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Bounded nonvanishing functions and Bateman functions. (English)

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New properties of the Bateman functions are deduced. These functions are the coefficients $F_n(t)$ of the power series at $z = 0$ of the generating function $\exp\{-t(1+z)/(1-z)\}$. Using the Laplace transformation a residual representation is given. From its differential equation an estimation of $F_n(t)$ is found which then is improved by using the connection with generalized Laguerre polynomials and a method from Szegoe. The generating function of the Bateman functions are superordinate to the bounded functions in the unit disk having $a_0 = \exp(-t)$ as first coefficient of its Taylor series at $z = 0$. The Krzyz conjecture for these functions can be shown to be true asymptotically. However, this conjecture can be shown to hold for big enough index n , i.e. for $n \geq n_0$ with n_0 explicitly given. Related estimations are discussed and a conjecture formulated which is tested by computer graphing.

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Keywords : Bateman functions; Krzyz conjecture

Classification:

- **30C50** Coefficient problems for univalent and multivalent functions
- **33C25** Orthogonal polynomials and functions
- **30C80** Maximum principle, etc. (one complex variable)
- **33C15** Confluent hypergeometric functions
- **68Q40** Symbolic computation, algebraic computation