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Foupouagnigni, M.; Koepf, W.; Ronveaux, A.

Factorization of fourth-order differential equations for perturbed classical orthogonal polynomials. (English)

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A sequence of monic polynomials $(M_n)_n$ which are orthogonal with respect to a regular linear functional $\mathcal{U}$ is said to belong to the Laguerre-Hahn class if the Stieltjes function

$$S(z) := -\sum_{n=0}^{\infty} \frac{\langle \mathcal{U}, x^n \rangle}{z^{n+1}},$$

satisfies a Riccati differential equation $\Phi S' = BS^2 + CS + D$, where $\Phi \neq 0$, B, C and D are polynomials. Each Laguerre-Hahn orthogonal polynomial sequence $(M_n)_n$ satisfies a common fourth-order differential equation

$$\mathbb{F}_n(y(x)) = \sum_{i=0}^{4} J_i(x,n) y^{(i)}(x) = 0$$

where the coefficients J_i are polynomials in x, with degree not depending on n. The authors factorize $\mathbb{F}_n$ as product of two second-order linear differential operators (with polynomial coefficients) for the case of Laguerre-Hahn sequences

$$M_n(x) = A_n(x) P^{(1)}_{n+k-1} + B_n(x) P_{n+k}$$

where $(P_n)_n$ is an arbitrary classical orthogonal polynominal sequence, $(P_n^{(1)})_n$ is the first associated of $(P_n)_n$ and A_n, B_n are polynomials of degree not depending on n. Moreover, the authors find four lineary independent solutions of the fourth-order differential equations $\mathbb{F}_n(y) = 0$ for the followig five perturbations of classical orthogonal polynomial sequences $(P_n)_n$: the rth associated, the generalized co-recursive, the generalized co-dilated, the generalized co-recursive associated and the generalized co-modified. Some results are also extended to the case of semi-classical $(P_n)_n$.

Aleksey A.Dovgoshey (Donetsk)

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∗33C45 Orthogonal polynomials and functions of hypergeometric type
33C47 Other special orthogonal polynomials and functions

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