649.30011

Koepf, W.

Close-to-convex functions, univalence criteria and quasiconformal extension. (English)

Ann. Univ. Mariae Curie-Sklodowska, Sect. A 40, 97-103 (1986). [ISSN 0365-1029]

Let f(z) be analytic in $|z| < 1$ with f'(z)$\neq 0$ there, and let

$$\{f, z\} = (f''/f')' - 1/2(f''/f')^2$$

denote the Schwarzian derivative of f with respect to z, and let

$$\sigma = \|S(f, z)\| = (1 - |z|^2)^2 |\{f, z\}|.$$

For $0 \leq \beta \leq 1$, $C(\beta)$ denotes the family of close-to-convex functions of order β, with $C(0)$ the well-known convex functions and C(1) the close- to-convex functions. Nehari has shown that if $\sigma \leq 2$ then f is univalent in $|z| < 1$, and that functions in C(0) satisfy this univalence criterion. For f in $C(\beta)$, $\beta \in (0, 1)$, the author obtains exact estimates for $\sigma = \|S(f, z)\|$.
Also considered is the Becker inequality

$$(1 - |z|^2)|f''(z)/f'(z)| \leq \lambda,$$

which implies univalence of f when $\lambda = 1$. The author shows that, for an m-fold symmetric function f_m in $C(\beta)$ with $\beta <$ and $m \geq 4/(1 - 2\beta)$ the Becker univalence criterion holds. In addition, if $m > 4/(1 - \beta)$, with $\beta \in [0, 1)$, then f_m has a quasiconformal extension.

M.T.McGregor

Keywords : Schwarzian derivative; close-to-convex functions of order β; Becker inequality

Classification:

- 30C45 Special classes of univalent and multivalent functions
- 30C55 General theory of univalent and multivalent functions