\beta - 1\right)$$
$$+ \alpha\left(\alpha q^{n+1} - 1\right)\left(q^{N} - q^{n}\right)\beta\left(\alpha q^{2n+4}\beta - 1\right)S(n)\, q^{n} q^{2}\left(q^{n} q - 1\right) = 0$$

```
> `recursion/compare`(RE1, RE4, S(n));
```

*Recursions are NOT identical!* **(174)**

```
> `recursion/compare`(RE2, RE4, S(n));
```

*Recursions are identical.* **(175)**

```
> `recursion/compare`(RE3, RE4, S(n));
```

*Recursions are NOT identical!* **(176)**

Obtaining the relations

```
> qhahnsummand(alpha, beta, N, x, q, n);
```

$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(\alpha\beta q^{n+1}, q, n)\ \mathit{qpochhammer}(x, q, n)\ q^n}{\mathit{qpochhammer}(\alpha q, q, n)\ \mathit{qpochhammer}(q^{-N}, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{177}$$

```
> denumh := qhahnsummand(alpha, beta, N, x, q, n)*(-1)^n*q^binomial
  (n, 2)/qpochhammer(x, q, n);
```

$$\mathit{denumh} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(\alpha\beta q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{\mathit{qpochhammer}(\alpha q, q, n)\ \mathit{qpochhammer}(q^{-N}, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{178}$$

```
> simplify(qhahnsummand(beta, alpha, -(ln(q)*N-ln(1/(alpha*beta*
  q^2)))/ln(q), q^N*beta*q*x, q, n));
```

$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(\alpha\beta q^{n+1}, q, n)\ \mathit{qpochhammer}(q^{1+N} x\beta, q, n)\ q^n}{\mathit{qpochhammer}(\beta q, q, n)\ \mathit{qpochhammer}(\beta\alpha q^{N+2}, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{179}$$

```
> numh21 := simplify(simplify(qhahnsummand(beta, alpha, -(ln(q)*N-
  ln(1/(alpha*beta*q^2)))/ln(q), q^N*beta*q*x, q, n))*(-1)^n*
  beta^n*q^((1/2)*(2*N+n+1)*n)/qpochhammer(q^(1+N)*beta*x, q, n));
```

$$\mathit{numh21} := \frac{(-1)^n\ q^{\frac{n\,(3+2N+n)}{2}}\ \beta^n\ \mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(\alpha\beta q^{n+1}, q, n)}{\mathit{qpochhammer}(q, q, n)\ \mathit{qpochhammer}(\beta q, q, n)\ \mathit{qpochhammer}(\beta\alpha q^{N+2}, q, n)} \tag{180}$$

```
> Ch21 := simplify(numh21/denumh);
```

$$\mathit{Ch21} := \frac{\beta^n\ q^{n\,(1+N)}\ \mathit{qpochhammer}(q^{-N}, q, n)\ \mathit{qpochhammer}(\alpha q, q, n)}{\mathit{qpochhammer}(\beta q, q, n)\ \mathit{qpochhammer}(\beta\alpha q^{N+2}, q, n)} \tag{181}$$

```
> simplify([seq(Ch21*add(qhahnsummand(alpha, beta, N, x, q, j), j =
  0 .. n), n = 0 .. 1)-seq(add(qhahnsummand(beta, alpha, -(ln(q)*N-
  ln(1/(alpha*beta*q^2)))/ln(q), q^N*beta*q*x, q, j), j = 0 .. n),
  n = 0 .. 1)]);
```

$$\Big[0, \Big(\mathit{qpochhammer}\Big(\frac{1}{q}, q, 1\Big)\ \big(\mathit{qpochhammer}(x, q, 1)\ \beta q^{N+2} - q\ \mathit{qpochhammer}(q^{1+N} x\beta, q, 1)\big)\ \mathit{qpochhammer}(\alpha\beta q^2, q, 1) + \mathit{qpochhammer}(q, q, 1)\ \big(\mathit{qpochhammer}(q^{-N}, q, 1)\ \mathit{qpochhammer}(\alpha q, q, 1)\ \beta q^{1+N} - \mathit{qpochhammer}(\beta q, q, 1)\ \mathit{qpochhammer}(\beta\alpha q^{N+2}, q, 1)\big)\Big) \Big/ \big(\mathit{qpochhammer}(\beta q, q, 1)\ \mathit{qpochhammer}(\beta\alpha q^{N+2}, q, 1)\ \mathit{qpochhammer}(q, q, 1)\big)\Big] \tag{182}$$

```
> NRE1 := qsumrecursion(Ch21*qhahnsummand(alpha, beta, N, x, q, k),
  q, k, S(n), recursion = up);
```

$$\begin{aligned} \mathit{NRE1} := {} & (\alpha q^{n+2}\beta - 1)\ (\beta\alpha q^{3+n+N} - 1)\ (\alpha q^{2n+2}\beta - 1)\ (q^{n+2}\beta - 1)\ S(n+2) \\ & + q\beta\,(\alpha q^{2n+3}\beta - 1)\ (\alpha^2 q^{N+4n+6} x\beta^2 - \alpha^2 q^{N+3n+5}\beta^2 - \alpha^2 q^{N+3n+5}\beta \\ & + \alpha^2 q^{N+2n+4}\beta - \alpha q^{N+2n+4} x\beta - \alpha^2 q^{3n+4}\beta + \alpha^2 q^{N+2n+3}\beta + \alpha q^{N+2n+4}\beta \\ & - \alpha q^{3n+4}\beta + \alpha q^{N+2n+3}\beta - \alpha q^{N+2n+2} x\beta + \alpha q^{2n+3}\beta - \alpha q^{N+n+2}\beta + \alpha q^{2n+2}\beta \\ & + \alpha q^{2n+3} - \alpha q^{N+n+2} + \alpha q^{2n+2} - \alpha q^{n+1} + x q^N - q^{n+1})\ S(n+1) + \alpha q^{n+3}\ (q^{n+1} \\ & - 1)\ (\alpha q^{n+1} - 1)\ (q^N - q^n)\ \beta^2\ (\alpha q^{2n+4}\beta - 1)\ S(n) = 0 \end{aligned} \tag{183}$$

```
> `recursion/compare`(NRE1, RE2, S(n));
```
*Recursions are identical.* **(184)**

```
> simplify(qhahnsummand(1/(q*q^N), alpha*beta*q*q^N, ln(1/(alpha*q))/ln(q), x, q, n));
```
$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(\alpha\beta q^{n+1}, q, n)\ \mathit{qpochhammer}(x, q, n)\ q^n}{\mathit{qpochhammer}(\alpha q, q, n)\ \mathit{qpochhammer}(q^{-N}, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{185}$$

```
> numh31 := simplify(qhahnsummand(1/(q*q^N), alpha*beta*q*q^N, ln(1/(alpha*q))/ln(q), x, q, n))*(-1)^n*q^binomial(n, 2)/qpochhammer(x, q, n);
```
$$\mathit{numh31} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(\alpha\beta q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{\mathit{qpochhammer}(\alpha q, q, n)\ \mathit{qpochhammer}(q^{-N}, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{186}$$

```
> Ch31 := numh31/denumh;
```
$$\mathit{Ch31} := 1 \tag{187}$$

Big q-Jacobi and q-Hahn

```
> qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, n);
```
$$\frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a b q^{n+1}, q, n)\ \mathit{qpochhammer}(x, q, n)\ q^n}{\mathit{qpochhammer}(a q, q, n)\ \mathit{qpochhammer}\left(q^{-\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}}, q, n\right)\ \mathit{qpochhammer}(q, q, n)} \tag{188}$$

```
> denumBH := qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, n)*(-1)^n*q^binomial(n, 2)/qpochhammer(x, q, n);
```
$$\mathit{denumBH} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a b q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{\mathit{qpochhammer}(a q, q, n)\ \mathit{qpochhammer}\left(q^{-\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}}, q, n\right)\ \mathit{qpochhammer}(q, q, n)} \tag{189}$$

```
> numBH := BJsummand(a, b, c, x, q, n)*(-1)^n*q^binomial(n, 2)/qpochhammer(x, q, n);
```
$$\mathit{numBH} := \frac{\mathit{qpochhammer}(q^{-n}, q, n)\ \mathit{qpochhammer}(a b q^{n+1}, q, n)\ q^n\ (-1)^n\ q^{\binom{n}{2}}}{\mathit{qpochhammer}(a q, q, n)\ \mathit{qpochhammer}(c q, q, n)\ \mathit{qpochhammer}(q, q, n)} \tag{190}$$

```
> CBH := simplify(numBH/denumBH);
```
$$\mathit{CBH} := 1 \tag{191}$$

```
> Rbh := qsumrecursion(qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, j), q, j, S(n), recursion = up);
```
$$\begin{aligned}\mathit{Rbh} := &-(q^{n+2} a - 1)\left(q^{n+1} - q^{\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}}\right)(q^{2n+2} a b - 1)(q^{n+2} a b - 1)\, S(n+2) \\ &- (q^{2n+3} a b - 1)\left(q^{4n+\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}+6} a^2 x b^2 - q^{3n+\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}+5} a^2 b^2 - q^{3n+\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}+5} a^2 b\right. \\ &\left. - q^{3n+4} a^2 b + q^{2n+\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}+4} a^2 b - q^{2n+\frac{\ln\left(\frac{1}{c q}\right)}{\ln(q)}+4} a x b - q^{3n+4} a b\right.\end{aligned} \tag{192}$$

$$+ q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+3} a^2 b + q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+4} a b + q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+3} a b - q^{2n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2} a x b$$
$$+ q^{2n+3} a b + q^{2n+2} a b + q^{2n+3} a - q^{n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2} a b + q^{2n+2} a - q^{n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2} a$$
$$- q^{n+1} a - q^{n+1} + x q^{\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}}\Bigg) S(n+1) + q^{n+1} a \left(q^{n+1} - 1\right) \Bigg(q^{n+\frac{\ln\left(\frac{1}{cq}\right)}{\ln(q)}+2} a b$$
$$-1\Bigg) \left(q^{n+1} b - 1\right) \left(q^{2n+4} a b - 1\right) S(n) = 0$$

```
> `recursion/compare`(RB1, simplify(Rbh), S(n));
```

*Recursions are identical.* **(193)**

```
> simplify([seq(add(qhahnsummand(a, b, ln(1/(c*q))/ln(q), x, q, j),
  j = 0 .. n), n = 0 .. 5)-seq(add(BJsummand(a, b, c, x, q, j), j =
  0 .. n), n = 0 .. 5)]);
```

$$[0, 0, 0, 0, 0, 0] \quad \textbf{(194)}$$

q-Hahn to Big q-Jacobi

```
> qhahnsummand(alpha, beta, N, x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(\alpha \beta q^{n+1}, q, n\right) qpochhammer(x, q, n)\, q^n}{qpochhammer(\alpha q, q, n)\, qpochhammer\left(q^{-N}, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(195)}$$

```
> numHB := qhahnsummand(alpha, beta, N, x, q, n)*(-1)^n*q^binomial
  (n, 2)/qpochhammer(x, q, n);
```

$$numHB := \frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(\alpha \beta q^{n+1}, q, n\right) q^n \left(-1\right)^n q^{\binom{n}{2}}}{qpochhammer(\alpha q, q, n)\, qpochhammer\left(q^{-N}, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(196)}$$

```
> BJsummand(alpha, beta, q^(-N-1), x, q, n);
```

$$\frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(\alpha \beta q^{n+1}, q, n\right) qpochhammer(x, q, n)\, q^n}{qpochhammer(\alpha q, q, n)\, qpochhammer\left(q^{-1-N} q, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(197)}$$

```
> denumHB := BJsummand(alpha, beta, q^(-N-1), x, q, n)*(-1)^n*
  q^binomial(n, 2)/qpochhammer(x, q, n);
```

$$denumHB := \frac{qpochhammer\left(q^{-n}, q, n\right) qpochhammer\left(\alpha \beta q^{n+1}, q, n\right) q^n \left(-1\right)^n q^{\binom{n}{2}}}{qpochhammer(\alpha q, q, n)\, qpochhammer\left(q^{-1-N} q, q, n\right) qpochhammer(q, q, n)} \quad \textbf{(198)}$$

```
> CHB := simplify(numHB/denumHB);
```

$$CHB := 1 \quad \textbf{(199)}$$

```
> RBH := qsumrecursion(BJsummand(alpha, beta, q^(-N-1), x, q, j),
  q, j, S(n), recursion = up);
```

$$RBH := \left(\alpha q^{n+2} \beta - 1\right) \left(-q^{n+1} + q^N\right) \left(\alpha q^{n+2} - 1\right) \left(\alpha q^{2n+2} \beta - 1\right) S(n+2) \quad \textbf{(200)}$$
$$- \left(\alpha q^{2n+3} \beta - 1\right) \Big(\alpha^2 q^{N+4n+6} x \beta^2 - \alpha^2 q^{N+3n+5} \beta^2 - \alpha^2 q^{N+3n+5} \beta + \alpha^2 q^{N+2n+4} \beta$$
$$- \alpha q^{N+2n+4} x \beta - \alpha^2 q^{3n+4} \beta + \alpha^2 q^{N+2n+3} \beta + \alpha q^{N+2n+4} \beta - \alpha q^{3n+4} \beta$$
$$+ \alpha q^{N+2n+3} \beta - \alpha q^{N+2n+2} x \beta + \alpha q^{2n+3} \beta - \alpha q^{N+n+2} \beta + \alpha q^{2n+2} \beta + \alpha q^{2n+3}$$

$$- \alpha q^{N+n+2} + \alpha q^{2n+2} - \alpha q^{n+1} + x q^{N} - q^{n+1}\Big) S(n+1) + \alpha q^{n+1} \left(q^{n+1}\right.$$
$$\left. - 1\right) \left(\alpha q^{2n+4} \beta - 1\right) \left(\alpha q^{N+n+2} \beta - 1\right) \left(\beta q^{n+1} - 1\right) S(n) = 0$$

```
> `recursion/compare`(RE1, simplify(RBH), S(n));
```

*Recursions are identical.* **(201)**

```
> simplify([seq(add(qhahnsummand(alpha, beta, N, x, q, j), j = 0 ..
  n), n = 0 .. 5)-seq(add(BJsummand(alpha, beta, q^(-N-1), x, q,
  j), j = 0 .. n), n = 0 .. 5)]);
```

$[0, 0, 0, 0, 0, 0]$ **(202)**

```
>
>
>
>
>
>
>
>
>
>
```