Introduction to Maple. *By André Heck*. Springer, New York, 1993. xiii + 497 pp. $ 39.00, hardcover. ISBN 0-387-97662-0.

I warmly recommend this book to everybody who honestly wants to use MAPLE. The use of computer algebra systems makes doing mathematics much easier in a certain sense. On the other hand, however, every user of such a system knows about certain difficulties arising with the use of this kind of program.

Here are some examples. A MAPLE user may define the function

```
f:= x->if x<0 then -x^2 else x^3 fi;
```

and try to plot it with the command

```
plot(f(x),x=-1..1);
```

and may be irritated about the error message

```
Error, (in f) cannot evaluate boolean
```

Or one may try to define the function

```
f(x):=x*sqrt(x);
```

and may be astonished about the fact that `f(a)` is not evaluated to `a*sqrt(a)`.

Finally, not everybody will understand the difference between the substitution commands

```
subs({x=y,y=x},x*y+x^2*y^3);
```

and

```
subs(x=y,y=x,x*y+x^2*y^3);
```

and their results $xy + y^2x^3$, and $x^2 + x^5$, right away, or can imagine immediately the result of the substitution

```
subs(-1=1,x^2-x+1/x-1/x^2);
```

(what is your guess?).

One of the main issues concerning computer algebra systems is the order in which evaluations occur. Each computer algebra system has its own internal structure, and evaluation mechanisms. All above examples are connected with this question.

What a computer algebra system like MAPLE gives you, is the computing power to do extremely complicated and tedious symbolic computations, but the price that you have to pay for this service is that you have to learn and understand much more about the internal structure of such a program than you probably originally thought of.

If you need any help with regard to this sort of questions, or if you just want to understand how MAPLE internally works, the book under review is a very good choice.

In the sequel we give a list of the book's chapters, and a short description of their contents:

1. Introduction to Computer Algebra
   In this chapter a brief introduction into the historical development of computer algebra and computer algebra systems is given. Then the use of MAPLE V Release 2 in a UNIX environment under the X-Windows system is discussed. The question of automatic simplification is raised, and discussed using examples from polynomial arithmetic. The advantages of computer algebra systems over purely numerical languages with regard to certain situations are discussed. As an example MAPLE is used to prove that a specific function satisfies a certain partial differential equation. A second, more advanced example concerning a generating function question is given. In a third example MAPLE is used to develop a conjecture concerning a certain determinant. Finally the limits of computer algebra are briefly discussed.
2. The First Steps: Calculus on Numbers
   This chapter gives an introduction how to use MAPLE in interactive mode, especially in an X-Windows environment. Syntactical errors and their messages are dicussed, command completion is mentioned, and the help facility is described in detail.
   Integers and rational numbers, and their internal representations, as well as floiting-point numbers, are next. MAPLE's standard functions and constants are listed. The first examples of MAPLE's graphical capabilities are given, and complex numbers are mentioned briefly.
3. Variables and Names
   Here assignments, unassignments, and the use of variables is discussed. Further a list of MAPLE's reserved names is given. The concept of full evaluation is described, and how to gain control over the evaluation process. Finally the basic data types are listed.
4. Getting Around with Maple
   This chapter describes how you can enter your input, and how you can control the output through the `printlevel`, `infolevel` and `interface` commands. MAPLE's library mechanism, and the interplay with the file system, including Input/Output formatting is discussed. The export of Fortran, and LaTeX code is mentioned.
5. Polynomials and Rational Functions
   Here an introduction to the work with polynomials and rational functions is given. Examples for the expansion, and factorization of polynomials, and the normalization of rational functions are given. Further continued fractions representations, and partial fraction decompositions are mentioned.
6. Internal Data Representation and Substitution
   This chapter is a highlight of the book. After having read it, you will understand some of the behavior of MAPLE much better. The internal representation of polynomials which is one of the most important issues in a computer algebra system, is studied in detail. Moreover substitution, subexpressions, and

the mapping of functions is described.

7. Manipulation of Polynomials and Rational Functions

   In this chapter a complete description of the `expand` and `factor` commands is given. Further expansion and factorization over finite fields, or algebraic function fields are covered. Then an introduction to the theory of canonical normal forms and normal forms is given that enables the reader to understand the simplification function `normal` of rational functions better. Finally grouping and sorting of polynomials is treated.

8. Functions

   The definition of functions in terms of array operators like `f:=x->x*sqrt(x);`, and procedures like `f:=proc(x) x*sqrt(x) end;` are discussed in detail. Recursive programming, and MAPLE's use of remember tables is treated. Finally functional operations, and anonymous functions are mentioned.

9. Differentiation

   Symbolic differentiation, i. e. the work with the `diff` and `D` operations, is the topic of this chapter. An example on implicit differentiation is covered, and the advantages of the technique of automatic differentiation are nicely demonstrated.

10. Integration and Summation

    Here MAPLE's capabilities to handle indefinite, definite, and numerical integration, and summation, are described. The techniques that are used by MAPLE's `integrate`, and `sum` procedures, are listed in detail. In particular, a brief introduction to the theory of elementary, and Liouvillian functions is given. Elliptic integrals, and integral transforms are covered, and an example of the use of the fast Fourier transform for data smoothing is given.

11. Truncated Series Expansions, Power Series, and Limits

    In this chapter MAPLE's capabilities of series approximations are described. Power series, Laurent series, Puiseux series, generalized series, as well as asymptotic series are covered, and some of the techniques used to get these results, are listed. A description of the `powseries` package follows, and finally limits are treated.

12. Composite Data Types

    Here the data types `sequence`, `set`, `list`, `vector` and `array`, the work with them, and the conversions between them are discussed in detail.

13. Simplification

    Advanced simplification techniques are discussed, and side effects of simplifications are mentioned. Some inconsistencies of MAPLE connected with this issue are discussed. In detail, the simplification commads `expand`, `combine`, `simplify`, and `convert`, and especially their use on expressions involving exponential, logarithmic, trigonometric, factorial and Γ functions, is addressed. The final topic is the algebraic simplification with respect to side relations with the aid of Gröbner base techniques.

14. Graphics

    The topic of this chapter are MAPLE's main plotting facilities, and their options. The internal structure of MAPLE's graphic objects are nicely described. A large number of concrete plot examples are given which show the immense capabilities of MAPLE. However, using the knowledge about the internal plotting algorithms, on p. 315 the author gives an example of a completely wrong picture, too! Again, the important issue of the evaluation order is addressed.

15. Solving Equations

    Here MAPLE's capabilities to solve equations, and systems of equations symbolically, are covered. Some disappointing, and even wrong results are discussed, as well. Gröbner bases techniques are discussed in more detail, now. Several examples of the direct use of Gröbner bases are given. Next numerical solving is mentioned. Integer and modular solving is briefly addressed, and examples of recurrence equations solving are given.

16. Differential Equations

    The solution of differential equations is the contents of this chapter. The capabilities and weaknesses of the general purpose solver `dsolve` are described in detail. Power series methods, and numerical solutions come next. Lengthy examples concerning numerical solutions, perturbation methods, and Lie symmetry methods, are given.

17. Linear Algebra: Basics

    Here the work with matrices is demonstrated. In particular, the special evaluation concept for matrices (last name evaluation) is discussed.

18. Linear Algebra: Applications

    In this chapter, matrix arithmetic is illustrated with five practical examples.

Undoubtedly the book is worth its price. Besides the successful exposition, each chapter contains a reasonable number of exercises. Many of the chapters are self-contained, so that one can read them without prior knowledge of other chapters. An exception are the chapters about polynomials, and rational functions, and those about MAPLE's internal data structure which in my opinion form the highlights of the book. Another important topic is the description of the main heuristics, and algorithms that are used by some of MAPLE's functions.

At the end I like to mention some minor irritations that I had with the text.

In my opinion, the author stresses the `alias`ing too much. On page 392, e. g., he himselm must admit that *Because of our aliases, we cannot easily compute* .... On pages 178–179 some aliases are incorrectly stated: The first alias command should be `alias(y = y(x)):`, and the second one `alias(y = y):`. (Moreover, in the text a colon is missing). Less would be more here. In my opinion the whole aliasing is unnecessary, and could be omitted, simplifying the presentation at several places.

The presentation of Chapters 16 on *Differential Equations* and 18 on *Linear Algebra: Applications* in my opinion does not fit properly in an *Introduction* to MAPLE. They include advanced material with lengthy MAPLE output. Besides the aliasing problem mentioned above, the examples are just too long to be of interest to readers who likes to get an introduction to MAPLE. Further, on page 404, some TEX command is printed, i. e., a backslash in the source file is missing. One may joke that even the author himself found those examples too lengthy to proofread them carefully enough.

Finally on page 150 the MAPLE output is corrupted: In two instances the output of the rational function considered contains unexpectedly logarithms and exponentials.

But let me repeat that these are only minor complaints about a very nice book.

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