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Recurrence equations and their classical orthogonal polynomial solutions. (English)

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Classical orthogonal polynomials of a continuous or a discrete variable can be classified using second order differential or (q-)difference equations of the following form

$$\sigma(x)y''(x)+\tau(x)y'(x)+\lambda_n y(x)=0,$$
$$\sigma(x)\Delta\nabla y(x)+\tau(x)\Delta y(x)+\lambda_n y(x)=0,$$
$$\sigma(x)D_qD_{1/q}y(x)+\tau(x)D_qy(x)+\lambda_{q,n}y(x)=0.$$

Here $\sigma(x),\tau(x)$ are polynomials of at most degree two and one respectively. The difference operators are defined by

$$\Delta y(x)=y(x+1)-y(x),\ \nabla y(x)=y(x)-y(x-1),\ D_qf(x)=\frac{f(qx)-f(x)}{(q-1)x},\ q\neq 1.$$

The authors now give a general method to express the coefficients A_n,B_n and C_n of the homogeneous recurrence equation

$$p_{n+1}(x)=(A_nx+B_n)p_n(x)-C_np_{n-1}(x),$$

satisfied by the classical orthogonal polynomials, in terms of the given polynomial coefficients of the differential/difference equation.
The classification theorems give explicit results and in three tables all the classical (and some 'degenerate') cases are given, along with algorithms using Maple. For the implementation of the algorithms (retode) as well as a worksheet with the examples from the paper (`retode.mws`), the reader is referred to the web-page of the first author (`http://www.mathematik.uni-kassel.de/~koepf/Publikationen`). Also the authors discuss the differences with the computer program `rec2ortho` as implemented by Koornwinder and Swarttouw (`http://turing.wins.uva.nl/~thk/recentpapers/rec2ortho.htm`
in 1996-1998.
A very nicely written paper.

Marcel G.de Bruin (Delft)

Keywords : differential equations; difference equations; q-differences; recurrence equations; polynomial solutions

Classification :

∗33C45 Orthogonal polynomials and functions of hypergeometric type
34B99 Boundary value problems for ODE

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