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Die reellen Zahlen als Fundament und Baustein der Analysis. (The real numbers as foundation and component of analysis). (German)

Muenchen: Oldenbourg. v, 190 S. DM 42.00 (2000). [ISBN 3-486-24455-8/pbk]

This textbook covers an area which is usually skipped over in a first course on real analysis: axiomatic and explicit constructions of the real numbers. The main focus of the first chapter (Characterization of the Real Numbers) is the axiomatic description of the real numbers. Basic concepts of algebra are introduced, ordered fields are discussed, completeness is defined (via the supremum property), and it is shown that up to isomorphism there is only one complete ordered field. The second chapter (Construction of the Real Numbers) discusses three explicit constructions of the reals from the rationals: the construction via Cauchy sequences due to Cantor, the construction via pairs of sets approximating a number from above and below due to Capelli, and the construction via nested intervals due to P. Bachmann. The third chapter (Imbedding into Complete Metric Spaces) generalizes the Cantor construction to metric spaces, and the fourth chapter (Decimal Representation of the Real Numbers) shows the correspondence between real numbers and decimal fractions.

The book is written on an elementary level and in a very clear and careful way which makes it suitable for interested first-year students. It supposes almost no prerequisites except some familiarity with mathematical reasoning. The book can also be recommended as the basis for a proseminar after a first course in analysis. Finally, because of its extremely thought-out and didactic structure, selected parts of the book (such as the discussion of the Euler number  $e$ ) can without changes be incorporated into any regular class on real analysis.

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*Keywords* : real numbers; ordered fields; completeness; Cantor construction; Capelli construction; Bachmann construction; metric spaces

*Classification*:

- 26-01 Textbooks (real functions)
- 26A03 Elementary topology of the real line