University of Kassel, Institute of Mathematics **Summer Term 2012**
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Numerical Methods for Partial Differential Equations

Homework 2

Problem 1
Show that the one-dimensional Euler equations can be written as

$$\rho_t + v\rho_x + \rho v_x = 0,$$
$$v_t + vv_x + \frac{1}{\rho}p_x = 0,$$
$$p_t + vp_x + \gamma p v_x = 0.$$

Is this a hyperbolic system? (4 P)

Problem 2
We consider the two-dimensional linear advection equation

$$u_t + \nabla \cdot (u\mathbf{a}) = 0, \quad u(x,y,0) = u_0(x,y),$$

in $\mathbb{R}^2 \times \mathbb{R}_0^+$. The velocity field $\mathbf{a}(x,y) = (a_1(x,y), a_2(x,y))^T$ is supposed to be divergence-free, i. e. $\nabla \cdot \mathbf{a} = 0$. Show that the solution of this equation is constant along the characteristics

$$x'(t) = a_1(x(t), y(t)), \quad x(0) = x_0,$$
$$y'(t) = a_2(x(t), y(t)), \quad y(0) = y_0.$$

Note that the divergence operator is understood with respect to the spatial coordinates, thus $\nabla \cdot (a_1, a_2)^T = \partial_x a_1 + \partial_y a_2$. (4 P)

Due on Monday, April 23, 2012.