Internet Resources for Mathematics

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Abstract

Topics of This Talk

- In this talk I would like to show you how the internet can be used as work bench for the researcher.
- As a mathematician I will concentrate on internet resources with mathematical contents which, however, might be also of interest for other people.
- These internet resources are open for the international public.

Summary

- Integer Sequences
- Decimal Numbers
- Orthogonal Polynomials
- Mathematical Encyclopedias
- Recreational Sites

Integer Sequences

Integer Sequences Resources

- The text book The Encyclopedia of Integer Sequences by Neil Sloane and Simon Plouffe offers a rich database of integer sequences.
- Using the book, you can answer questions like this: How does the sequence

$$1, 1, 2, 3, 5, 8, 13, \ldots$$

  continue?
- More importantly: Everybody has access to the internet based On-Line Encyclopedia of Integer Sequences.
- This database can be easily used to identify a given sequence. Let's try!

Integer Sequences

The Fibonacci Sequence

- The Fibonacci sequence given by $F_1 = 1$, $F_2 = 1$ and

$$F_n = F_{n-1} + F_{n-2}$$

  has many, many applications and origins.
- A mathematical journal is devoted entirely to these numbers: The Fibonacci Quarterly.
- The Fibonacci numbers in Wikipedia.
- They are connected with the golden ratio.
- Fibonacci numbers occur in the nature, for example in the sun flower and in many more flowers and fruits.
- Here is a short movie about this sequence.

Integer Sequences

Just Another Example

- Assume you come up in your research with the sequence

$$\frac{(2n)!}{n!\,(n+1)!}$$

  where $n! = 1 \cdot 2 \cdots n$ denotes the factorial function.
- These numbers can easily be calculated. However, the question is: Are these numbers known?
- We compute the starting sequence:

$$1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, \ldots$$

  and try to find out!

Identification of Decimal Numbers

Inverse Symbolic Calculator

- We connect with the Inverse Symbolic Calculator by Simon Plouffe.
- The item Simple Lookup and Browser searches in a database whether the unknown decimal number x is listed. Hence this works like an inverse telephone book.
- We look up the number $x = \ln(\pi\sqrt{2}) \approx 1.4913034761293$ by a Simple Lookup.
- The item Smart Lookup searches in a database whether x is listed, but not only for x, but also for $\ln x$, e^x, $\sin x$ etc.

Decimal Numbers

Integer Relations Algorithms

- The item Integer Relation Algorithms tries to find a polynomial relation between the constant given and other mathematical constants.
- For this purpose modern mathematical algorithms like the PSLQ Integer Relation Algorithm are used.
- If successful, this method can find complicated relations and identities.
- We try $x = \sqrt{2 + \sqrt{3}}$ using Integer Relation Algorithms.

A More Specialized Topic

Orthogonal Polynomials

- Using the *Maple* Computer Algebra System, jointly with René Swarttouw I have created a web site for orthogonal polynomials of the so-called Askey-Wilson scheme
- With CAOP (Computer Algebra and Orthogonal Polynomials) one can compute recurrence relations and differential equations for classical systems in the web.
- Calling CAOP...
- Here obviously also modern algorithms are used to generate these results.

Mathematical Encyclopedias

Wikipedia

- The web contains several important encyclopedias.
- We have already seen Wikipedia.
- This encyclopedia is created by the web users themselves and has rather good control mechanisms.
- Wikipedia is fast growing and incorporates informations about almost everything.
- Let us search Wikipedia for some beautiful results by Leonard Euler.

The Most Remarkable Formula of Mathematics

Euler's Formula

$$e^{i\pi}+1=0$$

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Euler's Formula

$$e^{i\pi}+1=0$$

Introduction of Graph Theory

Euler Solved Real World Problems

- Euler introduced Graph Theory into Mathematics.
- He solved the problem of the seven bridges of Königsberg.
- If a graph contains such a tour it is called an Euler graph.
- Euler investigated properties of polyhedra which are connected with the modern notion of planar graphs.
- His investigations resulted in the polyhedron formula: The number of vertices plus the number of faces minus the number of edges of a polyhedron always equals 2:

$$V + F - E = 2 \, .$$

- Planar graphs have the Four Color Property.

Mathematical Encyclopedias

MathWorld

- Another more specialized encyclopedia is MathWorld.
- This encyclopedia was created by Eric Weisstein and is hosted by Wolfram Research.
- MathWorld includes many mathematical formulae which can be created or checked with *Mathematica*.
- Let us search MathWorld for Euler.

Mathematical Encyclopedias

WebMathematica

- If you don't have a *Mathematica* license, you can nevertheless use some *Mathematica* materials, accessible on WebMathematica.
- Let's check the Polyhedron Explorer which deals with Platonic solids.
- The truncated icosahedron is an Archimedean solid and is well-known as a soccer ball.
- It also occurs in chemistry as one of the so-called Fulleren molecules C_{60}.

Mathematical Encyclopedias

Wolfram Demonstrations Project

- As last encyclopedic program we visit the Wolfram Demonstrations Project.
- The project comes as a byproduct of the newest software release *Mathematica 6*.
- As an example, let us create the Sierpinski triangle.
- With *Mathematica*, we can create the demonstration ourselves: Sierpinski source
- Sierpinski triangle after 10 steps.

Recreational Sites

An example from YouTube

- As final example a short movie from www.youtube.com: Mathematics Genius

Many Thanks for Your Interest!